Strength Evaluation of Reinforced Concrete Beam-Column Joints

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SUMMARY

Joint research by three countries (America, Japan, and New Zealand) has made remarkable improvements in joint design. In spite of this cooperative effort, the joint shear strength predictions of the ACI Recommendations and the AIJ Guidelines (Architectural Institute of Japan) are based on the concrete arch mechanism, while the prediction of the NZS (New Zealand Standard) Code is evaluated by means of both arch and truss mechanisms. In this study, a method was provided to predict the strength of reinforced concrete beam-column joints that fail in shear before the plastic hinges occur at both ends of the adjacent beams. In the proposed method, the softening effect of concrete in the joint area was evaluated by using both arch and truss mechanism. In order to verify the proposed method, the predicted results of the proposed equation were compared with experimental results of RC beam-column joints, as reported in the technical literature. Comparisons between the observed and calculated shear strengths of the tested beam-column assemblies, showed reasonable agreement.

Keywords: RC beam-columns joint, shear strength, J-failure, reinforcement steel, softening effect.

1. INTRODUCTION

Since the mid-1960s, the performance of reinforced concrete beam-column joints has been recognized as a factor that influences the behavior of ductile moment frames. Therefore, many theoretical and experimental studies have been carried out to evaluate the performance of beam-column joints subjected to lateral loading. Reinforced concrete beam-column joints are important structural members that should assure structural stability when the members are subjected to cyclic loading, such as gravity loading and seismic lateral loading. In seismic lateral loading, proper and safe behavior is possible, because energy dissipation ability and corresponding deformability are large when the plastic hinges of beams are well designed. Otherwise, joints show much more brittle failure in shear, and the failures cause severe damage of the overall structure. Therefore, it is important that failure in the joint should not occur until the deformation reaches the designed deformability, through producing plastic hinges in adjacent beams and providing adequate energy dissipation ability.

In United States and Japan, the shear strength of beam-column joints has been evaluated by the ACI Recommendations (ACI 352-R02 (2003)), and by the AIJ Guidelines (Architectural Institute of Japan (2010)). The ACI Recommendations divide joints into two categories: Type 1 for structures in non-seismic hazard areas and Type 2 in seismic hazard areas. The shape of joints is reflected in defining the capacity of beam-column joints for Type 1, in that ductility of joints is not particularly considered, and for Type 2, in that energy dissipation capacity is important because of strong seismic loading. Also, the AIJ Guidelines have a similar design purpose to the ACI Recommendations, and in ensuring that the structure remains enough deformation, even after developing yielding mechanisms. The strength evaluation of reinforced concrete beam-column joints could be distinguished by the methods used, whether based on arch mechanism or truss mechanism. In the ACI Recommendations and the AIJ Guidelines, the prediction of joint shear strength is based on the concrete arch mechanism while both arch and truss mechanisms are applied to joint shear strength estimation in the NZS (New

Zealand Standard (1995)) Code. The method based on arch mechanism is relatively simple to evaluate the shear strength with, but it is difficult in the estimation to reflect the effect of shear reinforcement in the joints and bondage. On the other hands, an equation for evaluation can be complicated, when arch and truss mechanisms are used simultaneously.

In this study, a method was provided to predict the strength of reinforced concrete beam-column joints that fail in shear before the plastic hinges occur at both ends of the adjacent beams. In the proposed method, the softening effect of concrete in joint area was evaluated by using both arch and truss mechanisms, and the equation including two mechanisms was suggested reflecting the compressive strength of concrete in the beam-column joints. In order to verify the shear strength of the proposed method, the predicted results were analyzed and compared with 54 experimental specimens that had been performed previously.

2. EXISTING DESIGN STANDARD OF BEAM-COLUMN JOINTS

In spite of cooperative research efforts between the United States, Japan and New Zealand, the three countries have proposed different views on shear mechanisms and joint strength, and unified design method does not yet exist. The standard of the ACI Recommendations is an empirical method based on experiments, and only concrete compressive strength is considered, even though the equation is simple and accurate, because of using a concrete compressive strut in the beam-column joint, as shown in Fig. 1(a).

In the AIJ Guidelines, similar to the ACI Recommendations, shear force in the joint is resisted by concrete, and the amount of the reinforcement is determined according to shear stress in the joint. In the NZS Code, the arch and truss mechanisms (Fig. 1(b)), which consists of horizontal and vertical steel and concrete compressive strut, are reflected in the design standard to evaluate joint shear strength. The predicted results in this study are compared with the values estimated by the ACI Recommendations and the AIJ Guidelines because, in the NZS Code, the required amount of reinforcement in the joint is evaluated by taking into account the yield strength of plastic hinges in the beams. In addition, the direct method to calculate joint strength has not been proposed in the NZS Code. D_c is the compressive force of the arch mechanism, and D_s is that of the truss mechanism. b_a is the strut width, and α is the crack angle.



Figure 1. (a) Arch action

(b) Truss action

2.1 ACI-ASCE Committee 352

In ACI-ASCE Committee 352, beam-column joints are classified into two groups. Type 1 is for members without significant inelastic deformation, and Type 2 is for members that should have enough ductility in the inelastic range with energy dissipation. The shear strength is defined by applying a correction factor to the basic equation, which considers concrete compressive strength as in Eqn. 2.1.

$$V_j = 0.083\gamma \sqrt{f_{ck}} b_j h_c \tag{2.1}$$

$$b_{j} = \min(\frac{b_{j} + b_{c}}{2}, b_{b} + \sum \frac{mh_{c}}{2}, b_{c})$$
(2.2)

Where, V_j : joint shear strength, γ : factor dependent on shape of beam-column joint and seismic zone or non-seismic zone, f_{ck} : concrete compressive strength (MPa), b_j : effective width of the joint and value satisfying Eqn. 2.2, h_c : column depth, b_b : beam width in longitudinal direction, b_c : joint width, m : 0.3 for the joint that eccentricity between center of beam and column exceeds $b_c/8$ and 0.5 for other cases.

2.2 AIJ 2010

The Architectural Institute of Japan recommends a nominal joint shear strength equation that is based on databases of experiments performed for interior and exterior connection assemblies from 1996 to 1988. Test results indicate that in shear failure, the effect of concrete compressive strength on the shear strength of beam-column joints is larger than that of the amount of transverse reinforcement. Therefore, the nominal joint shear strength is defined by considering the compressive strut mechanism in shear resistance mechanisms in the form of Eqn. 2.3.

$$V_{ju} = k\phi F_j b_j h_j \tag{2.3}$$

$$F_{i} = 0.8 \times f_{ck}^{0.7} (N/mm^{2})$$
(2.4)

$$b_{j} = b_{b} + b_{a1} + b_{a2} \tag{2.5}$$

where, k: the factor dependent on the shape of joints (1.0 for + shape, 0.7 for $|\cdot|$ and T shape, 0.4 for L shape) φ : 1.0 for when transverse beams exist on both sides and 0.85 for the others, F_j : modified concrete compressive strength expressed as Eqn. 2.4, b_j : effective width of beam-column joint according to Eqn. 2.5, h_j : joint depth (0.75 h_c for $|\cdot|$ and L type and h_c for |+| and T type), b_b : beam width, b_{ai} : the distance between column faces and both beam faces.

3. PROPOSED EQUATION OF JOINTS SHEAR STRENGTH

The horizontal shear strength resists the extent of the difference between tensile force and compressive force in adjacent beams. Each researcher has a different opinion about the nominal strength of beam-column joints, therefore, the ACI-ASCE Committee 352R-02 and the AIJ Guidelines consider the arch mechanism only, while the NZS Code considers the arch and truss mechanism. In this study, the horizontal shear strength is calculated according to Eqn. 3.1, which combines the arch and truss mechanisms. The study is based on the NZS Code being able to compare and evaluate shear and bond strength.

$$V_{jh} = V_{ch} + V_{sh} \tag{3.1}$$

 V_{jh} is the horizontal shear strength of the beam-column joint. V_{ch} and V_{sh} is the shear strength by the arch and truss mechanisms. V_{ch} is defined in Eqn. 3.2 as using compressive force D_c in Fig. 1(a). D_c is expressed with concrete stress (f_a) and area of cracked concrete compressive strut, given by Eqn. 3.2.

$$V_{ch} = f_a \left(b_s \, x \, b_a \right) \cos a \tag{3.2}$$

where, b_s : beam width, b_a : strut width, α : strut angle

Fig. 2 shows the stress model of truss mechanism in the beam-column joint. Shear stress, τ_{bc} from the horizontal shear stress V_{sh} allows equilibrium equations of force based on Fig. 2, such as Eqn. 3.3, 3.4, 3.5.



Figure 2. Stress of truss model in beam-column joint

$$f_{b} = f_{2}^{c} \cos^{2} \alpha + f_{1}^{c} \sin^{2} \alpha + \rho_{b} f_{bs}$$
(3.3)

$$f_{c} = f_{2}^{c} \sin^{2} \alpha + f_{1}^{c} \cos^{2} \alpha + \rho_{c} f_{cs}$$
(3.4)

$$\tau_{bc} = (-f_2^c + f_1^c) \sin \alpha \cos \alpha \tag{3.5}$$

where, f_b , f_c : direct stress in beam and column direction, f_1^c , f_2^c : principle tensile and compressive stress of concrete, ρ_b , ρ_c : reinforcement ratio of beam and column, f_{bs} , f_{cs} : steel stress of beam and column.

The following assumptions are used to apply equilibrium equations (Eqn. 3.3, 3.4, 3.5) of force to the truss model of the beam-column joint:

1) The directions of the principle compressive stress and diagonal crack of the concrete are identical. 2) The diagonal angle in the concrete within joint is constant, and the angle is defined by the geometric conditions of joints, and given by Eqn. 3.6, where, h_b : beam depth, h_c : column depth

$$\sin \alpha = h_b / \sqrt{h_b^2 + h_c^2} \qquad \cos \alpha = h_c / \sqrt{h_b^2 + h_c^2}$$
(3.6)

3) The tensile stress in the concrete joints after cracking is so small that it can be ignored. $(f_I^c=0)$ 4) Dowel action and aggregate interlock are not directly considered.

According to Assumption 3), V_{sh} is defined, as show in Eqn. 3.7, when the effective area of the joint is reflected on the shear stress, τ_{bc} .

$$V_{sh} = f_2^c (h_c \times b_j) \sin \alpha \cos \alpha \tag{3.7}$$

The beam-column joint fails when the concrete stress of the compressive strut (f_a) and that of the truss mechanism (f_2^c) reach the effective compressive strength (vf_{ck}) of the concrete given by Eqn. 3.8.

$$V f_{ck} = f_a + f_2^c \tag{3.8}$$

This concept is also used by Nielsen (1975) in the study of beam shear strength analysis, but the shear resistance ratio of the arch and truss mechanism is computed by the strength when the shear reinforcement yields. There are many cases where compressive failure occurs in the beam-column joint before the reinforcement yields, because of transverse reinforcement of the column and longitudinal reinforcement of the beam and column. Therefore, the connection between the horizontal shear strength by the arch mechanism and the total horizontal shear strength is expressed by Eqn. 3.9, because the portion of shear contribution by the two mechanisms cannot be calculated.

$$V_{ch} = 0.3(1+3.5n)V_{jh}$$
(3.9)

$$c = (0.25 + 0.85n)h_c \tag{3.10}$$

Park (1992) suggested Eqn. 3.9, based on experimental results to estimate the shear capacity of a beam-column joint to which a concrete strut contributes. The compression zone of cross section in the beam and column varies with the axial force ratio, thus Eqns. 3.9 and 3.10 consider the depth of the compression area, c and axial force ratio, n. Fig. 3 indicates that depth of compression zone, c, gets deeper as the axial force ratio, n increases. If axial force ratio is 0, 1/4 of the column depth is effective depth of the compression zone according to Eqn. 3.10. Also, the column depth corresponds to the depth of compression area when axial force ratio is 1.



Figure 3. Depth of compression zone

Eqn. 3.11 can be suggested from Eqns. 3.1 and 3.9.

$$V_{jh} = \frac{V_{sh}}{1 - 0.3(1 + 3.5n)} \tag{3.11}$$

In Eqn. 3.12, V_{jh} is expressed by substituting Eqn. 3.7 in Eqn. 3.11.

$$V_{jh} = \frac{1}{1 - 0.3(1 + 3.5n)} (\nu f_{ck} - f_a) h_c b_j \sin \alpha \cos \alpha$$
(3.12)

Eqn. 3.13 is the compressive strength by the arch mechanism, and is computed by substituting Eqn. 3.9 in Eqn. 3.2. Eqns. 3.2 and 3.9 are equations for horizontal shear strength.

$$f_a = \frac{0.3(1+3.5n)V_{jh}}{b_s b_a \cos \alpha}$$
(3.13)

Finally, Eqn. 3.14 for V_{jh} is proposed by substituting f_{a} , from Eqn. 3.13 in Eqn. 3.12.

$$V_{jh} = \frac{b_s b_a \times v f_{ck} h_c b_j \sin \alpha \cos \alpha}{(1 - 0.3(1 + 3.5n)) b_s b_a + 0.3(1 + 3.5n) h_c b_j \sin \alpha}$$
(3.14)

To calculate Eqn. 3.14, the reduction factor, v, of the effective compressive strength of the concrete cracked and subjected to biaxial stress (compressive and tensile stress) is important. The compressive strength of the concrete strut differs depending on whether the joint is subjected to uniaxial stress or biaxial stress. When biaxial stress loads on the joint, the concrete compressive strength decreases because of the tension force in a perpendicular direction; that is the softening effect.

In Euro Code-2, the softening factor, v is defined by the following Eqn. 3.15, by inversely analysing the beam shear strength from experimental results. In the Rotating Angle Softened Truss Model (RA-STM) of Belarbi and Hsu (1988) and Modified Compression Field Theory (MCFT) of Vecchio and Collins (1989), the softening factor is determined by reflection of the principle tensile strain, ε_1 which is perpendicular to the concrete crack, as shown in Eqns. 3.16 and 3.17. In addition, the effective concrete compressive strength is defined as vf_c .

$$f_{2,\max} = \nu f_{ck} = 0.6 \left[1 - \frac{f_{ck}}{250} \right] f_{ck}$$
(3.15)

$$v = \frac{0.9}{\sqrt{1 + 400\varepsilon_1}} \qquad (f_c \le 41.5Mpa) \tag{3.16}$$

$$v = \frac{5.8}{\sqrt{f_c}(MPa)} \frac{1}{\sqrt{1 + 400\varepsilon_1}} \qquad (f_c \ge 41.5MPa)$$

$$v = \frac{1}{0.8 + 170\varepsilon_1} \tag{3.17}$$

According to Kim et al. (2010), the study results in Fig. 4 when the effective compressive strength is computed by Eqns. 3.16 and 3.17. The effective concrete strength decreases as the principle tensile strain increases, and the compressive strength declines by nearly half when the tensile strain becomes about twice the steel yield strain. In normal practice, it is known that Eqns. 3.16 and 3.17 estimate the effective compressive strength more precisely than do other equations, because Eqns. 3.16 and 3.17 use concrete tensile strain.



Figure 4. Steel tensile strain vs. softening factor

However, the results of the three equations are very similar, as presented in Fig. 4. Kim et al. suggest Eqn. 3.18, which considers a reinforcement ratio based on comparison and estimation of existing models, and Eqn. 3.18 is applied to the softening factor, v of Eqn. 3.14 that is proposed in this study.

$$v = (0.85 - \frac{\rho_x f_{xy}}{\rho_y f_{yy}} (0.85 - v_0))$$

$$v_0 = 0.6(1 - \frac{f_{ck}}{250})$$
(3.18)

 ρ_x is the sum of the longitudinal beam reinforcement ratio and transverse column reinforcement ratio

referred to the x direction, and ρ_y is the longitudinal column reinforcement ratio referred to the y direction. f_{xy} is the yield strength of x direction reinforcement, and f_{yy} is that of y direction reinforcement. Between $\rho_x f_{xy}$ and $\rho_y f_{yy}$ in $\rho_x f_{xy} / \rho_y f_{yy}$ the larger one is the denominator, and the smaller one is the numerator.

4. SHEAR ESTIMATION OF THE PROPOSED EQUATION

4.1 Shear estimation results by the proposed equation

In this study, the proposed estimation results are compared with the experimental shear strength based on 54 specimens from preceding studies that failed in shear before adjacent beams yield occurred (J-failure). Collected data and results are presented in Table. 1, and V_{test}/V_{cal} shows to what extent test and calculation result correspond each other. There is more than a 30% difference between V_{test} and V_{cal} in Teraoka (1996)'s specimens, which is relatively high difference, but the difference is less than 20% in most cases.

Fig. 5 shows the relationship f_{ck} , $\rho_x f_{xy}$, and $\rho_x f_{yy}$ and V_{test}/V_{cal} to know which factor influences the results of estimation by the suggested method. V_{test}/V_{cal} has a similar tendency for concrete compressive strength and reinforcement ratio in Fig. 5(a), (b), and (c) and is influenced by the reinforcement ratio. In Fig. 5(d) and (e), a tendency is not evident, because the range of x direction reinforcement ratio is smaller than y direction reinforcement ratio. The proposed equation evaluates lesser shear strength when ρ_x is larger than 0.04, and when ρ_y is larger than 0.025. V_{test}/V_{cal} tends to decrease as the concrete compressive strength increases in Fig. 5(a), and increases as the x and y direction reinforcement ratio increases in Fig. 5(d) and (e). Therefore, the result shown in Fig. 5(b) and (c) is appropriate. Estimation by the proposed method for experimental value (V_{test}/V_{cal}) results in 1.14 for the average, and 20% for the coefficient of variation.



(a) Concrete compression strength (b) x-Direction reinforcement index (c) y-Direction reinforcement index



Figure 5. Effect of reinforcement ratio in shear strength

Table 1. Shear strength estimation and comparison

		Beam	Column	c.		V _{test} /V _{cal}		
	Specimen	section	section	f_{ck}	Axial force	Proposed	ACT	АП
	-	(mmx mm)	(mmxmm)	(Mpa)	ratio, n	equation	ACI	AIJ
	1			-33.6	40.25	1.47	1.48	1.79
Teraoka	2	240 X 300	300X300	-33.6	0.25	1.50	1.52	1.83
	3			-34.5	0.24	1.51	1.54	1.85
	4			-36.6	0.32	1.47	1.65	1.96
	5			-36.6	0.32	1.48	1.67	1.98
	6			-39.6	0.32	1.43	1.66	1.94
	/			-46./	0.25	1.29	1.55	1.70
	<u></u>			-40.7	0.23	1.30	1.37	1.78
	10	-		-30.5	0.25	1.31	1.37	1.07
	11			-32.2	0.33	1.30	1.47	1.79
	12			-32.2	0.33	1.34	1.51	1.85
Shiohara	j-1	240 X 300	300X300	-81.2	0.11	0.84	0.84	0.86
	j-2			-81.2	0.11	0.91	0.91	0.92
	j-4			-72.8	0.13	0.81	0.91	0.94
	j-5			-72.8	0.13	1.03	1.02	1.06
	<u>j-6</u>			-79.2	0.12	0.91	0.91	0.93
	J-8			-79.2	0.12	0.95	1.03	1.05
	j-10 i 11			-39.2	0.12	1.07	0.92	1.08
	<u>J-11</u>			-39.2	0.12	0.93	1.09	1.27
Meinheit	2	280 X 300 240 X 457.2	457.2X330 .2	-41.8	0.25	1.24	1.34	1.15
	3			-26.6	0.39	1.25	1.29	1.13
	4			-35.8	0.30	1.31	1.40	1.24
	5			-35.8	0.04	1.49	1.39	1.23
	7			-37.2	0.47	1.11	1.40	1.22
	9			-31.0	0.35	1.49	1.56	1.42
	10			-29.6	0.36	1.43	1.48	1.36
	11			-25.5	0.42	1.35	1.47	1.39
	13			-41.3	0.30	1.00	1.52	1.13
Noguchi	0KL-2	200 X 300	300X300	-33.1	0.30	1.51	1.02	1.39
	OKJ-2 OKJ-3			-107.0	0.12	1.14	1.02	0.99
	OKJ-5			-70.0	0.12	1.15	1.06	1.11
	OKJ-6			-53.5	0.12	1.20	1.08	1.19
Fuji	A1	160 X 250	220X220	-40.2	0.08	1.06	0.94	1.09
	A2			-40.2	0.08	1.14	0.86	1.01
	A3			-40.2	0.23	1.15	0.94	1.09
	A4			-40.2	0.23	1.20	0.96	1.12
Kawasaki	JS-30-18	250 X 335	285X285	-33.91	0.30	1.20	1.19	1.08
Hatamoto	JS-48-18 \$11	-		-43.32	0.30	1.10	1.19	1.01
Hatamoto	<u>S11</u>	325 X 400	425X425	-44.49	0.21	1.10	1.25	1.14
	S-60			-31.2	0.15	1.11	1.00	1.00
Danaka	S-0	300 X 350	350X350	-31.2	0.00	0.82	0.76	0.93
	L-60			-31.2	0.15	0.72	0.76	0.93
Teraoka, Kanoh & Tanaka	HNO-1	300 X 400	400X400	-88.72	0.17	0.70	1.22	1.22
	HNO-3			-88.72	0.17	0.85	1.61	1.60
	HNO-4			-88.72	0.17	1.00	1.89	1.88
	HNO-5			-116.98	0.13	0.63	1.27	1.20
Watenaba	HINU-6	200 V 200	2007200	-110.98	0.13	0.81	1./1	1.01
w ataliabe	WI-3	200 A 300	300A300	-29.0	0.07	1.00	1 14	1.21
Watanabe	WI-6	200 X 300	300X300	-29.0	0.07	1.30	1.36	1.69
				27.0	Average	1.25	1.35	1.14
					Coefficient	0.20	0.22	0.25
					of variation	0.20	0.22	0.25

4.2 Comparison between the proposed equation and the existing equation

Shear strength estimation of the beam-column joint is compared with the results by existing joint design standards, such as ACI and AIJ, to verify whether evaluation by the method suggested in this study is appropriate or not. Shear strength based on the ACI Recommendations is computed by Eqn. 2.1, and the coefficient γ for a non-seismic hazard area (Type 1) is used in the calculation. In addition, the coefficient γ includes the effect that there are not transverse beams, except in Meinheit (1981) and Hatamoto (1961) specimens. The shear strength in the AIJ Guidelines is estimated by Eqn. 2.3, and k, which refers to the shape of the joint, is 1.0 for all specimens. φ is 1.0 for Meinheit and Hatamoto specimens that have transverse beams, and 0.85 for the others. b_j is defined according to Eqn. 2.5, and h_i also follows the AIJ Guidelines.

Shear strength estimations by the ACI Recommendations and the AIJ Guidelines that consider only concrete strength show a larger average of V_{test}/V_{cal} than that proposed method shows. The coefficient of variation is 22% for the ACI Recommendations and 25% for the AIJ Guidelines, both of higher value than 20% for proposed method. Therefore, it is concluded that the proposed equation makes a reasonable estimates of the shear strength of beam-column joints. Fig. 6(a) indicates the results of V_{test}/V_{cal} by the ACI Recommendations, the AIJ Guidelines, and the proposed equation for concrete compressive strength. The x-direction reinforcement index (Fig. 6(b)) is included to help visualize which method closely estimates shear strength against the experimental results. The three methods tend to evaluate shear strength as smaller than the experimental results presented, and Fig. 6 confirms that the proposed equation is appropriate for shear strength estimation of J-failure beam-column joints. In the suggested method, the effects of both the concrete compressive strut and the truss mechanism on the horizontal and vertical reinforcement are considered. Additionally, the influence of the reinforcement is taken into account in defining a softening factor, v which determines the effective compressive strength in the development of shear strength. Finally, the proposed method shows relatively close results to experimental values.



Figure 6. Result comparison of each equation

5. CONCLUSION

In this study, the shear strength is estimated for interior beam-column joints that fail in shear, before beam reinforcement yields (J-failure). A new equation for estimating shear capacity is suggested based on the arch and truss mechanisms, and the equation evaluates shear strength using the sum of stress by the arch and truss mechanisms. To verify the performance of the proposed model, 54 specimens from previous research are used. The proposed equation results in improved performance, when it is compared with the calculation results by the ACI Recommendations and the AIJ Guidelines. Each coefficient of variance is 20% for the proposed equation (Eqn. 3.14), 22% for the ACI

Recommendations (Eqn. 2.1), and 25% for the AIJ Guidelines (Eqn. 2.3). It is expected that this new equation could evaluate the strength and ductility of a beam-column joint that fails in bondage reasonably well, because the equation takes into account both the arch and truss mechanisms.

REFERENCES

- Joint ACI-ASCE Committee 352. (2003). Recommendations for Design of Beam-Column Connections in Monolithic Reinforced Concrete Structures. American Concrete Institute. Farmington hills, Michigan.
- AIJ Standard for Structural Calculation of Reinforced Concrete structures. (revised 2010). 179-190.
- NZS 3101: part 1. (1995). Concrete structures standard (NZS 3101:1995), Standard association of New Zealand, Willington, New Zealand.
- M. P. Nielsen and M. W. Braestrup. (1975). Plastic Shear Strength of Reinforced Concrete Beams. Bygningsstatiske Meddelelser. Denmark. Vol. 46, No. 3, 61–69.
- T. Paulay and M. J. N. Priestley. (1992). Seismic Design of Reinforced Concrete and Masonry Buildings. Jogn Wiley and Sons. New York. 250–263.
- T. T. C. Hsu. (1988). Softened Truss Model Theory for Shear and Torsion. ACI Structural Journal 85:6, 624-635.
- F. J. Vecchio and M. P. Collins. (1989). The Modified Compression-Field Theory for Reinforced Concrete Elements Subjected to Shear. ACI Structural Journal 83:2, 219-231.
- K. Hayashi and M. Teraoka. (1961). The Study for the Mechanical Properties of Reinforced Concrete +Type Beam-Column Joints. Architectural Institute of Japan. 117-118.
- K. Oka and H. Siohara. (1992). Tests of High-Strength Concrete Interior Beam-Column Joint Subassemblages. Earthquake Engineering. 3211–3217.
- D. F. Meinheit and J. O. Jirsa. (1981). Shear Strength of R/C Beam-Column Connections. Proceedings of the ASCE 107:ST11, 2227-2245.
- H. Noguchi and T. Kasiwazaki. (1992). Experimental Studies on Shear Performances of RC Interior Column-Beam Joints with High-Strength Materials. Earthquake Engineering. Tenth World Conference. 3163-3168.
- S. Fujii and S. Morita. (1991). Comparison between interior and exterior RC beam-column joint behavior. American Concrete Institute. SP-123, 145-166.
- K. Kawasaki et al. (1991). Experimental Study on the Shear Performance of R/C Interior Beam-Column Joints with Ultra High-Strength materials. Architectural Institute of Japan. 579-580.
- H. Hatamoto et al. (1961). Earthquake Resistant Design of a 30 Story Reinforced Concrete Building. Architectural Institute of Japan. 91-92.
- M. Teraoka, Y. Kanoh, S. Sasaki and K. Hayashi. (1996). An estimation of Ductility in Interior Beam-Column Subassemblages of Reinforced Concrete Frames. The Society of Materials Science 45:9, 1033-1041.
- K. Watanabe, K, Abe, J. Murakawa and H. Noguchi. (1988). Strength and Deformation of Reinforced Concrete Interior Beam-Column Joints. Transactions of the Japan Concrete InstituteVol.10, 183-188.