Modeling Lateral Strength Degradation of Reinforced Concrete Columns

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SUMMARY:
A strength degradation model was implemented in the analytical software OpenSEES to simulate the lateral strength degradation of reinforced columns with limited ductility. During analyses, the model is capable of dynamically monitoring key column forces and deformations for critical combinations that trigger column strength degradation. Once triggered, the degrading behavior is simulated using a nonlinear constitutive model. The proposed model is implemented in two formats: 1) manual calibration, and 2) automatic calibration. In the manual calibration format, the user can define a limiting column deformation and a limiting shear force. Once the first of these limiting values is reached, a degrading lateral strength behavior is triggered. Strength degradation is enforced through a shear spring that is placed in series with a flexural beam-column element. In its general form, the proposed model can be used with any beam-column element that exhibits lateral strength degradation. It was however conceived with the behavior of shear-critical reinforced concrete columns in mind. In the automatic calibration format, the proposed model behaves as in the manual format except that it only requires user input of column materials and geometric properties to automatically calibrate. The model is capable of determining the limiting shear force and deformation that initiate lateral strength degradation. Once degradation is triggered, the model is capable of estimating all the necessary damage parameters that govern the cyclic behavior of the element to loss of lateral strength. To-date, the model has only been calibrated to the behavior of flexure-shear-critical reinforced concrete columns; i.e., columns that yield in flexure prior to sustaining shear failures under lateral loading. The proposed model provides the engineering community with: 1) a versatile, highly customizable tool for simulating lateral strength degradation in beam-column elements through the manual calibration format; and 2) a user-friendly, automatically calibrating analytical tool that can be used in nonlinear dynamic analysis of reinforced concrete columns that are at risk of severe damage and collapse, through the automatic calibration format.

Keywords: reinforced concrete, columns, analytical model, shear, degradation

1. INTRODUCTION

Numerous reinforced concrete (RC) buildings that are vulnerable to severe damage and collapse during seismic events were constructed prior to the 1970s in areas of high seismicity. As damage progresses in a structure during a seismic event, significant load redistributions occur between failing elements and the surrounding elements. The challenge for structural engineers is to accurately determine whether an existing structure can satisfy life-safety or collapse-prevention performance objectives after a severe earthquake; a task that is highly dependent on how well the degrading behavior of failing elements and the associated load redistributions can be simulated.

Several analytical models have been proposed that estimate either a critical shear strength or lateral deformation that triggers lateral-strength degradation in RC column (e.g., (Elwood and Moehle, 2005, Ghannoum and Moehle, 2011, Kato and Ohnishi, 2002, Priestley et al., 1994, Pujol et al., 1999, Sasani, 2007, Sezen and Moehle, 2006, Haselton et al., 2008, Liel, 2008). A limited number of these models have also considered the ensuing degrading behavior (e.g., (Haselton et al., 2008, Liel, 2008, Elwood and Moehle, 2005)). For computational efficiency, line elements are typically chosen in conjunction with such models. Models in the literature also require user calibration of most parameters prior to running an analysis and cannot adjust response due to varying boundary conditions. During seismic excitation however, beam yielding and overturning effects can significantly alter the boundary conditions applied to a column and often lead to significant departures from the initial conditions on which pre-analysis calibration was based. Given that shear strength of RC columns depends not only on column material and geometric properties but also on loading conditions and deformations, there is a critical need for an analytical model that can adjust column behavior as boundary conditions and loading vary. Moreover, a large number of damage parameters need to be defined when simulating the cyclic strength degradation of RC columns. Most available models do not provide sufficient guidance for simulating the degrading lateral-strength of RC columns. The proposed model was developed to addresses such key shortcomings of line-element-based models.

An analytical model was developed that is capable of simulating the cyclic degradation in lateral strength of RC columns prone to shear failure. The model was implemented in the analytical software OpenSEES ((McKenna et al., 2000)). During analyses, the model is capable of dynamically monitoring key column forces and deformations for critical combinations that trigger column strength degradation. Once triggered, the degrading shear behavior is simulated using a nonlinear constitutive model. The proposed model is implemented in two formats: 1) manual calibration, and 2) automatic calibration.

In the manual calibration format, the user can define a limiting column deformation and a limiting shear force. Once the first of these limiting values is reached, a degrading lateral strength behavior is triggered. Strength degradation is enforced through a shear spring that is placed in series with a flexural beam-column element. Several damage algorithms were implemented to provide users with the necessary tools to simulate a wide range of structural elements. Parameters that govern the damage algorithms are also user-defined in the manual calibration format. In its general form, the manually calibrated model can be used with any beam-column element that exhibits lateral-strength degradation. It was however conceived with the behavior of shear-critical reinforced concrete columns in mind.

In the automatic calibration format, the proposed model behaves as in the manual format except that it only requires user input of column materials and geometric properties to automatically calibrate. The model is capable of determining the limiting shear force and deformation that initiate lateral strength degradation. Once degradation is triggered, the model is capable of estimating all the necessary damage parameters that govern the cyclic behavior of the element to loss of lateral strength. To-date, the model has only been calibrated to the behavior of flexure-shear-critical RC columns; i.e., RC columns that yield in flexure prior to sustaining shear failures under lateral loading. In developing the model, a database of RC columns was compiled. Parameters extracted from database column-tests were scrutinized for trends and regression models relating damage parameters to column physical properties and boundary conditions were produced. The regression models were implemented in the proposed analytical model and form the basis for the automatic calibration capabilities.

2. ANALYTICAL MODEL – MANUAL CALIBRATION

The proposed model dynamically monitors beam-column elements for conditions that trigger degradation. It detects the initiation of lateral-strength degradation when user-defined limiting forces or deformations are reached. Upon initiation of the degrading behavior, a zero-length shear spring connected in series with a beam-column flexural element changes its constitutive properties to include pinching, strength degradation, and stiffness degradation.

The model was implemented through a zero-length shear spring for computational efficiency and for compatibility with the most commonly used line-element analytical framework. The proposed model addresses several shortcomings of shear-spring type models, namely: 1) it has the ability to continually monitor forces and deformations in flexural elements for conditions that trigger shear failure, 2) it has a built-in function that compensates for flexural deformation offsets that arise from the degrading behavior in shear springs, and 3) it is able to trigger shear failure through either a limiting shear force or element deformations (whichever is reached first). In addition, the proposed model introduces several functionalities that give users a high degree of control over strength degradation triggering and the ensuing cyclic degrading behavior. Damage functions were developed to control the degrading behavior and include pinching, reloading stiffness degradation, and strength degradation. The rate of damage accumulation can be controlled by cyclic, energy, and displacement damage algorithms. Several damage functions and damage accumulation algorithms were introduced such that the proposed model is able to simulate the behavior of a wide range of elements exhibiting lateral strength degradation. More details about the model can be found in (LeBorgne, 2012).

2.1. Triggering Lateral Strength Degradation

The proposed model monitors the difference in rotation between two user-defined nodes to trigger the degrading behavior of the zero-length shear spring; when the user-defined critical rotation value is reached. For RC columns that yield in flexure prior to failing in shear, nodes should be chosen to bracket the plastic hinge regions adjacent to the shear springs. The model is compatible with any flexural element (e.g., lumped plasticity, fiber-section elements). However, fiber-section based elements are recommended for use with the proposed model as they typically produce higher accuracy than lumped plasticity elements; particularly under varying axial loads and boundary conditions (Ghannoum and Moehle, 2012). It is necessary for rotation-tracking to introduce intermediate nodes in flexural elements at a distance from element ends the user deems sufficient to capture plastic rotations (Fig. 1). Alternatively, the model can monitor the drift between two user-defined nodes and trigger degradation at a user-defined critical drift value.

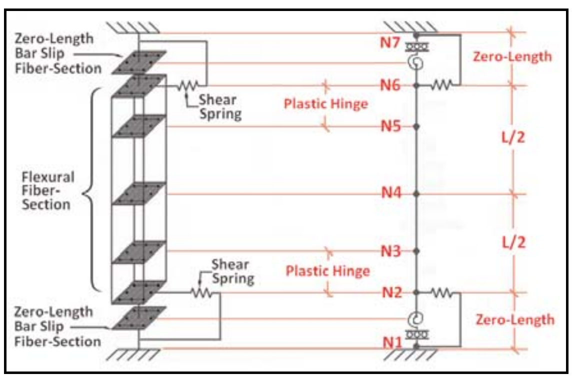

Figure 1. Analytical fiber-section and lumped plasticity models with zero-length shear springs in series (adapted from (LeBorgne, 2012))

2.2. Nonlinear Degrading Material Model

The nonlinear degrading material controls the deformation response of the zero-length shear spring (Fig. 2). Prior to the initiation of strength degradation, the material has a user-defined elastic slope ($K_{elastic}$). After degradation is triggered, a backbone is defined to limit the shear at a given deformation through a degrading slope K_{deg} and residual shear strength V_r. At load reversal, a tri-linear pinching curve is generated using six pinching factors. The behavior during reloading can be adjusted by stiffness and strength damage algorithms.

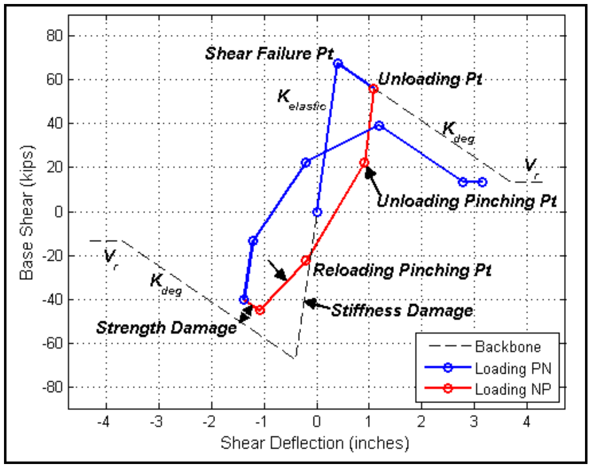

Figure 2. Nonlinear constitutive model governing the zero-length shear springs

The shear spring controls pinching behavior using three user-defined pinching factors for each loading direction. Upon load reversal (A_t, *A'*, *A* in Fig. 3), the points that form the tri-linear pinching segments are evaluated. The initial unloading stiffness is set equal to the elastic stiffness. Pinching point coordinates at *B'* and *C'* (Fig. 3) are defined as fractions of the unloading point *A'*. Based on experimental evidence from RC column tests exhibiting shear failures, once a certain loss in shear capacity is experienced (A_t), the column cannot exceed that capacity during reloading (D_t). Hence, once the unloading pinching point (*A'*) is reached, the reloading stiffness of the shear spring is automatically adjusted to ensure that the shear at backbone contact (*D'* in Fig. 3) is at or below the point which unloading started (in absolute value).

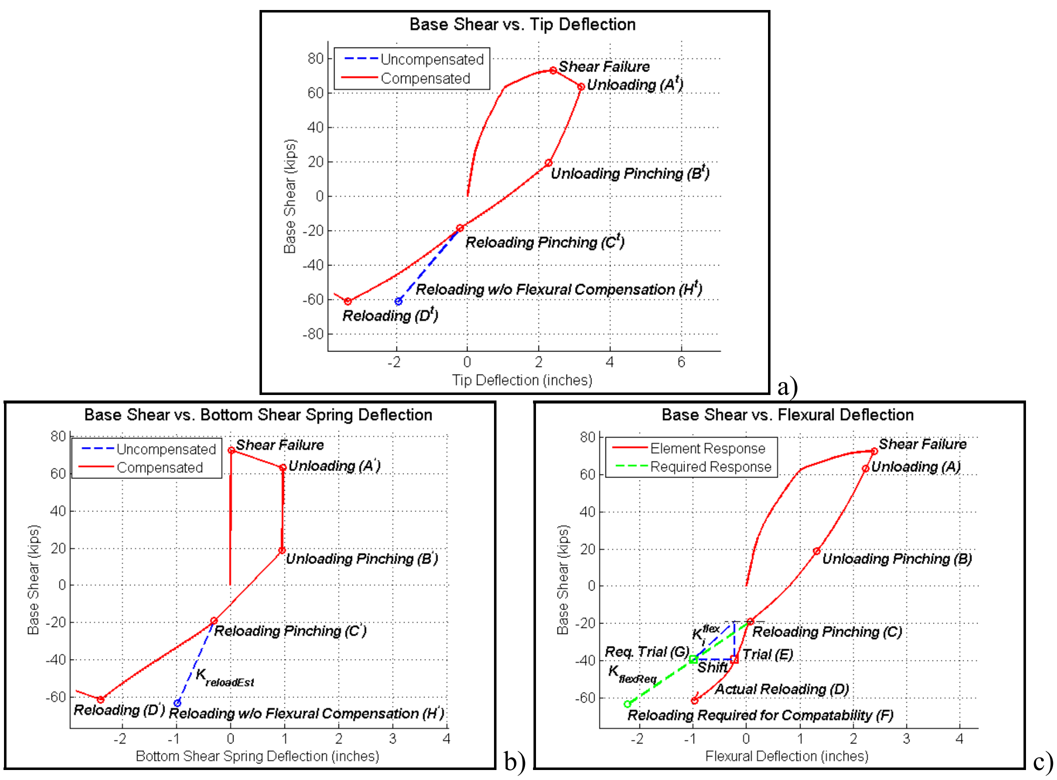

Figure 3. Illustration of degrading behavior. a) Element total deformation measured at tip (e.g., at Node 7 in Fig. 1), b) shear spring response, c) element flexural response (e.g., measured between nodes N2 and N7 in Fig. 1 (adapted from (LeBorgne, 2012))

An important limitation in current shear-spring-based degrading models is that a symmetrically defined degrading shear model (symmetric in both loading directions) can result in an unsymmetrical element behavior. Differences between the unloading and reloading stiffnesses of the flexural element cause such an effect (Fig. 3). To remedy that limitation, a flexural compensation algorithm was implemented in the shear spring constitutive model. The model monitors the flexural deformations of flexural element and automatically adjusts the shear-spring reloading stiffness (from H' to D' in Fig. 3 b)) to obtain the desired symmetric global behavior. More detail on the flexural compensation algorithm can be found in (LeBorgne, 2012).

Stiffness damage algorithms were introduced to reduce the reloading slope at each half cycle. A cyclic damage algorithm was implemented that has the ability to account for damage due to repeated loading at the same deformation and does not require knowing the displacement or monotonic energy at collapse. The user-defined cyclic damage index has a range of zero to unity where unity provides the most aggressive stiffness degradation at each half cycle. Reductions in reloading stiffness are applied by multiplying unity minus the cyclic damage index by the reloading stiffness of the previous half-cycle. In addition to cyclic damage, displacement- and energy-based damage algorithms were implemented using a more general form of the (Park and Ang, 1985) damage model as proposed by (Mitra and Lowes, 2007):

$$D_{stiffness} = \alpha_1 \cdot \left(\frac{\delta_{max}}{\delta_{collapse}} \right)^{\alpha_3} + \alpha_2 \cdot \left(\frac{E_i}{E_{mono}} \right)^{\alpha_4} \le D_{lim} \quad \text{(Eq. 1)}$$

Where:

$$E_i = \int_0^{\delta_{max}} V \, d\delta \text{ and } D_{stiffness} \le 1$$

The reloading stiffness is a product of one minus the damage index ($D_{stiffness}$) and the initial reloading stiffness. A damage limit (D_{lim}) is user-defined to ensure the damage index does not increase above a threshold value. δ_{max} is the maximum deformation previously reached by the spring element. The user defined parameters α1 through α4 govern the rate of damage accumulation. The energy and displacement damage equation requires the estimation of the deformation at axial collapse ($\delta_{collapse}$) prior to initiating the analysis. When degradation is triggered, the area under the backbone of the shear spring is computed. E_{mono} is the computed area under the backbone curve up the deformation the residual shear strength V_r is reached. During analysis, the nonlinear material calculates the accumulated energy (E_i) by integrating the area under the shear (V) versus lateral drift (δ) curve; but only during backbone contact.

Strength damage is applied by shifting the shear-force intercept of the backbone once the reloading pinching point is reached (Fig. 2). Damage algorithms using cyclic, energy, and displacement measures were implemented. Cyclic strength degradation is enforced by subtracting from the shear intercept the product of the shear at unloading and the user-defined damage factor. The cyclic damage factor has a range of zero to unity where unity would provide the most aggressive strength degradation at each half cycle. The displacement and energy based damage index is computed for strength damage in the same fashion as the reloading stiffness damage using the (Mitra and Lowes, 2007) damage model. The Mitra and Lowes damage model is applied by multiplying one minus the damage index by the shear intercept of the shear-spring backbone. Once the material shear force degrades below the residual shear strength, strength degradation damage is terminated.

2.3. Illustration of Model Capabilities

The proposed lateral strength degradation model was fitted to a variety of quasi-static cyclic tests of RC columns. The flexural element used in series with the proposed zero-length shear spring model was based on the fiber-section flexural representation proposed by (Ghannoum and Moehle, 2012). The fiber-section representation consisted of a beam-column element discretized using fiber-sections and coupled with zero-length fiber-sections that account for longitudinal bar-slip from anchorage regions adjacent to the columns (illustrated in Fig. 1). The fiber-section representation was used as it can simulate the flexural behavior of column element more accurately than the lumped-plasticity representation.

The column data used to highlight model capabilities were taken for columns that sustained flexural yielding followed by shear-strength degradation and eventually axial collapse. Adequate analytical fit to the experimental test data was achieved by properly selecting a limiting column end-rotation at initiation of shear failure, pinching terms, and cyclic strength degradation damage. No reloading stiffness damage was necessary for the columns considered as the automatic reloading stiffness adjustments sufficed.

The shear force versus lateral deflection of Specimen 1 tested by (Sezen and Moehle, 2006) is shown in Fig. 4. It can be seen that this specimen exhibits significant strength and stiffness degradation after the initiation of shear failure. This column is modeled by applying moderate strength degradation and using positive reloading pinching factors (see (LeBorgne, 2012) for sample OpenSEES input). The resulting strength degradation can be seen in Fig. 4, where the backbone is contacted five times before the nonlinear shear material reaches its residual shear capacity. Specimen 3CLH18 tested by (Lynn, 2001) also experiences significant strength damage after the initiation of shear failure; however this column maintained reloading stiffness for a greater number of cycles. To maintain reloading stiffness, moderate cyclic strength damage and negative reloading displacement pinching factors were used.

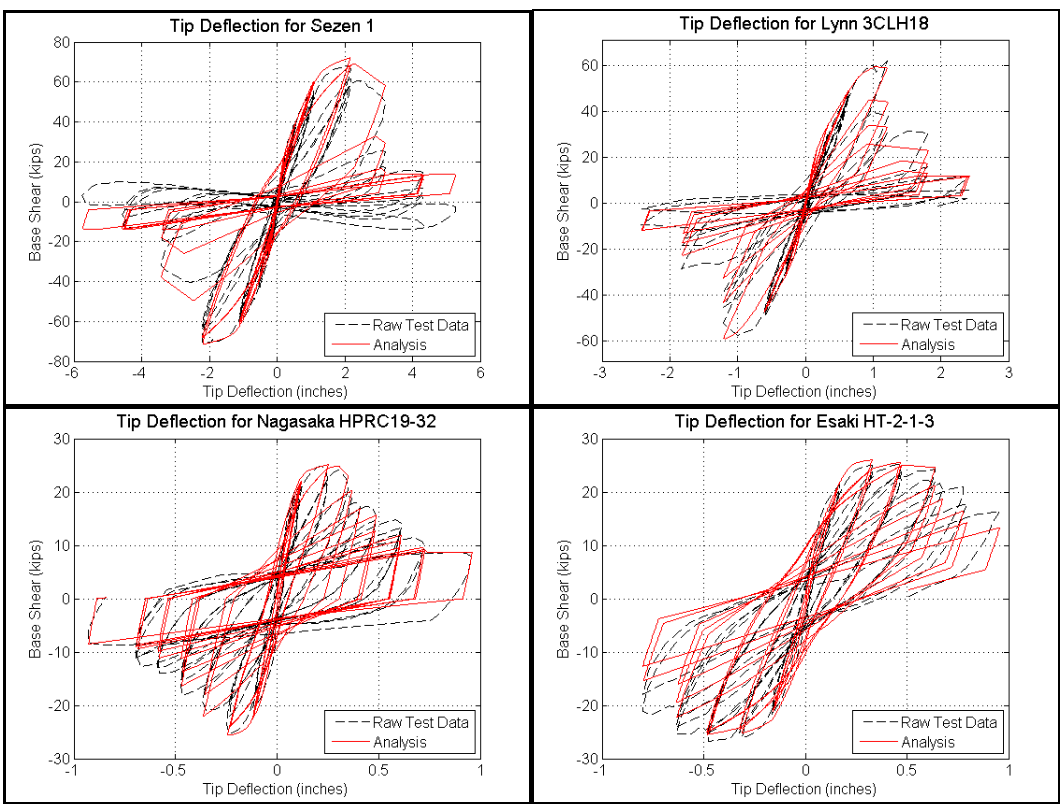

Figure 4. Manual calibration of proposed model to test results (adapted from (LeBorgne, 2012)).

The response of the specimen tested by (Nagasaka, 1982) exhibits light pinching with little strength and stiffness degradation. The shear material is adjusted to reflect that behavior by decreasing the reloading displacement pinching factors and increasing the reloading shear force pinching factor. Additionally, the unloading pinching factor is reduced to zero resulting in the open hysteresis loops. A column test by (Esaki, 1996) has moderate reloading stiffness degradation and tight pinching loops. It can be seen that during the test, the column was not pushed to significant deformations that would enable contact with the backbone curve (Fig. 4). This type of behavior is modeled by defining a shallow degrading slope and utilizing reloading displacement pinching factors that are less than zero and reloading shear force factors that are near zero. No strength degradation is defined since there is little backbone contact during the displacement history.

3. ANALYTICAL MODEL – AUTOMATIC CALIBRATION

The automatically calibrating model provides the engineering community with a user-friendly analytical tool that only requires the user to input column material and geometric properties to simulate the degrading lateral strength behavior of reinforced concrete columns. To-date the model has been calibrated to the behavior of flexure-shear-critical RC columns; i.e., columns that sustain flexural yielding before shear failure. The manual-calibration model is used as the basis for the automatically calibrating model. A calibration framework was developed to extract the necessary damage parameters from an experimental test database. Extracted damage parameters were related to pertinent predictor variables comprised of column physical properties and boundary conditions using stepwise regressions. The final regression equations were implemented into the column analytical framework to define model damage parameters using only user-defined column physical properties.

An experimental database of tests on columns that yielded in flexure prior to sustaining shear failure was compiled (Fig. 5) to calibrate the shear spring constitutive model. The database contains columns with a minimum shear span to depth ratio (*a/d*) of 1.5 and a constant axial load throughout their deformation history.

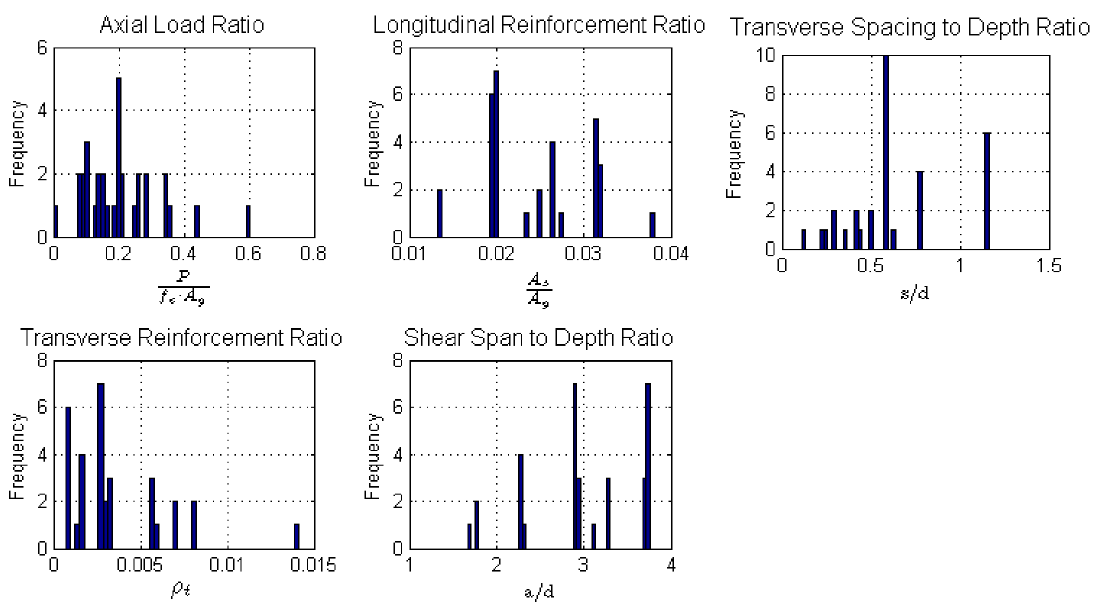

Figure 5. RC column database (adapted from (LeBorgne, 2012))

Column tests in the database provided experimental results in the form of a global lateral load versus deformation relation. Each global response includes both shear and flexural deformations. In order to calibrate the parameters that govern shear deformations in the shear spring, it was necessary to de-couple shear deformations from flexural ones. Approximate shear deformations were extracted by creating a model of the flexural behavior using OpenSEES and subtracting the flexural response from the experimental deformations. The flexural framework developed by (Ghannoum and Moehle, 2012) was used to estimate column flexural deformations as it was shown to capture flexural deformation of RC column with high accuracy (e.g., Fig. 4). Fig. 6 illustrates the extraction of flexural and shear deformations for a column in the database.

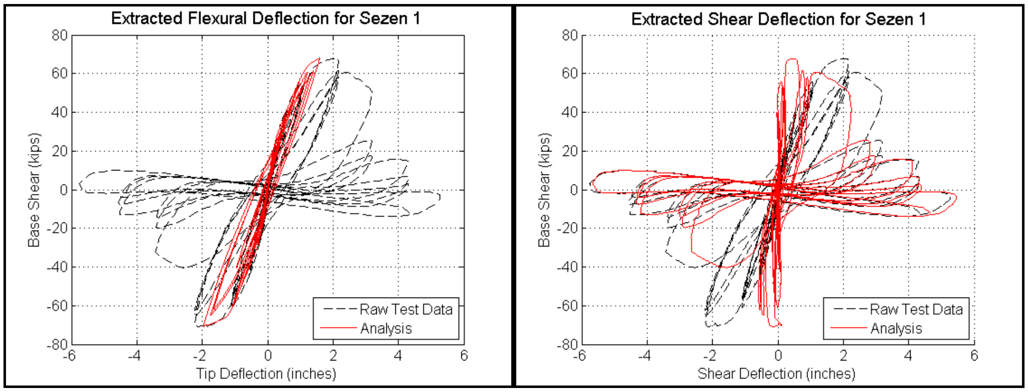

Figure 6. Extracted flexural and shear deformations for Specimen 1 from (Sezen and Moehle, 2006); adapted from (LeBorgne, 2012)

Relevant parameters were extracted from the estimated shear deformations. Extracted parameters (target variables) for the shear spring constitutive model were related to column properties and boundary conditions (predictor variables) so damage parameters could be estimated during analysis. Normalized predictor variables were chosen from a variety of available column properties that were suspected of influencing the shear degrading behavior. A stepwise linear regression was conducted to relate multiple predictor variables to each target variable. Shear failure initiation was defined in the model through a limiting column-end rotation. Key predictor variables were: Column dimensions including the shear span, concrete material properties, reinforcement material properties, transverse reinforcement details, development length of longitudinal bars, area of concrete core, and axial load. More detail about the regression fits can be found in (LeBorgne, 2012).

Despite some scatter between regression estimates and extracted model parameters, overall, the calibrated model produces a good fit with column global deformations. Analytical results of the calibrated model are shown for two experimental tests in Fig. 7. In these tests, the nonlinear flexural behavior is accurately modeled and the column-end rotation at initiation of shear failure is well estimated. The degrading behavior is adequately represented by the degrading slope, pinching, and strength damage parameters.

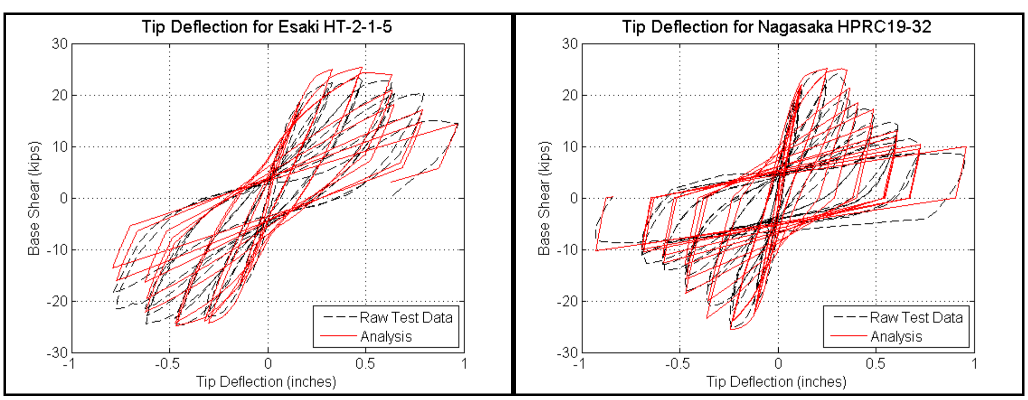

Figure 7. Results of calibrated analytical model showing accurate triggering of degradation; for (Esaki, 1996) and (Nagasaka, 1982) specimens; adapted from (LeBorgne, 2012)

The most critical parameter affecting the accuracy of deformation estimates during degradation was found to be the column-end rotation at which shear failure initiates. When shear-strength degradation is triggered at lower deformations than experimentally recorded, faster degradation than recorded will be estimated (Fig. 8). Alternatively, triggering shear-strength degradation at larger deformations causes slower degradation and over-predicts column strength at a given deformation (Fig. 8).

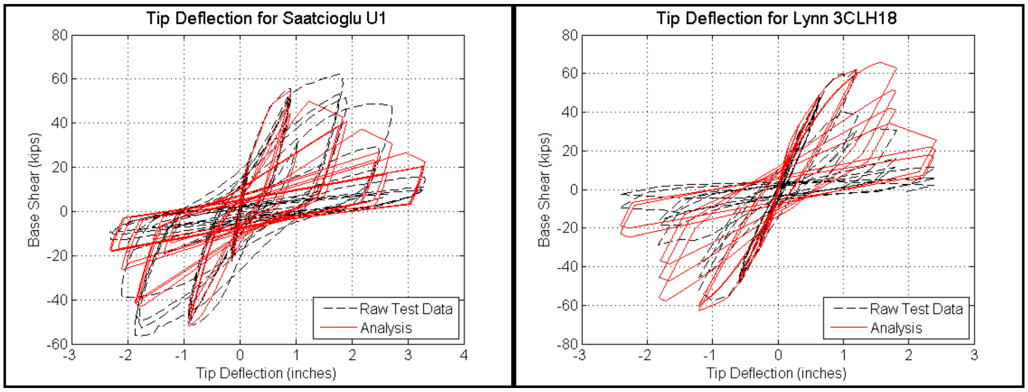

Figure 8. Results of calibrated analytical model showing inaccurate triggering of degradation; for (Saatcioglu and Ozcebe, 1989) and (Lynn, 2001) specimens; adapted from (LeBorgne, 2012)

The predictor variables used to calibrate this model have a finite range as shown in Fig. 5. Caution should be exercised if using the calibrated model outside of the range of experimental parameters as analytical results could be erroneous. It should be noted that the calibrated shear model was defined in conjunction with the flexural fiber-section model described earlier. Best results will be obtained by using the shear model with a similar flexural model. Plots comparing the calibrated model results with all column experimental results of the database can be found in (LeBorgne, 2012). Sample OpenSEES input scripts for the calibrated model are also given in (LeBorgne, 2012).

3. SUMMARY AND CONCLUSIONS

A strength degradation model was implemented in the analytical software OpenSEES to simulate the lateral strength degradation of reinforced columns with limited ductility. During analyses, the model is capable of dynamically monitoring key column forces and deformations for critical combinations that trigger column strength degradation. Once triggered, the degrading shear behavior is simulated using a nonlinear constitutive model. The model was implemented through a zero-length shear spring for computational efficiency and for compatibility with the most commonly used line-element analytical framework. The proposed model addresses several shortcomings of shear-spring type models, namely: 1) it has the ability to continually monitor forces and deformations in flexural elements for conditions that trigger shear failure, 2) it has a built-in function that compensates for flexural deformation offsets that arise from the degrading behavior in shear springs, and 3) it is able to trigger shear failure through either a limiting shear force or element deformations (whichever is reached first). In addition, the proposed model introduces several functionalities that give users a high degree of control over shear failure triggering and the ensuing cyclic degrading behavior.

The proposed model is implemented in two formats: 1) manual calibration, and 2) automatic calibration. Through manual calibration, it was demonstrated that the proposed model can simulate the shear-strength degrading behavior of reinforced concrete columns with high accuracy. The diverse damage algorithms that were implemented provide the necessary tools to model wide range of beam-column elements. The automatic calibration model provides the engineering community with a user-friendly analytical tool that only requires the user to input column material and geometric properties to simulate the degrading lateral strength behavior of reinforced concrete columns. To-date the model has been calibrated to the behavior of flexure-shear-critical RC columns. Extension of the calibration to a wide range of reinforced concrete column behaviors is the subject of future work. An experimental database of tests on columns that yielded in flexure prior to sustaining shear failure was compiled to calibrate the shear spring constitutive model. Despite some scatter between regression estimates and extracted model parameters, overall, the calibrated model produced a good fit with column global deformations. The most critical parameter affecting the accuracy of deformation estimates in the automatic calibration model was found to be the column-end rotation at which shear failure initiates.

The proposed model thus provides the engineering community with: 1) a versatile, highly customizable tool for simulating lateral strength degradation in beam-column elements through the manual calibration format; and 2) a user-friendly, automatically calibrating analytical tool that can be used in nonlinear dynamic analysis of RC columns that are at risk of severe damage and collapse, through the automatic calibration format.

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