Seismic Response of Asymmetric Building with Semi-Active Stiffness Dampers

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SUMMARY:

The seismic response of single-storey, one-way asymmetric building with semi-active variable stiffness dampers is investigated. The switching and resetting control laws are considered for the semi-active devices. The governing equations of motion are derived based on the mathematical model of asymmetric building. The seismic response of the system is obtained by numerically solving the equations of motion using state space method under different system parameters. The important parameters selected are eccentricity ratio of superstructure, uncoupled time period and ratio of uncoupled torsional to lateral frequency. The effects of these parameters are investigated on peak responses of lateral, torsional and edge deformations as well as on damper control forces. The comparative performance is investigated for asymmetric building installed with passive stiffness and semi-active stiffness dampers. It is shown that the semi-active stiffness dampers reduce the earthquake induced deformations significantly as compared to passive stiffness dampers. Also, the effects of torsional coupling on effectiveness of passive system in reducing edge deformations are found to be more sensitive to the variation of eccentricity as compared to semi-active control system.

Keywords: seismic response, torsionally coupled, eccentricity, passive damper, semi-active stiffness damper

1. INTRODUCTION

Asymmetric buildings are more vulnerable to severe damage during seismic event. The uneven distribution of mass and/or stiffness of the structural components cause the asymmetry in buildings. The prime focus of the structural engineer is to reduce the torsional response mainly by avoiding the eccentricity which is produced due to irregular distribution of mass and/or stiffness. However, due to stringent architectural and functional requirements, many times it is not possible to avoid the superstructure eccentricity and hence in such cases, use of structural control techniques proves to be an effective solution to minimize the lateral-torsional response of buildings. In past many researchers had investigated the performance of base isolation, passive control as well as active control for asymmetric buildings (Jangid and Datta, 1994; Goel, 1998; Date and Jangid, 2001; Lin and Chopra, 2003). A semi-active control system generally originates from passive system and combines the best features of both passive and active control systems (Symans and Constantinou, 1999). The development of various semi-active systems for structural application is in primitive stage as compared to other control systems. Further, limited research has been done to investigate the seismic response of asymmetric building with semi-active systems (Yoshida et al., 2003; Li and Li, 2009). Although, above studies reflect the effectiveness of passive and some of the semi-active systems in controlling the torsional responses, however, no specific study has been done to investigate the effectiveness of semi-active stiffness dampers for asymmetric buildings. Also, a comparative study to investigate the performance of passive stiffness and semi-active stiffness dampers for torsionally coupled building has not been done so far. Further, the effects of torsional coupling on the effectiveness of stiffness dampers for the asymmetric systems are also not studied.

In this paper, the seismic response of single storey, one-way asymmetric building is investigated under various earthquake ground motions. The objectives of the study are summarized as (i) to investigate the comparative seismic response of asymmetric building installed with passive and semi-active stiffness dampers in controlling lateral, torsional and edge displacements as well as accelerations and (ii) to study the effects of torsional coupling on the effectiveness of passive and semi-active control systems for asymmetric system as compared to the corresponding symmetric system.

2. STRUCTURAL MODEL

The system considered is a linearly elastic idealized one-storey building consists of a rigid deck supported on columns as shown in Fig. 2.1. The mass of deck is assumed to be uniformly distributed and hence the centre of mass (CM) coincides with the geometrical centre of the deck. The columns are arranged in a way such that it produces the stiffness asymmetry with respect to CM in one direction and hence, the centre of rigidity (CR) is located at an eccentric distance, e_x from CM in x-direction. The system is symmetric in x-direction and therefore, two degrees-of-freedom are considered for model namely the lateral displacement in y-direction, u_y and torsional displacement, u_θ . The governing equations of motion of the system in the matrix form are expressed as

$$M\ddot{u} + C\dot{u} + Ku = -M\Gamma\ddot{u}_{\varrho} + \Lambda F \tag{2.1}$$

where M, C and K are mass, damping and stiffness matrices of the system, respectively; $u = \left\{u_y \ u_\theta\right\}^T$ is displacement vector; Γ is the influence coefficient vector; $\ddot{u}_g = \left\{\ddot{u}_{gy} \ 0\right\}^T$ is ground acceleration vector; \ddot{u}_{gy} is ground acceleration in y-direction; Λ is matrix that defines the location of control devices; $F = \left\{F_{dy} \ F_{d\theta}\right\}^T$ is the vector of control forces; and F_{dy} and $F_{d\theta}$ are resultant control forces of dampers along y- and θ - direction, respectively.

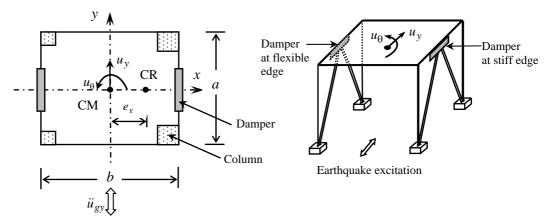


Figure 2.1. Plan and isometric view of asymmetric building showing arrangements of dampers

The mass matrix can be expressed as,

$$\mathbf{M} = \begin{bmatrix} m & 0 \\ 0 & mr^2 \end{bmatrix} \tag{2.2}$$

where m represents the lumped mass of the deck; r is mass radius of gyration about a vertical axis through CM which is given by, $r = \sqrt{\left(a^2 + b^2\right)/12}$; where a and b are plan dimensions of building.

The stiffness matrix of the system is modified as follows (Goel, 1998)

$$\mathbf{K} = K_y \begin{bmatrix} 1 & e_x \\ e_x & e_x^2 + r^2 \Omega_\theta^2 \end{bmatrix}$$
 (2.3)

$$e_x = \frac{1}{K_y} \sum_i K_{yi} x_i$$
 and $\Omega_\theta = \frac{\omega_\theta}{\omega_y}$ (2.4)

$$\omega_{\theta} = \sqrt{\frac{K_{\theta r}}{mr^2}} \text{ and } \omega_{y} = \sqrt{\frac{K_{y}}{m}}$$
(2.5)

$$K_{\theta r} = K_{\theta \theta} - e_x^2 K_y$$
 and $K_{\theta \theta} = \sum_i K_{xi} y_i^2 + \sum_i K_{yi} x_i^2$ (2.6)

where K_y denotes the total lateral stiffness of system in y-direction; e_x is structural eccentricity between CM and CR of the system; Ω_{θ} is the ratio of uncoupled torsional to lateral frequency of the system; K_{yi} indicates the lateral stiffness of i^{th} column in y-direction; x_i is the x-coordinate distance of i^{th} element with respect to CM; ω_y is uncoupled lateral frequency of the system; ω_{θ} is uncoupled torsional frequency of the system; $K_{\theta r}$ is torsional stiffness of the system about a vertical axis at CR; $K_{\theta \theta}$ is torsional stiffness of the system about a vertical axis at CM; K_{xi} indicates the lateral stiffness of i^{th} column in x-direction; and y_i is the y-coordinate distance of i^{th} element with respect to CM.

The damping matrix of the system is not known explicitly and it is constructed from the Rayleigh's damping considering mass and stiffness proportional as,

$$C = a_0 M + a_1 K \tag{2.7}$$

in which a_0 and a_1 are the coefficients depends on damping ratio of two vibration modes. For the present study 5 % damping is considered for both modes of vibration of system. The governing equations of motion for the model are solved using the state space method.

3. MODEL OF DAMPER AND CONTROL LAWS

Semi-active stiffness devices are utilized to modify the stiffness and thus natural vibration characteristics of the structure. These devices are engaged or released so as to include or not include, the stiffness of the bracing systems of the structure (Symans and Constantinou, 1999). The damper consists of a cylinder-piston system with a valve in bypass pipe connecting two sides of the cylinder. Fig. 3.1 shows the schematic and mathematical model of stiffness damper. When the valve is closed, the damper serves as a stiffness element in which the stiffness (k_f) is provided by the bulk modulus of the fluid in the cylinder. When the valve is open, the piston is free to move and the damper provides only a small damping without stiffness. The effective stiffness of the device consists of damper stiffness (k_f) and bracing stiffness (k_h) and is given by (Yang et al. 2000)

$$k_{hi} = \frac{k_{fi} k_{bi}}{(k_{fi} + k_{bi})} \tag{3.1}$$

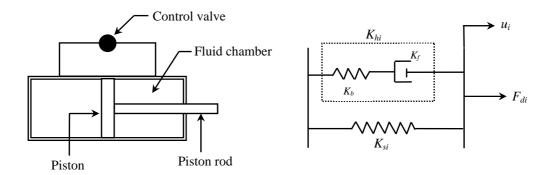


Figure 3.1. Schematic and mathematical model of semi-active variable stiffness damper (Kori and Jangid, 2007)

3.1. Switching control law

This control law has been derived based on sliding mode control by Kamagata and Kobori (1994) and Yang et al. (1996). In this control, the valve of hydraulic damper is pulsed to open during a certain time interval and close during another time interval, which can be referred as switching semi-active stiffness damper (SSASD). When a valve of the i^{th} damper is closed, the effective stiffness, k_{hi} is added to the story unit and when a valve is open, the effective stiffness, k_{hi} is zero. When the valve is switched off from on, a certain amount of energy is taken out of the structural system and when it is on, energy is added to the structural system. The control force of i^{th} SSASD can be obtained as

$$F_{di} = k_{hi} v_i u_i \tag{3.2}$$

where k_{hi} is the effective stiffness of i^{th} damper; u_i is the relative displacement at the location of i^{th} damper; and v_i is the switching parameter of i^{th} damper which is based on the switching control law expressed as (Yang et al. 2000)

$$v_i(t) = \begin{cases} 1 & \text{if } u_i \dot{u}_i \ge 0\\ 0 & \text{otherwise} \end{cases}$$
 (3.3)

When $v_i(t) = 1$, indicates that the i^{th} SSASD is locked (i.e. valve is closed) and $v_i(t) = 0$, indicates the i^{th} SSASD is unlocked (i.e. valve is open).

3.2. Resetting control law

In this control, the valve of hydraulic damper is closed for most of the time. Hence, the energy is stored in the damper-bracing systems in form of potential energy. At appropriate time instants, the valve is pulsed to open and close quickly. The position of the piston of damper at that moment is referred as resetting position, u_{ri} and energy is released during this stage. The hydraulic damper in resetting mode is referred as resetting semi-active stiffness damper (RSASD) (Yang et al. 2000). The control force of i^{th} RSASD can be obtained as

$$F_{di} = k_{bi}(u_i - u_{pi}) (3.4)$$

where u_{ri} is resetting position of i^{th} damper. When the RSASD is reset (valve is pulsed to open and close), $u_{ri} = u_i$. At that instant, the applied damper force is zero. Yang et al. (2000) derived a resetting control law considering the Lyapunov function V as follows

$$V = 0.5 \mathbf{u}^{T} \mathbf{K} \mathbf{u} + 0.5 \dot{\mathbf{u}}^{T} \mathbf{M} \dot{\mathbf{u}} + \alpha_{I} \mathbf{u}^{T} \mathbf{M} \dot{\mathbf{u}}$$
(3.5)

where α_L is constant such that the Lyapunov function is positive definite as follows

$$\begin{bmatrix} \mathbf{K} & \alpha_L \mathbf{M} \\ \alpha_L \mathbf{M} & \mathbf{M} \end{bmatrix} > 0 \tag{3.6}$$

Based on this, by minimizing \dot{V} , Yang et al. (2000) derived the resetting control law as follows

$$u_{ri} = u_i \text{ when } \dot{u}_i + \alpha_I u_i = 0 \tag{3.7}$$

3.3. Passive control

In passive mode of control, the valve is either always open or always closed. When the valve is always closed, the switching parameter, v_i is always considered equal to unity and the damper force of passive stiffness damper (PSD) is calculated as expressed in Eqn. 3.2.

4. NUMERICAL STUDY

The seismic response of linearly elastic, single-storey, one-way asymmetric building installed with passive and semi-active stiffness dampers is investigated by numerical simulation study. The response quantities of interest are lateral and torsional displacements of the floor mass obtained at the CM (u_y and u_θ), displacements at stiff and flexible edges of building (u_{ys} and u_{yf}), lateral and torsional accelerations of the floor mass obtained at the CM (\ddot{u}_y and \ddot{u}_θ), accelerations at stiff and flexible edges of building (\ddot{u}_{ys} and \ddot{u}_{yf}), control forces of dampers installed at stiff edge (F_{ds}) and at flexible edge (F_{df}) of building as well as resultant damper force, F_{dy} (= $F_{ds} + F_{df}$). The responses are obtained for four earthquake ground motions namely, Imperial Valley (19th May, 1940, El Centro), Loma Prieta (18th October, 1989, LGPC), Northridge (17th January, 1994, Sylmar CS) and Kobe (16th January, 1995, JMA) with corresponding peak ground acceleration values of 0.31g, 0.96g, 0.89g and 0.82g. For the study carried out herein, the aspect ratio of plan dimension is kept as unity and the mass and stiffness of system are considered such as to have required lateral time period. Further, total two stiffness dampers (one at each edge) are installed in building as shown in the Fig. 2.1.

In order to study the effectiveness of control system and effects of torsional coupling, the responses are expressed in terms of indices, R_e and R_t defined as follows:

$$R_e = \frac{\text{Peak response of controlled asymmetric system}}{\text{Peak response of corresponding uncontrolled system}}$$
(4.1)

$$R_t = \frac{\text{Peak response of controlled asymmetric system}}{\text{Peak response of corresponding symmetric system}}$$
(4.2)

The value of R_e less than unity indicates that the control system is effective in reducing the responses. On the other hand, the value of R_t greater than unity indicates that the response of asymmetric system increases due to torsional coupling and hence the effectiveness of control system is less for asymmetric system as compared to corresponding symmetric system.

For the stiffness damper, the effective damper stiffness (k_{hi}) plays an important role while designing the control system. For the present study, the stiffness ratio, (k_r) is defined as follows

$$k_r = \frac{k_{hi}}{k_{si}} \tag{4.3}$$

where k_{si} is the story stiffness.

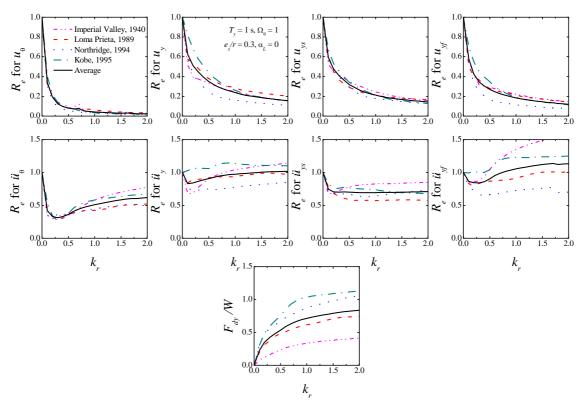


Figure 4.1. Effect of stiffness ratio (k_r) on ratio, R_e for various responses for system with RSASD

In order to arrive at the optimum value of stiffness ratio (k_r), a parametric study is carried out for the strongly coupled (Ω_0 =1) asymmetric system with lateral time period, T_y = 1 s and intermediate eccentricity ratio, $e_x/r=0.3$ installed with RSASDs. The response ratios, R_e are obtained for various displacements and accelerations and plotted against k_r (which is varied from 0 to 2) in Fig. 4.1. The constant α_L of resetting control law is considered as zero (Yang et al. 2000). It is observed from the various earthquakes as well as from average trends that with the increase in k_r , the ratio, R_e for displacement responses decreases continuously. This means the effectiveness of control system is more in reducing displacement with higher values of k_r . On the other hand, R_e for various accelerations decreases initially with increase in k_r and then increases with further increase in k_r . This implies that there exists an optimum range of stiffness ratio, k_r in order to achieve the optimum reduction in torsional, lateral and edge accelerations. Moreover, the variation of peak resultant damper force against ratio, k_r are also shown. The damper forces are normalized with the weight of deck, W. It is observed that for larger values of k_r , the control forces developed in the dampers are more. Hence, to achieve the optimum compromise between the reduction in various responses as well as damper capacity, the suitable value of stiffness ratio, k_r is considered as 0.5 for the further study.

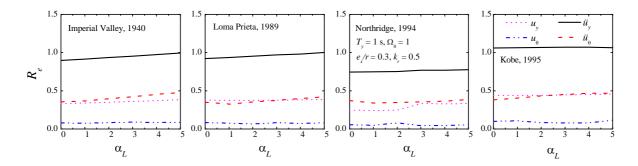


Figure 4.2. Effect of parameter, α_{I} on response ratio, R_{e} for various responses

The constant α_L used for resetting control also plays an important role in performance of system. Fig. 4.2 shows the variations of ratio, R_e for lateral and torsional displacements and accelerations against α_L . Initially the constant α_L is considered as zero and varied as long as the check for Lyapunov function holds good. For the considered structural model, the Lyapunov function does not holds good beyond the value of $\alpha_L = 5$. It is observed from the figure that the ratios, R_e for various responses mildly increase with increase in α_L , in general. However, the variation of R_e for u_0 is little more sensitive to the change in α_L . Thus, for the study carried out herein, the constant α_L is considered as zero which led to higher reduction in various responses for structural system under consideration.

To study the comparative performance of PSD, SSASD and RSASD, various responses are obtained for the system with $T_y=1$ s, $\Omega_\theta=1$ and $e_x/r=0.3$ by considering the optimum value of stiffness ratio. The time histories of various uncontrolled and controlled responses like lateral displacement and acceleration at CM as well as torsional displacement and acceleration obtained under Imperial Valley, 1940 earthquake are shown in Figs. 4.3 and 4.4. It can be observed from the figures that RSASD is more effective in reducing u_θ , u_y , \ddot{u}_θ and \ddot{u}_y as compared to SSASD and PSD. On the contrary, the installation of PSD increases the accelerations as compared to that of uncontrolled system.

The ratio, R_e for peak values of various displacements and accelerations are obtained for each case under considered earthquakes and shown in Table 4.1. It is observed that ratio, R_e for torsional, lateral and edge displacements as well as their acceleration counterparts is more for PSD as compared to SSASD and RSASD implying the effectiveness of semi-active systems. It is further observed that R_e for various accelerations responses for system installed with PSD is more than unity indicating that the PSD is not effective in reducing accelerations. Moreover, the bold numbers in parentheses for the cases of SSASD and RSASD indicate the percentage reduction in R_e as compared to passive case. It is noticed that nearly all numbers in bold are positive indicating that the higher reduction can be achieved with semi-active devices as compared to passive case. Furthermore, the percentage reduction for RSASD case is more as compared to SSASD. The last column of table represents the average values of percentage reduction. In addition, the last set of rows shows the normalized peak resultant damper force which shows that the control force developed for RSASD is less than the corresponding force for PSD. Thus, the RSASD is quite effective in reducing lateral, torsional and edge displacement and acceleration responses as compared to SSASD and PSD for strongly coupled asymmetric building.

Fig. 4.5 shows the normalized damper force-displacement hysteresis loops for PSD, SSASD and RSASD for system with $T_v = 1$ s, $\Omega_0 = 1$ and $e_x / r = 0.3$ under Imperial Valley, 1940 earthquake.

The effects of torsional coupling also play an important role while designing the control system.

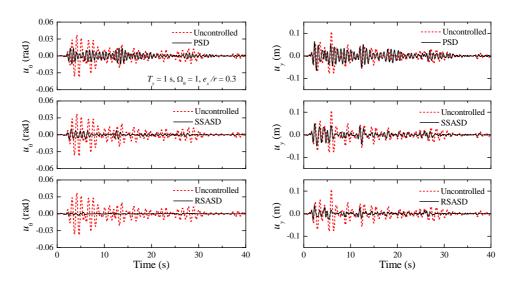


Figure 4.3. Time histories for uncontrolled and controlled displacements under Imperial Valley, 1940

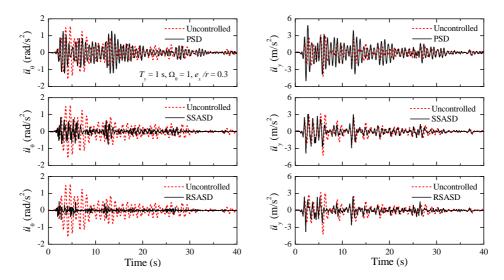


Figure 4.4. Time histories for uncontrolled and controlled accelerations under Imperial Valley, 1940

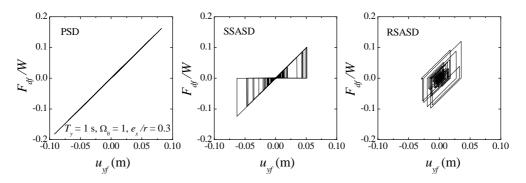


Figure 4.5. Hysteresis loops for normalized damper force for flexible edge damper under Imperial Valley, 1940

In order to study this, the ratio, R_t (which is between the peak response of controlled asymmetric and corresponding symmetric system) is obtained for lateral and edge displacement responses for the system with $T_y=1$ s and $\Omega_\theta=1$ and plotted against e_x/r in Fig. 4.6. It can be observed that the ratio, R_t for lateral displacement at CM (u_y) and edge displacements (u_{ys}) and u_{yf} varies significantly with change in e_x/r for the system installed with PSD as compared to RSASD.

Table 4.1. Response ratio, R_e for peak responses for different control strategies ($T_y = 1$, $\Omega_\theta = 1$, $e_x / r = 0.3$)

Ratio, R_e	Control Strategy	Imperial Valley, 1940	Loma Prieta, 1989	Northridge, 1994	Kobe, 1995	Average
иθ	PSD	0.391	0.765	0.206	0.404	
	SSASD	0.259 (33.84 %)	0.271 (64.55 %)	0.168 (18.04 %)	0.271 (32.89 %)	37.33 %
	RSASD	0.080 (79.85 %)	0.084 (89.05 %)	0.057 (72.51 %)	0.101 (75.07 %)	79.07 %
uy	PSD	0.597	1.238	0.519	0.810	
	SSASD	0.464 (22.26 %)	0.566 (54.34 %)	0.401 (22.88 %)	0.732 (9.65 %)	79.07 %
	RSASD	0.334 (44.17 %)	0.377 (69.53 %)	0.247 (52.52 %)	0.441 (45.58 %)	52.95 %
uys	PSD	0.588	0.639	0.604	0.482	
	SSASD	0.492 (16.25 %)	0.503 (21.25 %)	0.430 (28.71 %)	0.529 (-9.72 %)	14.12 %
	RSASD	0.352 (40.03 %)	0.291 (54.49 %)	0.294 (51.29 %)	0.343 (28.97 %)	43.70 %
uyf	PSD	0.755	1.325	0.354	0.791	
	SSASD	0.517 (31.53 %)	0.417 (68.50 %)	0.362 (-2.35 %)	0.617 (21.96 %)	29.91 %
	RSASD	0.291 (61.47 %)	0.278 (79.02 %)	0.166 (53.11 %)	0.347 (56.16 %)	62.44 %
ü _θ	PSD	0.813	1.583	0.472	0.910	
	SSASD	0.545 (32.96 %)	0.666 (57.95 %)	0.498 (-5.48 %)	0.607 (33.24 %)	29.67 %
	RSASD	0.352 (56.71 %)	0.349 (77.96 %)	0.370 (21.63 %)	0.380 (58.25 %)	53.64 %
üy	PSD	1.187	2.489	1.119	1.525	
	SSASD	0.914 (23.05 %)	1.041 (58.18 %)	0.803 (28.25 %)	1.364 (10.55 %)	30.01 %
	RSASD	0.899 (24.31 %)	0.920 (63.03 %)	0.746 (33.32 %)	1.060 (30.51 %)	37.79 %
ü _{ys}	PSD	1.018	1.064	1.040	0.776	
	SSASD	0.866 (14.97 %)	0.933 (12.32 %)	0.750 (27.86 %)	0.907(-16.79%)	9.59 %
	RSASD	0.780 (23.35 %)	0.588 (44.77 %)	0.665 (36.02 %)	0.769 (0.96 %)	26.27 %
ü _{yf}	PSD	1.830	3.505	0.865	1.874	
	SSASD	1.295 (29.23 %)	1.108 (68.38 %)	0.922 (-6.64 %)	1.478 (21.15 %)	28.03 %
	RSASD	1.049 (42.68 %)	0.867 (75.26 %)	0.663 (23.31 %)	1.044 (44.31 %)	46.39 %
F _{dy} /W	PSD	0.253	1.179	0.784	0.929	
	SSASD	0.188 (25.73 %)	0.442 (62.53 %)	0. 578 (26.35 %)	0.797 (14.19 %)	32.20 %
	RSASD	0.235 (6.84 %)	0.478 (59.42 %)	0.672 (14.30 %)	0.749 (19.38 %)	24.98 %

(Numbers in parentheses indicate percentage reduction as compared to the passive (PSD) case. Positive numbers correspond to a reduction in response ratio)

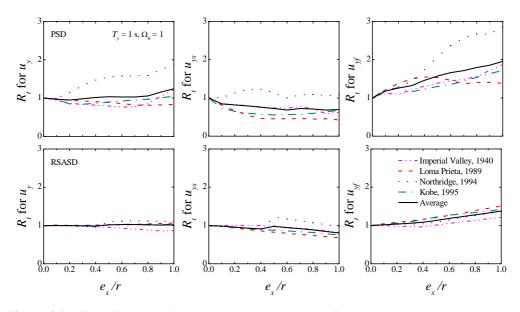


Figure 4.6. Effect of eccentricity ratio on response ratio, R_t for various displacement responses

Further, from the average trend, it is observed that the ratio, R_t for stiff edge displacement, u_{ys} decreases and remains less than unity with increase in e_x/r . This indicates that the effectiveness of control system is more for asymmetric system in reducing u_{ys} as compared to the corresponding symmetric system. Thus, the effectiveness will be underestimated by ignoring the effects of eccentricity. On the other hand, the opposite trend is observed for flexible edge displacement, u_{yf} . Thus, the difference between various displacement responses of asymmetric and corresponding symmetric system is significantly higher for system installed with PSD and it is comparatively very less for the system installed with RSASD and the difference increases with increase in superstructure eccentricity.

5. CONCLUSIONS

The seismic response of single-storey, one-way asymmetric building installed with passive and semiactive stiffness dampers subjected to earthquake ground motions is investigated. The responses are obtained by considering switching and resetting control laws to study the effectiveness of control system and effects of torsional coupling. From the trend of the results of the present study, following conclusions can be drawn:

- 1. For asymmetric buildings, the torsional, lateral and edge displacements decrease with the increase in stiffness ratio. On the other hand, there exists an optimum value of stiffness ratio for the torsional, lateral and edge accelerations.
- 2. The RSASD is quite effective in reducing lateral, torsional and edge displacement and acceleration responses as compared to SSASD and PSD for strongly coupled asymmetric building.
- 3. The difference between various displacement responses of asymmetric and corresponding symmetric system is significantly higher for system installed with PSD and it is comparatively very less for the system installed with RSASD and the difference increases with increase in superstructure eccentricity.

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