Preliminary Study of Response Prediction of Semi-active Controlled Structures

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SUMMARY:
This study elucidates a method used to ascertain control variables such as the performance index and weight coefficient, at the beginning of structural design, without the need for dynamic response analysis. The authors study magneto-rheological fluid dampers (MR dampers) controlled by an optimal regulator system which is a linear control system, and use the damping factor to evaluate the structure response. First, the relation between weight coefficient and damping factor are shown and the physical meanings of the performance index and weight coefficient are discussed. Secondly, the influence that MR damper's controllable range has on the maximum response is considered. And, then appropriate compensation is proposed.

Keywords: Semi-active control, LQR regulator, MR damper, Damping factor

1. INTRODUCTION

Structural vibration control has improved rapidly. Recently, excessive modification control of base-isolated systems in case of large earthquakes is performed by passive control. If design structures are intended for excessive modification control during a large earthquake, then the structures transmit a short period element to the superstructure during a minor earthquake, creating anxiety about spoiling the base-isolated system's effect. Therefore, some studies have been conducted to apply semi-active control to base-isolated systems (Fujitani, H. et al. (2004), Ramello, J. C. et al. (2002), Yoshioka, H. et al. (2002),). Semi-active systems present two distinct advantages. They can perform structural vibration control rather than passive control. Furthermore, it requires little energy compared with active control. However, semi-active control also presents the difficulty that construction of a control system and grasping a damping performance are difficult.

The authors studied damping performance of the MR damper and optimal regulator systems as a semi-active control system. The MR damper has been anticipated for control of the response of civil and other structures in recent years because of its large force capacity and variable force characteristics. Optimal regulator system is a fundamental method among control methods. In this study, relations between control variables (weight coefficient) and control effects were examined by making a damping factor into an evaluation index as a preliminary study of the response prediction of semi-active controlled structures.

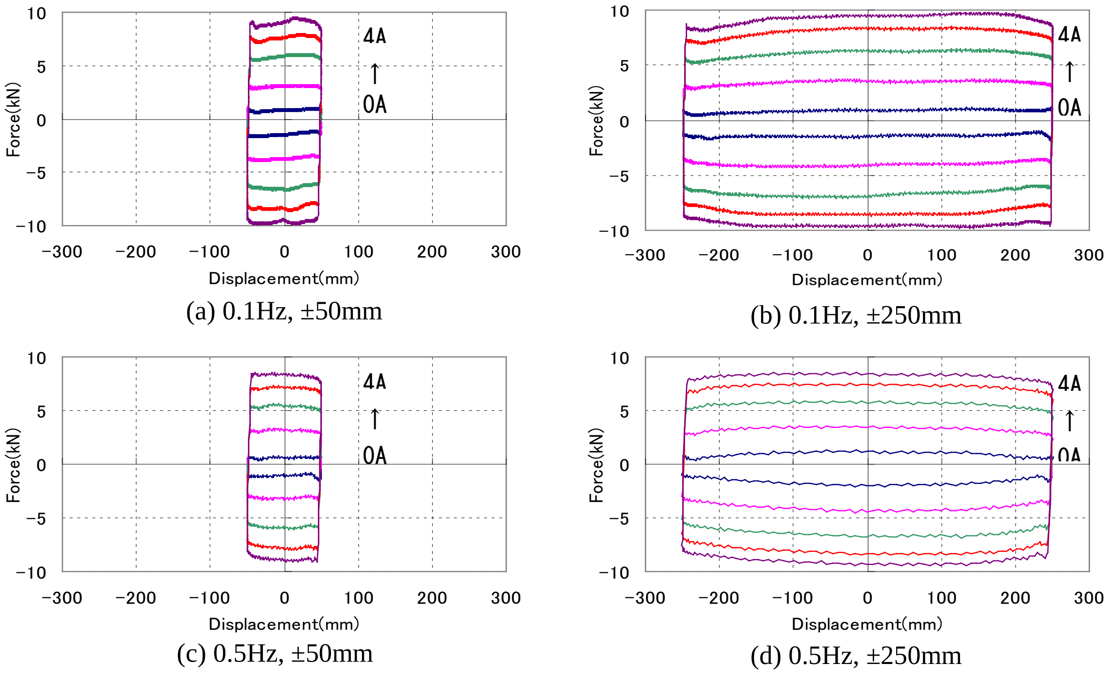
2. COMPONENTS OF A SEMI-ACTIVE SYSTEM

2.1 Magneto-Rheological Damper (MR damper)

The maximum damping force of the MR damper used for this study is 10 kN. Its stroke is +/-300 mm as shown Table 2.1. Fig. 2.1 shows the force-displacement relation of the MR damper. The damping force can shift by the volume of electric current. It can therefore be controlled using a PC. The MR damper is modelled simply by a Bingham plastic model, as shown in Fig. 2.2, and the relation between the damper force and the electric current is approximated as Eqn. 2.1.

Table 2.1. Dimensions of MR damper

Maximum Force		10kN
Storoke		±300mm
Electromagnet	MAX.Current	5.0A
MagnetoRheological fluid		Bando : #230



(a) 0.1Hz, ±50mm

(b) 0.1Hz, ±250mm

(c) 0.5Hz, ±50mm

(d) 0.5Hz, ±250mm

Figure 2.1. Force-displacement relationship of MR damper according to induced electric current

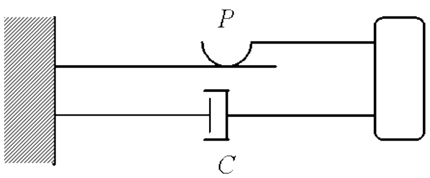


Figure 2.2. Bingham plastic model

$$F = \mathrm{sign}(v)(-1807I - 822.4) + 2.07v \tag{2.1}$$

F: damper force (N), I: electric current (A), v: velocity (mm/s)

2.2 Optimal Regulator System (Yang,J.N. et al. (1975))

For optimal control theory, the authors apply the performance index shown in Eqn. 2.2 α, β, γ, are weight coefficients respectively related to displacement, velocity, and acceleration.

$$J = \int \frac{1}{2}\left(\alpha(\ddot{x} + E\ddot{z})^2 + \beta\dot{x}^2 + \gamma x^2 + u^2\right) dt \tag{2.2}$$

By increasing each coefficient, the corresponding quantity of state can be shrunk. For example, by increasing α, acceleration can be reduced. In optimal control, it is necessary to set up the weight coefficients in Eqn. 2.2 appropriately.

Optimal regulator theory is used for the flexibility system model shown with the equation of state of Eqn. 2.3 (M, mass; K, stiffness). Control power (u) is a form of another formula (Eqn. 2.4). The feedback gain is given to the response displacement and velocity (S1, S2).

$$\dot{X} = AX + Bu \quad (2.3)$$
$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} \quad X = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$
$$u = [S1 \;\; S2]X \quad (2.4)$$

The equivalent viscous damping factor of the entire control drive is computed by considering that the feedback gain which starts displacement is stiffness and that velocity is a viscous damping coefficient: considering S1 is the equivalent stiffness and S2 is the viscous damping coefficient.

3. RELATION BETWEEN THE WEIGHT COEFFICIENT AND DAMPING FACTOR

3.1. Discussing the Weight Coefficient of Acceleration (α)

The performance index is set to Eqn. 3.1, when the effect of the weight coefficient of acceleration (α) is discussed.

$$J = \int (\alpha\ddot{x}^2 + u^2)\, dt \quad (3.1)$$

Eqn. 3.2 is derived from Eqn. 3.1, when variables are changed to the general format.

$$J = \int \left(X^T Q X + u^T R U + 2X^T N u\right) dt \quad (3.2)$$
$$Q = \begin{bmatrix} \alpha\omega^4 & 0 \\ 0 & 0 \end{bmatrix}, \; R = \frac{\alpha}{M^2} + 1, \;\; N = \begin{bmatrix} -\frac{\alpha\omega^2}{M} \\ 0 \end{bmatrix}, \; \omega = \sqrt{\frac{K}{M}}$$

The control force is given by Eqn. 3.3 in this equation.

$$u = [S1 \;\; S2]X = -R^{-1}(B^T P + N^T)X \quad (3.3)$$

P is a solution of the Ricatti equation (Eqn. 3.4).

$$A^T P + PA - (PB + N)R^{-1}\left(B^T P + N^T\right) + Q = 0 \quad (3.4)$$

P is set with Eqn. 3.5 (P is a positive-define and symmetric matrix).

$$P = \begin{bmatrix} P11 & P12 \\ P12 & P22 \end{bmatrix} \quad (3.5)$$

From Eqn. 2.3, Eqn. 3.2, and Eqn. 3.4, the solution of the Ricatti equation is Eqn. 3.6.

$$\begin{aligned} P11 &= \omega^3 M\sqrt{2M}\sqrt{\sqrt{M^2+\alpha} - M} \\ P12 &= \omega^2\left(M\sqrt{M^2+\alpha} - M^2\right) \\ P22 &= \omega\sqrt{2(M^2+\alpha)}\sqrt{M\sqrt{M^2+\alpha} - M^2} \end{aligned} \quad (3.6)$$

Then, the control force is expressed by Eqn. 3.7 using Eqns. 3.3-3.6.

$$u = [S1 \;\; S2]X = -M\omega^2\left(\frac{M}{\sqrt{M^2+\alpha}} - 1\right)x - \frac{M\omega\sqrt{2}\sqrt{M\sqrt{M^2+\alpha} - M^2}}{\sqrt{M^2+\alpha}}\dot{x} \quad (3.7)$$

Therefore, the equivalent stiffness (K_{eq}), equivalent damping coefficient (C_{eq}), equivalent circular frequency (ω_{eq}), and equivalent damping factor (h_{eq}) is Eqn. 3.8, Eqn. 3.9, Eqn. 3.10, and Eqn. 3.11.

$$K_{eq} = M\omega^2 + M\omega^2\left(\frac{M}{\sqrt{M^2+\alpha}} - 1\right) = \frac{M\omega^2}{\sqrt{1+\frac{\alpha}{M^2}}} \tag{3.8}$$

$$C_{eq} = \frac{M\omega\sqrt{2}\sqrt{\sqrt{1+\frac{\alpha}{M^2}}-1}}{\sqrt{1+\frac{\alpha}{M^2}}} \tag{3.9}$$

$$\omega_{eq}{}^2 = \frac{\omega^2}{\sqrt{1+\frac{\alpha}{M^2}}} \tag{3.10}$$

$$h_{eq} = \frac{Ceq}{2\sqrt{MKeq}} = \sqrt{\frac{1}{2} - \frac{1}{2\sqrt{1+\frac{\alpha}{M^2}}}} \tag{3.11}$$

From them, K_{eq} is expected to be smaller, ω_{eq} is also expected to be smaller, and damping will become larger if α is increased. In addition, h_{eq} has a limit value. It converges to $\frac{1}{\sqrt{2}}(=0.707)$ if α is infinity. Furthermore, it is determined only by the structure’s mass and weight coefficient of acceleration (α).

3.2. Discussing the Weight Coefficient of Velocity (β)

The performance index is set to Eqn. 3.12, when the effect of the weight coefficient of velocity (β) is discussed.

$$J = \int(\beta\dot{x}^2 + u^2)\,dt \tag{3.12}$$

Equivalent stiffness (K_{eq}), equivalent damping coefficient (C_{eq}), circular frequency (ω_{eq}), and equivalent damping factor (h_{eq}) are determined respectively as shown in Eqn. 3.13, Eqn. 3.14, Eqn. 3.15, and Eqn. 3.16 if the same development of section 3.1 is performed.

$$K_{eq} = K \tag{3.13}$$

$$C_{eq} = \sqrt{\beta} \tag{3.14}$$

$$\omega_{eq} = \omega \tag{3.15}$$

$$h_{eq} = \frac{C_{eq}}{2\sqrt{MKeq}} = \frac{\sqrt{\beta}}{2M\omega^2} \tag{3.16}$$

Contrary to the acceleration weight coefficient, when β is infinity, h_{eq} is also infinity. It is determined by the structure’s mass, circular frequency, and weight coefficient. However, K_{eq} and ω_{eq} do not change, even if β is increased.

3.3. Discussing the Weight Coefficient of Displacement (γ)

The performance index is set to Eqn. 3.17, when the effect of the weight coefficient of displacement (γ) is discussed.

$$J = \int(\gamma x^2 + u^2)\,dt \tag{3.17}$$

Equivalent stiffness (K_{eq}), equivalent damping coefficient (C_{eq}), circular frequency (ω_{eq}), and equivalent damping factor (h_{eq}) are determined respectively as shown in Eqn. 3.18, Eqn. 3.19, Eqn.

3.20, and Eqn. 3.21 if the same development of section 3.1 is performed.

$$K_{eq} = \sqrt{M^2\omega^4 + \gamma} \tag{3.18}$$

$$C_{eq} = \sqrt{2M(\sqrt{M^2\omega^4 + \gamma} - M\omega^2)} \tag{3.19}$$

$$\omega_{eq} = \sqrt{\omega^4 + \frac{\gamma}{M^2}} \tag{3.20}$$

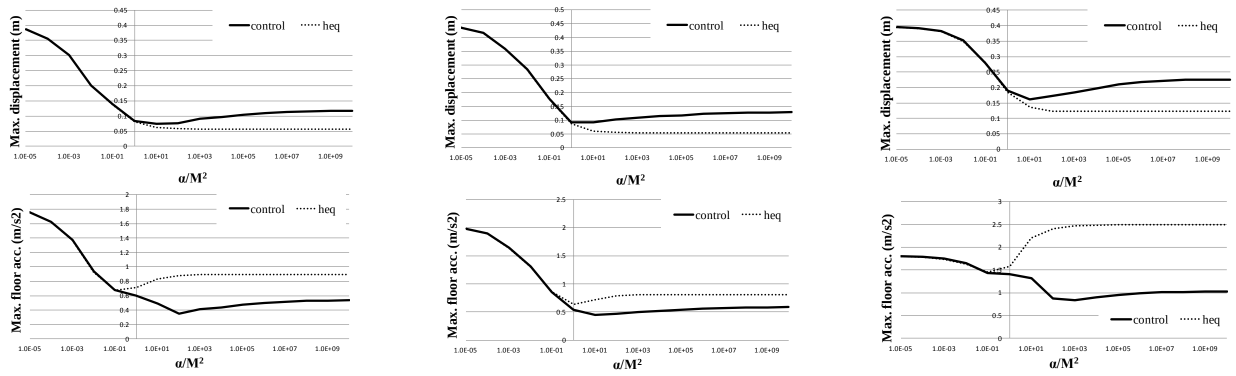
$$h_{eq} = \frac{C_{eq}}{2\sqrt{MK_{eq}}} = \sqrt{\frac{1}{2} - \frac{M\omega^2}{2\sqrt{M^2\omega^2+\gamma}}} \tag{3.21}$$

As with the acceleration weight coefficient, when γ is infinity, h_{eq} converges on $\frac{1}{\sqrt{2}}$(=0.707). However, different from the acceleration weight, it is determined by the structure's mass, circular frequency and weight coefficient. Furthermore, K_{eq} and ω_{eq} is only increased if γ is increased.

4. RESPONSE EVALUATION OF SEMI-ACTIVE CONTROLLED STRUCTURE

4.1. Evaluation by the Equivalent Damping Factor

A characteristic exists by which the control force cannot be applied in a direction that adds vibration: the MR damper can output only resistant force. Therefore, when an equivalent viscous damping factor estimates the control effect simply, maximum responses differ from actual controlled responses. For example, the maximum response displacements and floor accelerations were calculated for time history analysis against three actual observed ground motions. It is shown in Fig. 4.1. Then, the solid line "control" in Fig. 4.1 shows the maximum values which were produced by regarding Eqn. 3.1 as a performance index and the dot line "h_{eq}" shows those which were calculated by using h_{eq}. α was normalized by α/M^2. There are some differences between "control" and "h_{eq}" when α/M^2 is larger than 1.0.



a) El Centro 1940 NS b) Hachinohe 1968 NS c) JMA Kobe 1995 NS

Figure 4.1. Maximum responses in time history analysis

4.2. Correction Coefficient

The authors propose a correction coefficient to an equivalent damping factor. As for a modelled MR damper into the Bingham Plastic model, if a characteristic of MR damper is expressed with an expression focused on the relation between controlling force and response velocity, then it can be expressed as Eqn. 4.1.

$$u\dot{x} = (S1x + S2\dot{x})\dot{x} \geq 0 \tag{4.1}$$

If this equation is expressed on a phase plane, then it conforms to Fig. 4.2. The shaded areas of the figure show areas that MR damper can output the control force. Other areas show ranges that MR damper cannot output control force.

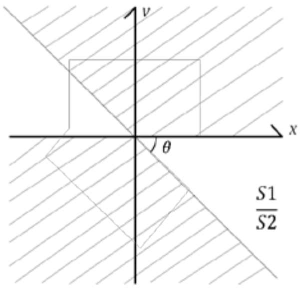


Figure 4.2. Phase Plane

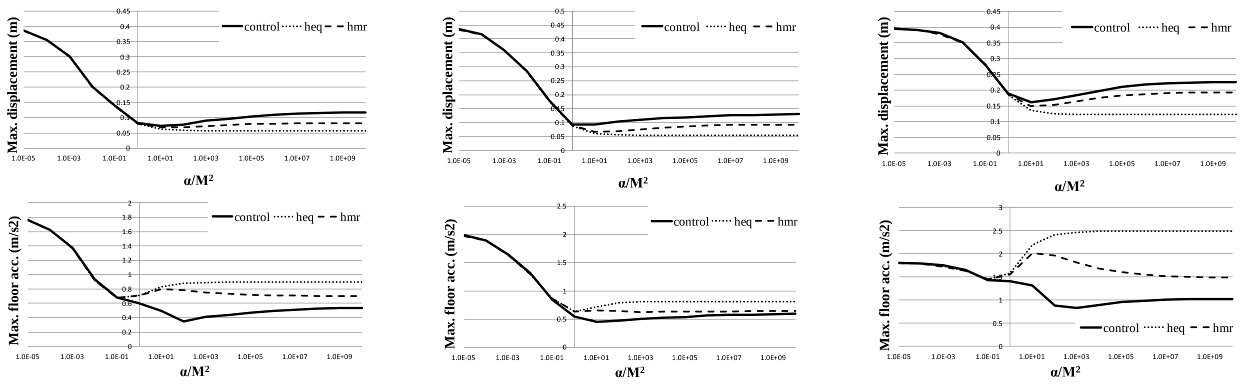
A correction coefficient (**p)** is determined from the ratio of the shaded area to whole area of Fig. 4.2. For example, when we specifically examine a performance index related to acceleration, then Eqn. 4.2 applies if the angle formed by the x-axis and the straight line of the inclination S1/S2 constitute is set to θ. It will be set to Eqn. 4.3 if multiplied by Eqn. 3.11 and Eqn. 4.2.

$$\mathbf{p} = \frac{(\boldsymbol{\pi}-\boldsymbol{\theta})^2}{(\boldsymbol{\pi}-\boldsymbol{\theta})^2+\boldsymbol{\theta}^2} \tag{4.2}$$

$$h_{mr} = ph_{eq} = \frac{\sin\theta}{\sqrt{3+\cos 2\theta}\left\{1+(\frac{\theta}{\pi-\theta})^2\right\}} \tag{4.3}$$

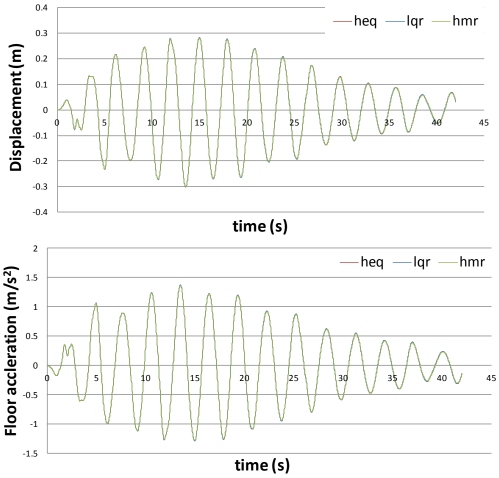
3.3. Verification by Time History Analysis

The maximum responses in time history analysis are shown in Fig. 4.3. h_{mr} is a damping factor after compensation. The dashed line “h_{mr}” in Fig 4.3 shows maximum values which were calculated by using h_{mr}. Fig 4.3 shows that the “ h_{mr}” is nearer to “control” than the “h_{eq}”. In addition, Figs. 4.4-4.5 portray the results of time history analysis. They were performed, respectively, in the series of α/M^2=0.0001 (Fig. 4.4), 1 (Fig. 4.5), and 1000 (Fig. 4.6). The values of α/M^2 respectively mean a small difference, and a medium difference, and a large difference of maximum responses. The the results show that "h_{mr}" waveforms are nearer to the “control” waveforms than “h_{eq}” waveforms. Especially for α/M^2=1000, periods of response waveforms differ under the influence by control which can make periods longer.



a) El Centro 1940 NS b) Hachinohe 1968 NS c) JMA Kobe 1995 NS

Figure 4.3. Maximum responses in time history analysis



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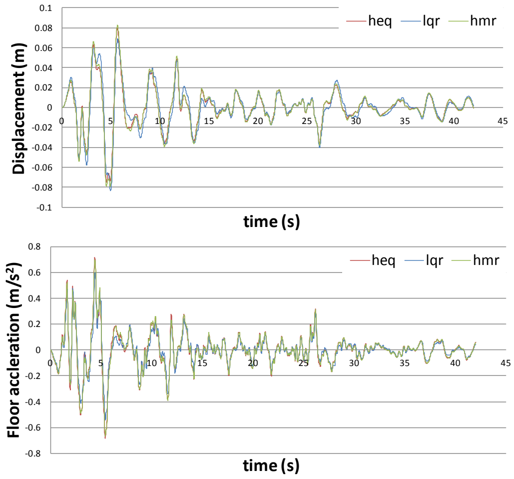
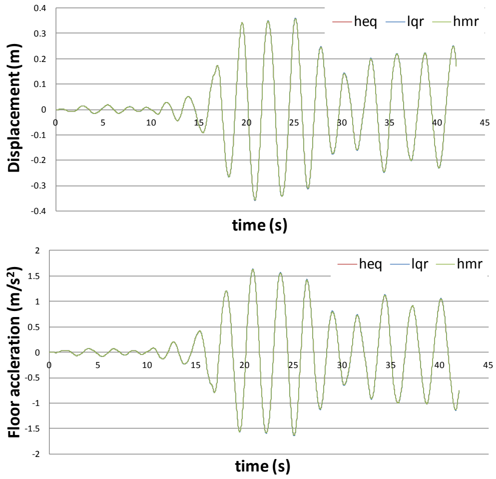
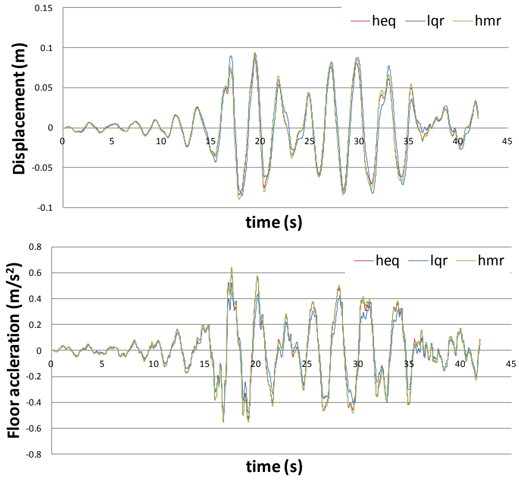


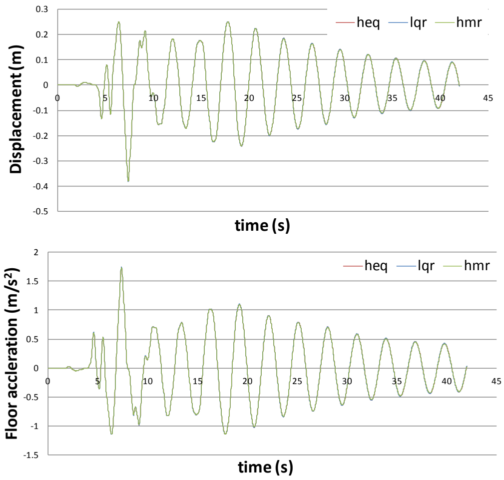
a) El Centro 1940 NS



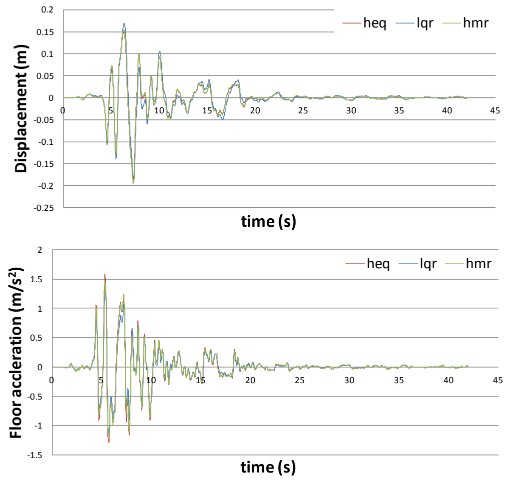
b) Hachionhe 1968 NS



b) Hachionhe 1968 NS



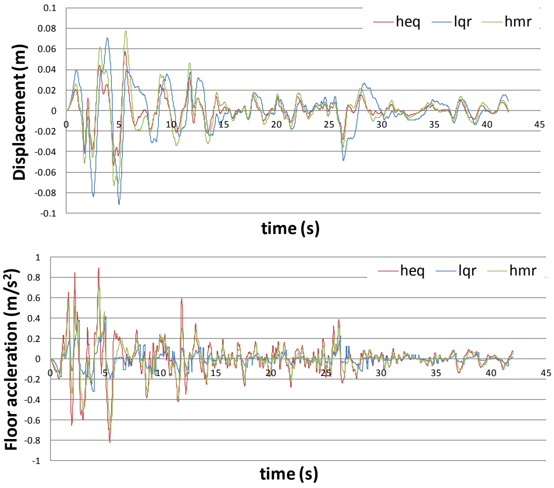
c) JMA Kobe 1995 NS



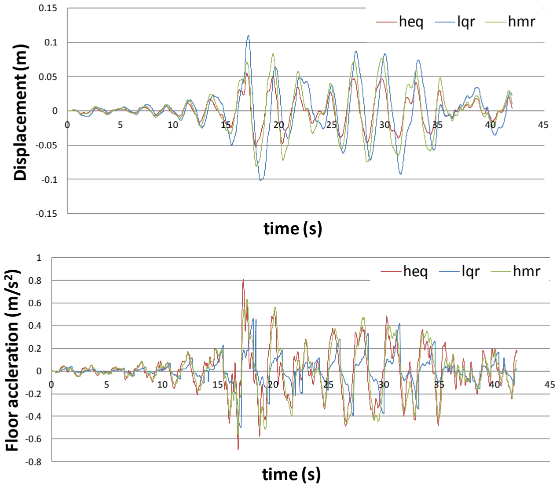
c) JMA Kobe 1995 NS

Figure. 4.4. The result of time history analysis (α/M^2=0.0001)

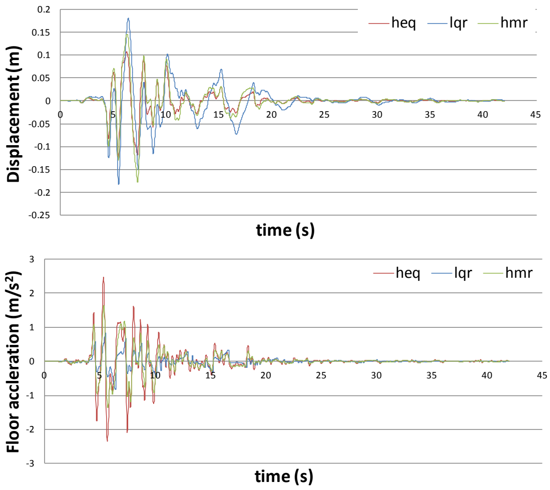
Figure. 4.5. The result of time history analysis (α/M^2=1)



a) El Centro 1940 NS



b) Hachionhe 1968 NS



c) JMA Kobe 1995 NS

Figure. 4.6. The result of time history analysis (α/M^2=1000)

5. CONCLUSIONS

Relations between control variables and control effects were studied by making a damping factor into an evaluation index.

1. The relations between the weight coefficient and damping factor are shown in section 3. Furthermore, physical meanings of performance index and weight matrix were discussed.

$J = \int (\alpha \ddot{x}^2 + u^2)\,dt$: The period and damping of a structure are increased if α increases. The damping factor converges to $1/\sqrt{2}$.

$J = \int (\beta \dot{x}^2 + u^2)\,dt$: The period of a structure is not changed and damping is increased if β increases.

$J = \int (\gamma x^2 + u^2)\,dt$: The period of a structure is shortened and damping is increased if γ increases. The damping factor converges to $1/\sqrt{2}$.

2. Examination of a controllable range was performed. If the weight coefficient is increased, then when an equivalent viscous damping factor estimates the control effect simply, the maximum responses differ from semi-active controlled responses. Therefore, we suggest that h_{eq} requires compensation. We proposed the correction coefficient which can reduce differences between elucidated responses from damping factors and real controlled responses. The h_{mr} which are compensated h_{eq} by the correction coefficient p explains expressed control effect.

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