Ductility Reduction Factor for Multi-Degree-of-Freedom Systems with Soil-Structure Interaction

B. Ganjavi & H. Hao
School of Civil and Resource Engineering, The University of Western Australia,
35 Stirling Highway, Crawley, WA 6009, Australia.

SUMMARY
Most recent studies considered soil-structure interaction (SSI) in inelastic response analysis are mainly based on idealized structural models of SDOF systems. However, an SDOF system might not be able to well capture the SSI and structural response characteristics of real MDOF systems. In this paper, through a comprehensive parametric study of large numbers of MDOF and its equivalent SDOF (E-SDOF) systems subjected to an ensemble of 30 earthquake ground motions recorded on alluvium and soft soils effects of SSI on ductility reduction factor (DRF) of MDOF systems are investigated. It is concluded that generally SSI reduces the DRF of both MDOF and more intensively SDOF systems. However, depending on the number of stories, soil flexibility, aspect ratio and inelastic range of vibration the DRF of MDOF systems could be significantly different from that of E-SDOF systems. Simplified equation is proposed to estimate DRF of MDOF soil-structure systems for practical purpose.

***Keywords**: Ductility reduction factor, Soil-Structure Interaction, MDOF systems, nonlinear dynamic analysis*

1. INTRODUCTION

In strong earthquake ground motions, the design base shear strength recommended in seismic provisions are typically much lower than the base shear strength that are required to sustain the structure in the elastic range. The primary seismic design of buildings in most of the conventional seismic codes is based on force-based procedure. These codes permit structures to behave inelastically during moderate and severe earthquake. Strength reductions from the elastic strength demand are prevalently accounted for through the use of strength reduction factor, *R*, which is one of the most controversial issues in the seismic-resistant design provisions. This factor, strongly dependent on the energy dissipation capacity of the structural systems, is used to reduce the elastic design force spectra in earthquake-resistant design. For an idealized elasto-plastic SDOF system, *R* corresponds to the seismic force at the predefined design level and can be considered as a product of the conventional reduction factor R_μ, reflecting the nonlinear hysteric behavior in a structure, and R_ω that account for other reduction factors such as reductions due to element overstrength, redundancy, strain hardening and etc. During the past four decades, extensive studies have been conducted on R_μ. The pioneering investigations performed by Veletsos and Newmark (1960) and Newmark and Hall (1973) may be regarded as the first renowned studies on R_μ. Based on elastic and inelastic response spectra of NS component of El Centro earthquake as well as previous studies on SDOF systems to pulse-type excitations, Newmark and Hall (1973) proposed simplified expressions for R_μ as a function of target period and ductility ratio of the structure. In another study, based on mean inelastic spectra of 20 artificial ground motions compatible with the Newmark-Hall (1973) elastic design spectra, Lai and Biggs (1980) proposed alternative expressions as a function of also the target ductility, period as well as period ranges. Many more studies were made by researchers to propose simplified equations for strength reduction factor of fixed-base SDOF systems (Elghadamsi and Mohraz, 1978; Fischinger *et al*., 1994; Miranda and Bertero, 1994; Lam *et al*., 1998; Ordaz and Perez-Rocha, 1998; Karmakar and Gupta, 2007). Elghadamsi and Mohraz (1987) may be one of the first researchers who studied the influence of soil condition on R_μ. With further investigations Krawinkler and Rahnama (1992) and

Miranda (1993) demonstrated the significant effect of soil conditions, especially for the case of soft soils, on strength reduction factors. However, effect of SSI on R_μ has not been considered in their works. Recent studies on elastic and inelastic responses of SDOF soil-structure systems indicated that SSI could have significant effects on ductility demand of structures (Aviles and Perez-Rocha, 2003 and 2005; Ghannad and Jahankha, 2007; Mahsuli and Ghannad, 2009). Ghannad and Jahankha (2007) investigated the effect of site condition and SSI on R_μ of SDOF systems. They concluded that SSI reduces the R_μ values, especially for the case of buildings located on soft soils; therefore, using the fixed-base strength reduction factors for soil-structure systems lead to underestimation of seismic design forces. These studies are mainly based on the dynamic response of SDOF systems while real structures have MDOF and more realistic representation of real structures needs MDOF models. The relationship between MDOF and SDOF system responses of fixed-base systems was first studied by Veletsos and Vann (1971) by considering some shear-beam models with equal story masses connected by weightless springs in series from one degree of freedom (DOF) to five DOFs. They concluded that for systems having more than three DOFs the proposed design regulations for SDOF systems were not sufficiently accurate and could lead to non-conservative estimates of the required inelastic lateral strength, and that errors tended to increase as the number of degrees of freedom increased. Another study was conducted by Nassar and Krawinkler (1991) on three types of simplified fixed-base MDOF models to estimate the modifications required to the inelastic strength demands obtained from bilinear SDOF systems in order to limit the story ductility demand in the first story of the MDOF systems to a predefined value. They found that the deviation of MDOF story ductility demands from the SDOF target ductility ratios increased with structural vibration period and target ductility ratio. More examples of the works conducted on the subject can be found in the reference (Seneviratna and Krawinkler, 1997; Santa-Ana and Miranda, 2000; Moghaddam and Mohammadi, 2001). However, all of the works were performed on fixed-base systems, i.e. based on a presumed assumption that soil beneath the structure is rigid. In a more recent study, Ganjavi and Hao (2011) through intensive parametric calculations investigated the effect of SSI on the strength and ductility demands of MDOF systems as well as its equivalent SDOF models considering both elastic and inelastic behaviours and concluded that the common SDOF systems might not lead to accurate estimation of the strength and ductility demands of MDOF soil-structure systems, especially for the cases of mid- and high-rise buildings, due to the significant contributions from high vibration modes.

In the present study, an intensive parametric study has been performed to investigate the effects of SSI on R_μ values of MDOF and its equivalent SDOF (E-SDOF) systems using simplified soil-structure model for shallow foundations. This is carried out for a wide range of structural dynamic characteristics and non-dimensional key parameters to investigate the relationship between R_μ values of MDOF and SDOF soil-structure systems.

2. SOIL- FOUNDATION-STRUCTUR MODEL

2.1. Specifications of Superstructure Models

To model MDOF systems the well-known shear-beam model is utilized in this study. In the MDOF shear-building models utilized in the present study, each floor is assumed as a lumped mass to be connected by elasto-plastic springs. Story heights are 3 m and total structural mass is considered as uniformly distributed along the height of the structure. A bilinear elasto-plastic model with 2% strain hardening in the force-displacement relationship is used to represent the hysteretic response of story lateral stiffness. This model is selected to represent the behaviour of non-deteriorating steel-framed structures of different heights. In all MDOF models, lateral story stiffness is assumed as proportional to story shear strength distributed over the height of the structure in accordance with the 2009 IBC load pattern (IBC, 2009). Five percent Rayleigh damping was assigned to the first mode and the mode in which the cumulative mass participation was at least 95%. For each MDOF building an E-SDOF is introduced. The properties of these E-SDOF systems are set such that the mass of the SDOF system is the same as the total mass of the MDOF building; similarly, the period of vibration, damping ratio and effective height of the E-SDOF systems are the same as the corresponding fundamental mode of the MDOF building.

2.2. Soil-Structure Model

The soil-foundation element is modelled by an equivalent linear discrete model based on the cone model with frequency-dependent coefficients and equivalent linear elastic properties (Wolf, 1994). Cone model based on the one-dimensional wave propagation theory represents circular rigid foundation with mass m_f and area moment of inertia I_f resting on a homogeneous half-space. The simplified cone model can be used with sufficient accuracy in engineering practice (Wolf, 1994). A typical MDOF soil-structure system and the corresponding E-SDOF system are shown in Fig. 1. The sway and rocking DOFs are defined for translational and rotational motions of the foundation, while the vertical and torsional movement of the foundation are neglected. The stiffness and energy dissipation of the supporting soil are modelled by springs and dashpot, respectively. Soil material damping is assumed as commonly used viscous damping so that more intricacies in time-domain analysis are avoided. All coefficients of springs and dashpots for sway and rocking motions used to define the soil-foundation model in Fig. 1 are summarized as follows:

$$k_h = \frac{8\rho v_s^2 r}{2-\upsilon}, \qquad c_h = \rho v_s A_f \ , \ k_\varphi = \frac{8\rho v_s^2 r^3}{3(1-\upsilon)}, \qquad c_\varphi = \rho v_p I_f \tag{1}$$

where k_h, c_h, k_φ and c_φ are sway stiffness, sway viscous damping, rocking stiffness, and rocking viscous damping, respectively. Equivalent radius and area of cylindrical foundation are denoted by r and A_f . Besides, ρ, υ, v_p and v_s are respectively the specific mass density, Poisson's ratio, dilatational and shear wave velocity of soil. To consider the soil material damping, ζ_0, in the soil-foundation element, each spring and dashpot is respectively augmented with an additional parallel connected dashpot and mass.

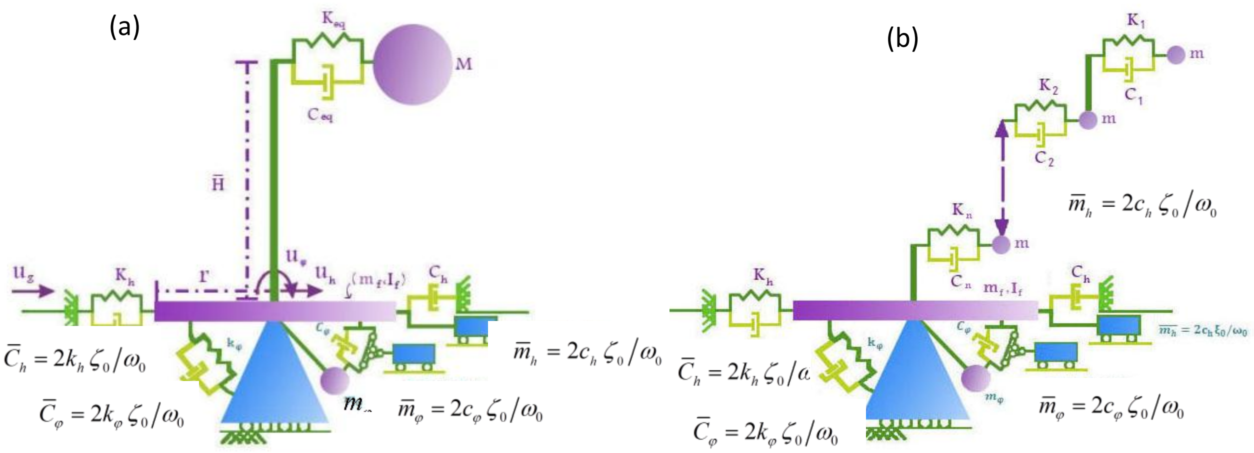

Figure 1: Soil-structure models for sway and rocking motions (a) E-SDOF system (b) Typical MDOF system

2.3. Key Parameters and Selected Earthquake Ground Motions

For a specific earthquake ground motion, the dynamic response of the structure can be interpreted based on the property of the superstructure relative to its underlying soil. It has been shown that the effect of these factors can be best described by some dimensionless parameters (Veletsos, 1977). In this study, dimensionless frequency $a_0 = \omega_{fix} \bar{H} / v_s$, and aspect ratio $\bar{H}/r$, are two factors participating with high powers in the equation of motion, and thus are considered as the key parameters which define the main SSI effect. ω_{fix} is the natural frequency of the fixed-base structure; v_s is the shear wave velocity of soil; r is the equivalent foundation radius, and $\bar{H}$ is the effective height of the structure. Other parameters, having less importance, may be set to some typical values for conventional buildings (Veletsos and Meek, 1974; Wolf, 1994). In the present study, the foundation mass ratio is assumed to be 0.1 of the total mass of the MDOF buildings. The Poisson's ratio is considered to be 0.4 for the alluvium soil and 0.45 for the soft soil. Also, a damping ratio of 5% is assigned to the soil material. An ensemble of 30 earthquake ground motions with different characteristics recorded on alluvium and soft soil deposits (soil type *C*, with shear wave velocity between 180 and 360 *m/s*, and *D*, with shear wave velocity lower than 180 *m/s*, based on the USGS site classification) are compiled and utilized in the nonlinear dynamic time history analyses. All selected ground motions are obtained from earthquakes with magnitude greater than 6 having closest distance to fault rupture more than 15 km without pulse type characteristics.

3. METHODOLOGY AND PROCEDURE FOR ANALYSIS

The adopted soil-structure models introduced in the previous sections are used directly in the time domain for nonlinear dynamic analysis. Step-by-step solution scheme in which dynamic imposed loads are incrementally applied to the model of the structure is utilized for all MDOF and E-SDOF models. Variable load increments by considering events within steps are defined in order to control the equilibrium errors in each analysis step. An event is considered as any kind of state change that causes a change in the structural stiffness. To conduct parametric studies for both MDOF and SDOF systems with consideration of SSI effects subjected to a given earthquake ground motion, a computer program, "*OPTSSI"*, has been written specifically for this study. The software has the capabilities of computing many parameters such as elastic and inelastic strength demand, maximum drift, residual drift, strength reduction factors, MDOF modifying factor as well as optimization based on uniform damage distribution over the height of the structure. Many verification processes have been conducted, and the results have been compared with those generated by OPENSEES. A series of 3-, 5-, 7-, 10-, 15- and 20-story MDOF shear buildings and also their equivalent SDOF models are considered to investigate the effect of SSI on strength reduction factors of both MDOF and E-SDOF systems. In this regard, for a given earthquake ground motion, a large family of different soil-structure models including MDOF as well as E-SDOF models and various predefined key parameters are considered. This includes MDOF and E-SDOF models with 30 fundamental periods of the corresponding fixed-base structures, ranging from 0.1 to 3 sec with intervals of 0.1, three values of aspect ratio ($\bar{H}/r$ =1, 3, 5), four values of dimensionless frequency (a_0 = 0, 1, 2, 3), and five values of target interstory displacement ductility ratio (μ_t =1, 2, 4, 6, 8) where μ_t =1 corresponds to the elastic state. It should be noted that the range of the fundamental period and aspect ratio, considered in the present study, are wider than those of the most practical structures. They are considered here, however, to cover all possible conditions and to compare the results obtained from MDOF systems of different number of stories with those obtained from their equivalent SDOF systems. For each earthquake ground motion, the total normalized elastic and inelastic shear strength of the MDOF and E-SDOF system are computed by a proposed iterative procedure in order to reach the μ_t in the structure, as a part of the soil–structure system, within a 0.5% error. Total normalized shear strength is defined as the total shear strength demands divided by the total structural mass and then normalized to the peak ground acceleration (PGA). Therefore, strength reduction factors of both MDOF and E-SDOF soil-structure models can be computed by dividing the elastic shear strength to the inelastic shear strength corresponding to the presumed target ductility ratio. In contrary to SDOF systems, strength demands of an MDOF system are also dependent on the presumed design lateral load distribution. In other words, considering the same total base shear strength demand any predefined lateral load distribution may change the amount of maximum ductility ratio ($\mu_{\max}$). Therefore, for an MDOF system ductility demand for each story needs to be computed and the greatest value among all stories is then considered as the ductility demand of the MDOF system.

4. EFFECT OF SSI ON DUCTILITY REDUCTION FACTOR

4.1. Effect of Number Of Stories and Dimensionless Frequency

To study the effect of number of stories and dimensionless frequency on DRF (R_μ) for fixed-base and flexible-base structures, shear buildings of 3, 5, 10, 15 and 20 stories as well as the corresponding E-SDOF systems are considered which represent the common building structures from low- to high-rise models. Results illustrated in Fig. 2 are mean values to 30 earthquake ground motions for systems with $\bar{H}/r$ = 3, corresponding to two ductility ratios (μ_t =2, 6) representing respectively low and high inelastic behaviours, and soil-structure system with two dimensionless frequencies (a_0 = 1 and 3), as well as the fixed-base structures. As stated before, a_0 is an index for the structure-to-soil stiffness ratio controlling the severity of SSI effects, and also the value of 3 for this parameter corresponds to significant SSI effect. It is observed that for fixed-base systems, regardless of the level of nonlinearity, increasing the number of DOFs (stories) always results in a reduction in the averaged values of R_μ. For soil-structure systems, the effect of the number of stories are, however, very different from the fixed-base models. For the cases with significant SSI effect, R_μ spectra become less sensitive to the variation of the number of stories. This is more apparent in cases with low level of inelasticity. In addition, an interesting point can be observed for the case of E-SDOF soil-structure systems with severe SSI effect (a_0 = 3) in which R_μ values are significantly lower than those of the MDOF systems in almost all ranges of period. Therefore, it can be concluded that the modifying factors for DRFs of MDOF soil-structure systems could be completely different from those of the fixed-base systems. For fixed-base structures, it has been proposed to multiply R_μ of SDOF systems by a modifying factor that takes into account the possible concentration of displacement ductility demands in specific floors (Miranda, 1997; Santa-Ana and Miranda, 2000) for use of the reduction factor in seismic analysis of MDOF systems. This factor was defined by Santa-Ana and Miranda (2000) for fixed-base systems as:

$$R_M = \frac{V_{SDOF}(\mu = \mu_i)}{V_{MDOF}(\mu = \mu_i)} \tag{2}$$

where V_{SDOF} and V_{MDOF} are the strength demands of SDOF and MDOF systems subjected to a given ground motion and presumed target ductility demand, respectively. Also R_M, represents a modification factor to the DRF of SDOF systems so it can be applied to MDOF structures. Therefore, the DRF of MDOF systems ($R_{\mu(MDOF)}$) can be computed from the following equation:

$$R_{\mu(MDOF)} = R_{\mu(SDOF)} . R_M \tag{3}$$

As seen, this modification factor just considers the difference between the inelastic demands of MDOF and the corresponding SDOF systems. Santa-Ana and Miranda (2000) and Moghaddam and Mohammadi (2001) showed that for fixed-base systems the values of this factor are approximately equal to one regardless of the number of stories. This means that for μ =1 the lateral strength of the MDOF systems is, on average, nearly equal to that of the SDOF system. However, results of this study indicate that this finding is not correct for soil-structure systems. To show the importance of this problem, the averaged ratios of strength demands on MDOF to those on E-SDOF systems for different ranges of nonlinearity are computed and the results are depicted in Figure 3 for both the fixed-base and soil-structure systems of a 10-story building.

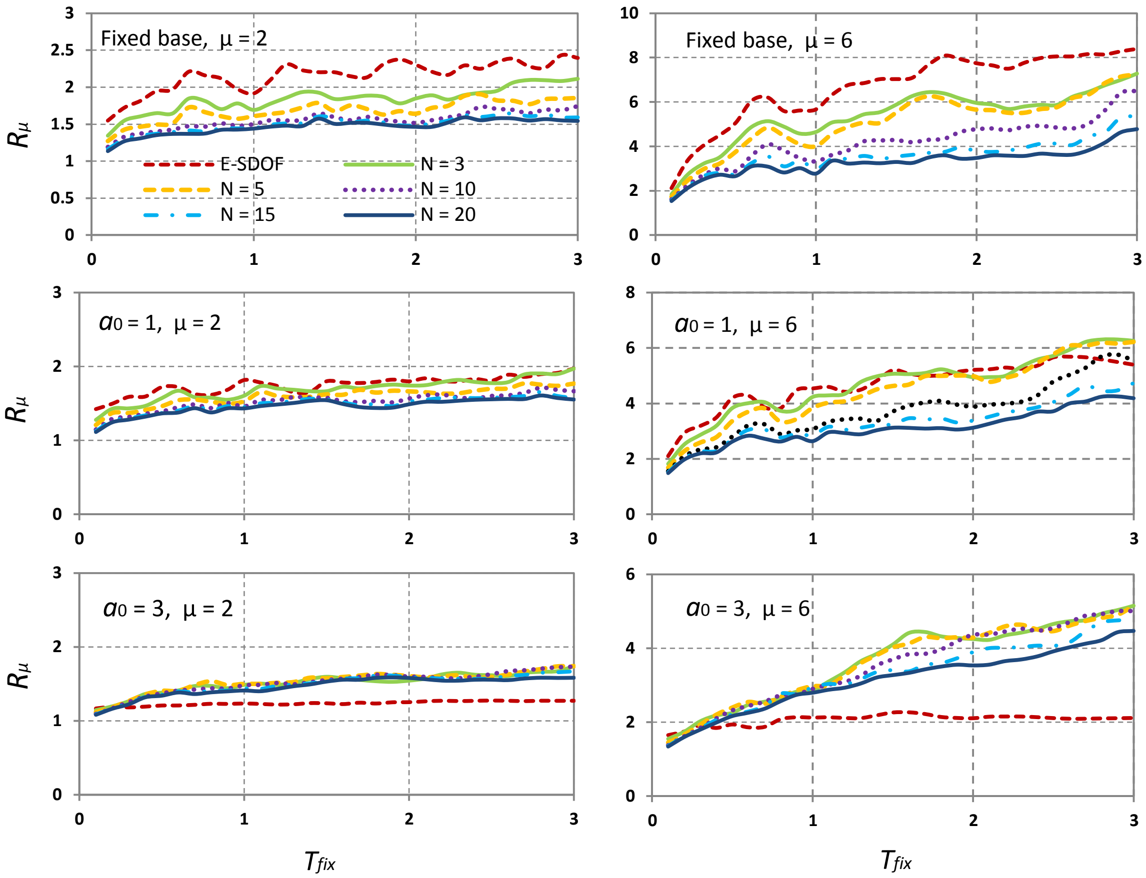

Figure 2: Effect of the number of stories on averaged strength reduction factor spectra of fixed-base and soil-structure systems ($\bar{H}/r$ = 3)

As seen, different from the fixed-base systems, the ratios of strength demands in elastic range of response (i.e., μ =1) is significant for soil-structure systems. In fact, in elastic range of response the ratios remarkably increase with SSI effect such that the more SSI effect (larger values of a_0), the more significant difference between the strength demands of MDOF and SDOF systems. As an instance, for the structure with long period of vibration, the value of this ratio can be greater than 5 when SSI effect is predominant while it is about 1.3 for the fixed-base system. Results of this study show that this phenomenon is more pronounced as the value of aspect ratio ($\bar{H}/r$) increases. For inelastic range of response, however, the effect of SSI becomes less important in a way that in high level of inelasticity the averaged ratios of strength demands are approximately insensitive and thus independent of the soil flexibility. It can be concluded that for soil-structure systems the values of both elastic and inelastic strength demands must be taken into account for calculation of the modification factor to the strength reduction factor. Therefore, the modification factor for soil-structure systems or for more precise analyses of fixed-base systems should be defined as:

$$\tilde{R}_M = \frac{R_{\mu(MDOF)}}{R_{\mu(SDOF)}} \tag{4}$$

where $\tilde{R}_M$ represents a modification factor to the strength reduction factor of SDOF systems so it can be applied to both MDOF fixe-base and soil-structure systems. To parametrically examine this modification factor for both fixed-base and soil-structure systems, results for a 10-story building with three levels of nonlinearity (μ_t = 2, 4, 8) corresponding to three values of dimensionless frequency (a_0 = 1, 2, 3) as well as the fixed-base structures with aspect ratio of 3 are shown in Fig. 4. It is seen that, regardless of the level of nonlinearity, the values of $\tilde{R}_M$ are generally less than one for the case of fixed-base systems. However, these factors increase as SSI effect increases (i.e. increasing the amount of a_0). It is also obvious that different from the fixed–base systems, $\tilde{R}_M$ values are sensitive to the level of nonlinearity for soil-structure systems such that they increase with ductility ratio and are generally larger than one especially for the structures with longer periods and severe SSI effects. As an example, for the case with high level of inelasticity (μ_t =8) and fundamental period of 2 sec, the values of $\tilde{R}_M$ are 0.78, 0.94, 1.71 and 2.4 for fixed-base, and soil-structure system with a_0 = 1, a_0 = 2, and a_0 = 3, respectively. As discussed above (Figure 3), the large differences among the $\tilde{R}_M$ values are caused by the large difference between the values of elastic strength demands of soil-structure systems and fixed-base models.

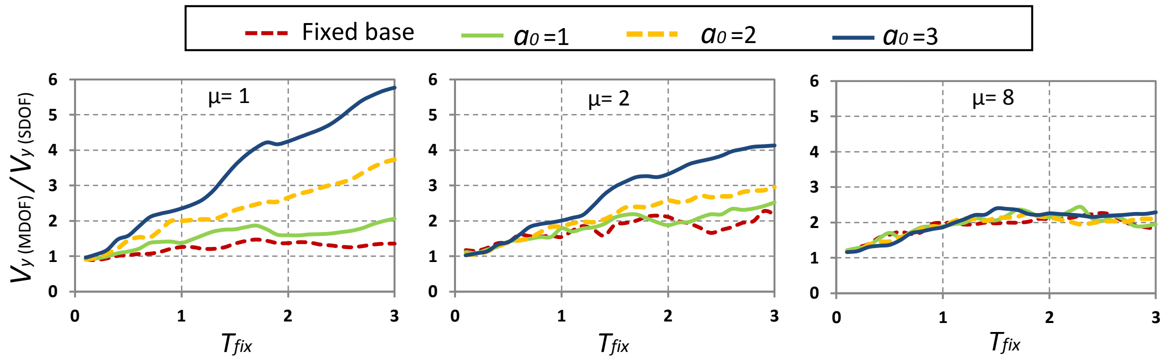

Figure 3: Averaged ratios of shear strength demands on MDOF system to those on E-SDOF system for different ranges of nonlinearity (10-story building; $\bar{H}/r$ = 3)

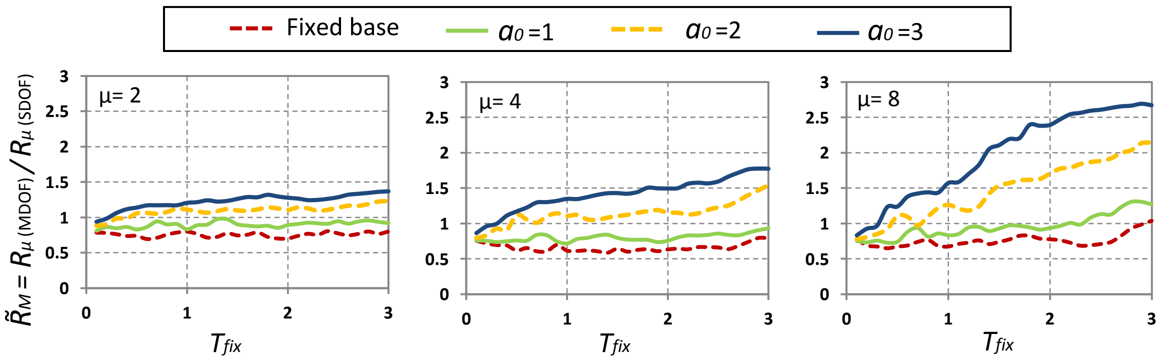

Figure 4: Averaged modifying factor for MDOF fixed-base and soil-structure systems (N= 10; $\bar{H}/r$ = 3)

4.2. Effect of Aspect Ratio

In order to examine the effect of aspect ratio on DRF of MDOF-soil structure systems a 10-story building with three values of aspect ratio ($\bar{H}/r$ = 1, 3, 5) and with three ductility ratios (μ_t = 2, 4, 8) as well as two dimensionless frequencies (a_0 = 1, 3) is considered and analyzed subjected to the selected ground motions. The results are plotted in Fig. 5. It is clear that for the case of less SSI effect, the values of averaged R_μ are insensitive to the variation of aspect ratio but significant for SDOF systems as reported by Ghannad and Jahankhah (2007). For the case with severe SSI effect and high inelastic response, except in short period ranges, the values of mean R_μ increase with the aspect ratio, which is completely different from the results obtained for the SDOF system by Ghannad and Jahankhah (2007) , where increasing the aspect ratio is always accompanied by decreasing the R_μ values. This finding indicates that SSI affects the strength reduction factors of MDOF and E-SDOF systems in a different manner. The same results have been observed in this study for MDOF soil-structure systems with different number of stories.

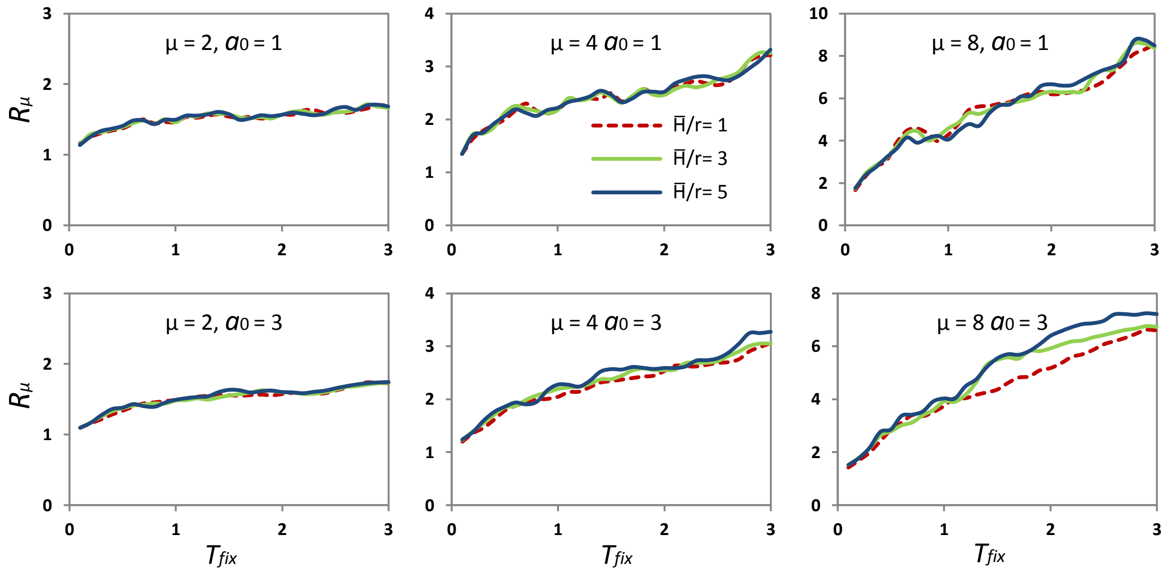

Figure 5: Effect of aspect ratio on averaged strength reduction factor spectra of MDOF soil-structure systems (10-story building)

5. ESTIMATION OF THE DUCTILITY REDUCTION FACTORS FOR MDOF SOIL-STRUCTURE SYSTEMS

In earthquake-resistant design and, in general, for practical purpose it is desirable to have a simplified expression to estimate strength reduction factors of MDOF systems. Here, based on nonlinear dynamic analyses of 10800 MDOF soil-structure systems the following simple equation is proposed:

$$R_{\mu(MDOF)} = a_i T_{fix}^{b_i} \tag{5}$$

where T_{fix} is the fundamental period of the corresponding fix-based structure; a_i and b_i are constants depending on the interstory displacement ductility ratio, number of stories, aspect ratio, and dimensionless frequency and can be obtained from the Tables reported by Ganjavi and Hao (2012). To show the capability of the proposed equation in estimating the DRF for MDOF soil-structure systems Fig. 6 is provided. This figure shows the comparison of the proposed equation in predicting the DRFs of 5- and 20-story buildings with different ranges of nonlinearity obtained from Eq. (5) with the averaged numerical results. As seen, there is a good agreement between Eq. (5) and the averaged numerical results for DRFs of MDOF soil-structure systems.

6. CONCLUSIONS

An intensive parametric study has been performed to investigate the effect of SSI on ductility reduction factor for E-SDOF and MDOF fixed-base and soil-structure systems. The results of this study are summarized in the following:

- For fixed-base MDOF systems, regardless of the level of nonlinearity, increasing the number of DOFs (stories) always reduces the averaged values of R_μ. This phenomenon is more pronounced for low- to mid-rise buildings. However, for soil-structure systems, as SSI effect becomes more significant, R_μ spectra become less sensitive to the number of stories, especially in the low inelastic response range.
- With severe SSI effect the R_μ values of E-SDOF systems are significantly lower than those of the MDOF systems in almost all ranges of periods. The MDOF modifying factors for strength

reduction factors of soil-structure systems could be completely different from those of fixed-base systems. The more significant is the SSI effect, the more difference between the elastic strength demands of MDOF and SDOF systems. The phenomenon is more pronounced as aspect ratio ($\bar{H}/r$) increases. A new modification factor ($\tilde{R}_M$) for soil-structure and fixed-base systems that account for both elastic and inelastic strength demands has been introduced.

- MDOF modification factor values are sensitive to the level of nonlinearity for soil-structure systems such that they increase with ductility ratio and are generally larger than one especially for structures with long periods and severe SSI effects.
- For the case with severe SSI effect and high inelastic response, except for short period ranges, the values of mean R_μ increase with the aspect ratio, which is completely different from the SDOF results in which increasing the aspect ratio is always accompanied by decreasing the R_μ values, indicating the SSI can affect strength reduction factors of MDOF and E-SDOF systems in a different manner.
- A new simplified equation which is functions of fixed-base fundamental period, ductility ratio, the number of stories, aspect ratio and dimensionless frequency has been proposed to estimate the strength reduction factors of MDOF soil-structure systems.

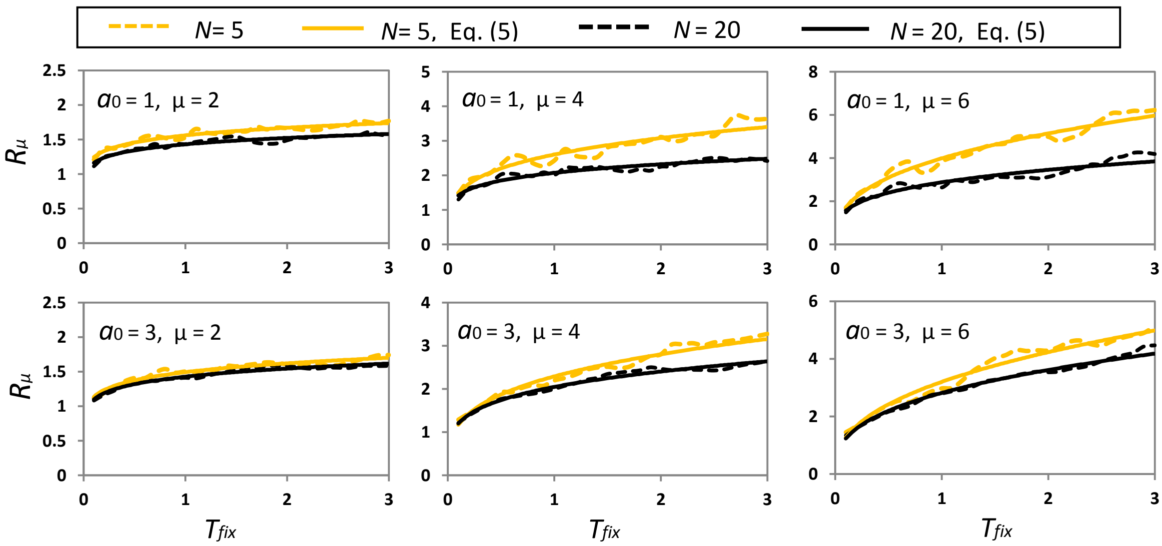

Figure 6: Correlation between Eq. (5) and averaged numerical results for strength reduction factors of MDOF soil-structure systems ($\bar{H}/r$ = 3)

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