Identification of Structural Systems with Full Characteristic Matrices under Single Point Excitation

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SUMMARY
The aim of "System Identification" is to determine the modal and system properties of structural systems. Because of various constraints in practice only single excitation and partial measurement at selected degrees of freedom is possible. In this paper, to identify a structural system, dynamic load was applied only along one of the degrees of freedom of the structure and the responses corresponding to a few degrees of freedom have been measured. To identify characteristic matrices of a system with this sort of restricted information, a new approach was intrtoduced. Taking into account the significant effect of noise reduction in improving the system identification accuracy levels, a noise reduction technique was also proposed. It was shown that as noise level increases, identification errors will also increase though to an acceptable range. The method's efficiency and precision were examined through the application of a "closed loop solution" to a six-storey flexural structure.

Keywords: System identification; Single force; Identification vector; Tree-coefficient-matrices; Noise reduction

1. INTRODUCTION

Structural Health Monitoring (SHM), as a need for reliable assessment of structural safety under service or extreme loads such as earthquake, requires system identification and damage detection. System Identification (SI) determines structural dynamic characteristics such as modal properties (frequencies, mode shapes and damping ratios) and system properties (mass, damping, and stiffness matrices). System identification methods that directly identify the characteristic matrices of a system (M, C, and K), have been investigated over several years as follows:

Potter and Richardson in 1974, Richardson in 1977 as well as Richardson and Shye in 1987 developed an approach to identify mass, damping, and stiffness matrices of a linear elastic system. In the method, using Laplace Transform of the measured input, and the displacements along the system's degrees of freedom (DOFs) under arbitrary loadings, and the "Transfer Matrix" (as a binomial function in term of Laplace value with coefficient of mass, damping, and stiffness matrices), they proposed a formulation for the direct identification of the system's characteristic matrices. They applied full force vector on each DOF. The result for noise free case was exact and correct but their method did not include a discussion of the identification errors due to noise (Potter and Richardson, 1974, Richardson, 1977, Shye and Richardson, 1987).

Masri, Miller, Saud and Caughey in 1987, and Agbabian, Masri, Miller and Caughey in 1991 studied the direct identification of characteristic matrices of linear and nonlinear structural systems, and presented a detailed formulation which was based on the inverse solution of the problem of identifying the system's characteristic matrices in time domain. The investigated structure in their method was a special case of shear structure, and hence its matrices of stiffness and damping were trigonal. In their method, the mass matrix was assumed to be known, and by this unreal assumption, the identification errors in two other matrices (damping and stiffness) largely decreased (Masri et al., 1987, Agbabian et al., 1991).

Zabel in 2002 proposed a method of Wavelet Transform in which the wavelet mother function was a Gaussian function, and the mass matrix was known. He applied his method on linear time-invariant systems by solving an over-determined system of equations that can be solved for damping and stiffness parameters vector by a least-squares method. The identification approach was verified by means of a numerical simulation (Zabel, 2002).

Ashtiany and Khanlari in 2011 proposed a method called "Added Mass Method", in which by adding a controlled change in dynamic properties of the structure, i.e. as masses of stories, the arrays of stiffness, mass, and damping matrices of both shear and tortional unknown structures are estimated. This method is also successfully drawn on for calculating mass, damping, and stiffness matrices of ASCE-SHM Benchmark structure and the results show that the identification errors in matrices are within acceptable range (Ashtiany and Khanlari, 2011).

In all previous studies, the vital task of reduction of the measurement points was missing. In the present paper, however, a method has been proposed that can be applied to any kind of classically or non-classically damped structural system. Compared to previous studies, this method is advantageous since here the input force is applied only along one of the DOFs of the structure, and each measured response (displacement, velocity, and acceleration) is independently mixed with instrumental noise. The existing methods of system property matrices estimation basically require exciting and measurement at all DOFs of the system. In order to obtain input and output data, one needs to perform an experiment/test on the system/structure under study. For instance, in modal testing, it is a common practice to excite the test structure by applying measurable excitations at several points, and then collect response data at the sensor locations (Ewins, 2000). However, many civil engineering structures are difficult to excite artificially due to their large size, geometry and location. Equally, a large amount of external energy is needed to excite an entire structure at a desired level of vibration. Thus, it is essential to reduce the number of the measuring points. This causes mathematical problems in the inverse solution of the equation of motion of the structure. This is the main reason why authors have proposed a new method in this paper as described below.

2. PROPOSED METHODOLOGY

The proposed method is based on the inverse solution of the single input problem of identifying the system's characteristic matrices of mass, damping and stiffness (M, C and K). One of the problems affecting the accuracy of the inverse solution of the system's motion equation is the existence of unknown and inevitable noise in the measured input and output data, which has an adverse effect on most of the existing identification methods. Filtering noise in the frequency domain, though leading to smooth measured data, may also alter the frequency content, and accordingly reduce the reliability levels of the identification process; thus, the proposed method conducts the noise reduction process in the time domain.

2.1. Input/output Measurements

The equation of motion for the DOFs of a structure in N_0 time steps can be written as:

$$M.\ddot{x}(t)+C.\dot{x}(t)+K.x(t) = f(t) \tag{2.1}$$

Where *f(t)* is the input force, and $\ddot{x}(t), \dot{x}(t)$ and *x(t)* are the acceleration, velocity, and displacement response, along the DOFs, respectively. Further, matrices *M*, *C*, and *K* are the mass, damping, and stiffness matrices respectively. The measured outputs are mixed with random noise, and are shown in N_0 time steps as:

$$u(t) = x(t) + noise(x(t)) \tag{2.2}$$

Fig. 2.1 presents the proposed system identification method, which is based on the inverse solution of the equation of motions of a Multi Degree of Freedom (MDOF) structure with known measured displacements $u_e(t)$ ($u_e = x_e + noise(x_e)$), velocities $\dot{u}_e(t)$, and accelerations $\ddot{u}_e(t)$ responses corresponding to known DOFs subjected to a known periodic impulsive type force $f_e(t)$ which is a Kronecker delta function, defined by Eqn. 2.3, and shown in Fig. 2.2 (as an example), applied along only one of the DOFs of the structure.

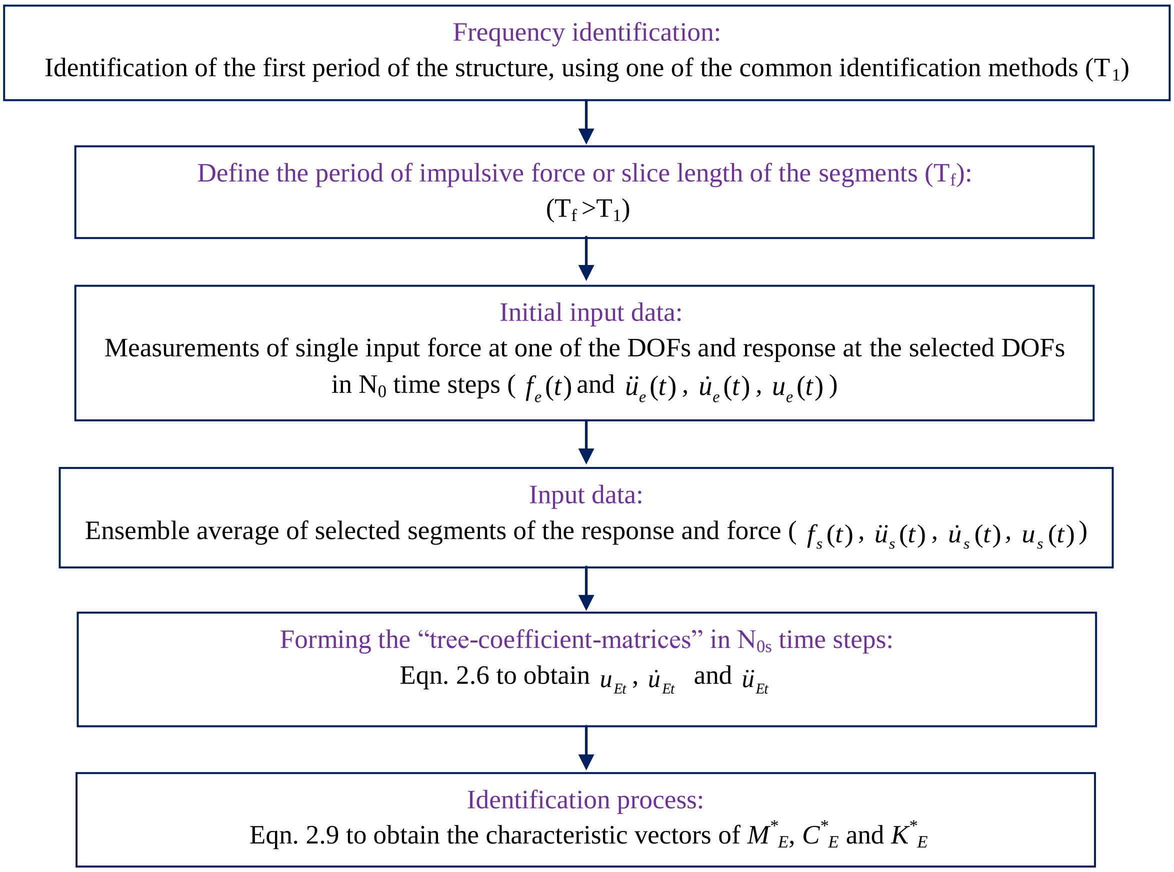

Figure 2.1. The proposed system mass, damping, and stiffness matrices identification process

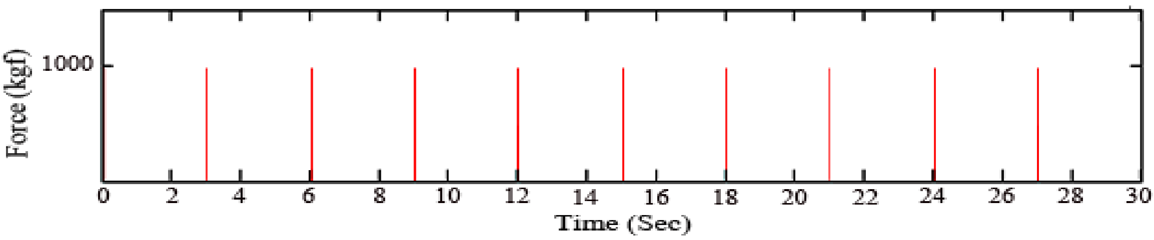

Figure 2.2. An example of the periodic impulsive force

$$f(t) = \begin{cases} g & : \quad \dfrac{t}{N_{0S} \cdot \Delta t} \in \left\{1,2,\cdots,\dfrac{N_0}{N_{0S}}\right\} \\ 0 & : \quad \dfrac{t}{N_{0S} \cdot \Delta t} \notin \left\{1,2,\cdots,\dfrac{N_0}{N_{0S}}\right\} \end{cases} \tag{2.3}$$

Where g is a constant value of the force, and Δt, N_0, and N_{0S}, are the time step, the number of total time steps, and the number of time steps of the force period respectively. Fig. 2.2 shows the force for Δt=0.005sec, N_0=6000, and N_{0s}=600.

2.2. Noise Reduction Process

Existence of noise gives way to inequality in Eqn. 2.1. It ill-conditions the inverse problem and distorts the identified mass, stiffness, and damping matrices. To reduce such noises, a noise reduction process on the measured response and the force was introduced. Considering the repeating characteristic of the input load, the shape of the response function will be repeated throughout the specific time equal to the period of the force (T_f) as shown in Fig. 2.3 for noisy displacement response (with 5% noise) at the second storey of a six-storey flexural structure, under a periodic impulsive force applied at the first storey. Thus, by breaking down the input and output signals into segments with the duration equal to T_f (which must be higher than the first period of the structure, in order to cover all the system periods), and taking the ensemble average of segmented parts of $f_e(t)$, $\ddot{u}_e(t)$, $\dot{u}_e(t)$, $u_e(t)$ (shown as $f_s(t)$, $\ddot{u}_s(t)$, $\dot{u}_s(t)$, $u_s(t)$ in N_{0s} time steps with the duration of one segment), the noise will be substantially reduced.

As shown in Fig. 2.3, the amplitude of the first segment of the responses is lower than other segments – since at the initial time of each segment, unlike the first segment, velocity will not be zero –. Since the instrumental noises are assumed to be white noise, the ensemble average of the segments of a white noise time history approaches to zero. Applying the aforementioned process will reduce the effect of noise on the responses of the structure. As an example, Fig. 2.4 shows the difference between the exact and noisy displacement responses for the first storey of an eight storey flexural structure for the case of 5% noise level, before and after applying the averaging process. Obviously, the duration of the cleaned response is reduced to the duration of one segment.

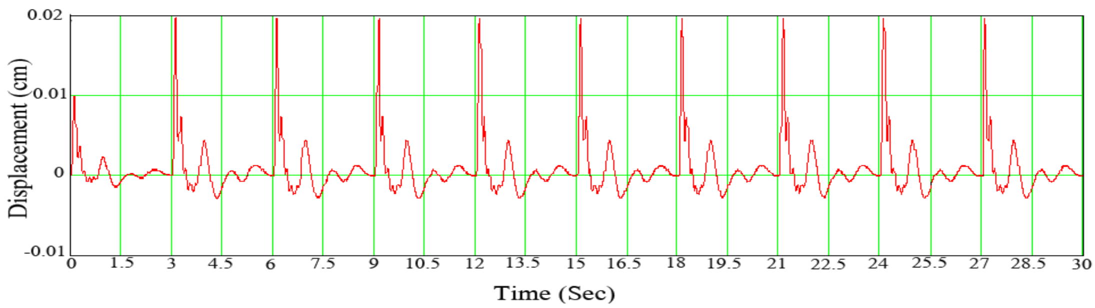

Figure 2.3. Noisy displacement response (with 5% noise) at the second storey of a six-storey structure, under a periodic impulsive force applied at the first storey

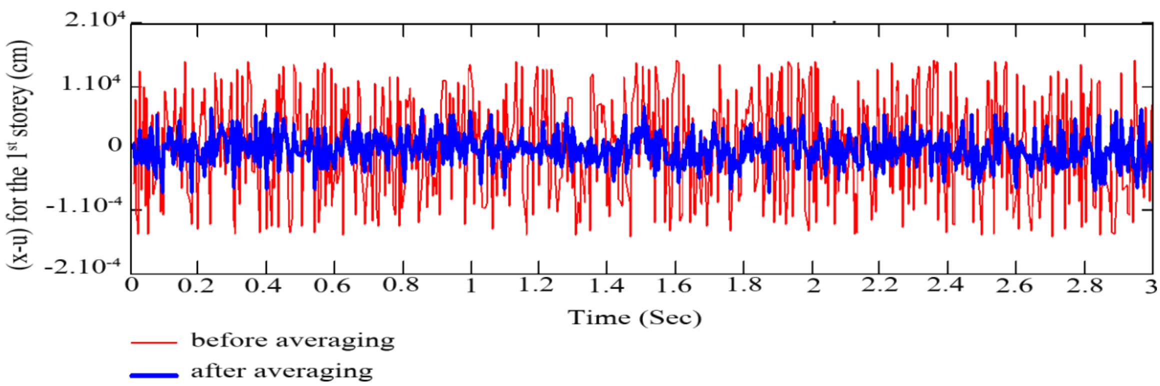

Figure 2.4. The difference between exact and noisy displacement responses for the first storey of an eight storey flexural structure, before and after applying the averaging process (5% noise)

2.3. System Identification Process

Below, the proposed method of system identification is described using the averaged force and response. In the case the force is applied along only one of the DOFs of the structure, the common inverse matrix solution method is not able to identify the elements of the characteristic matrices. Because when the zero values of the rows of the force tensor pre-multiplied to the inverted matrix, the row elements of the identified matrices are obtained zero. Obviously this is undesirable, and should be resolved by a suitable mathematical method. The equation of motion for the lateral or vertical DOFs of the structure, respectively for flexural structure or truss (DOFs 1 to N as shown in Fig. 2.5), in N_{0S} time steps can be written as:

$$\begin{Bmatrix} \ddot{u}_{1t}M_{11}+\ddot{u}_{2t}M_{12}+\ldots+\ddot{u}_{Nt}M_{1N}+\dot{u}_{1t}C_{11}+\dot{u}_{2t}C_{12}+\ldots+\dot{u}_{Nt}C_{1N}+u_{1t}K_{11}+u_{2t}K_{12}+\ldots+u_{Nt}K_{1N} \\ \ddot{u}_{1t}M_{12}+\ddot{u}_{2t}M_{22}+\ldots+\ddot{u}_{Nt}M_{2N}+\dot{u}_{1t}C_{12}+\dot{u}_{2t}C_{22}+\ldots+\dot{u}_{Nt}C_{2N}+u_{1t}K_{12}+u_{2t}K_{22}+\ldots+u_{Nt}K_{2N} \\ \vdots \\ \ddot{u}_{1t}M_{1N}+\ddot{u}_{2t}M_{2N}+\ldots+\ddot{u}_{Nt}M_{NN}+\dot{u}_{1t}C_{1N}+\dot{u}_{2t}C_{2N}+\ldots+\dot{u}_{Nt}C_{NN}+u_{1t}K_{1N}+u_{2t}K_{2N}+\ldots+u_{Nt}K_{NN} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ f_{Nt} \end{Bmatrix} \tag{2.4}$$

Assuming that the characteristic matrices are symmetric, the number of unknown elements in each matrix is equal to $\frac{N(N+1)}{2}$. The mass matrix can be accumulated in the "unknown identification vector" formed as:

$$M_E^* = \{M_{11} \quad M_{12} \quad \ldots \quad M_{1N} \quad M_{22} \quad M_{23} \quad \ldots \quad M_{2N} \quad \ldots \quad M_{N-1,N-1} \quad M_{N-1,N} \quad M_{NN}\}^T \tag{2.5}$$

Similar characteristic vectors of C^*_E and K^*_E can be written respectively for damping and stiffness, made from the characteristic matrices. For each time step *t*, using the value of displacement response of different storeys of the structure, the "tree-coefficient-matrices" of u_{Et} of order $N \times [\frac{N(N+1)}{2}]$ can be formed as:

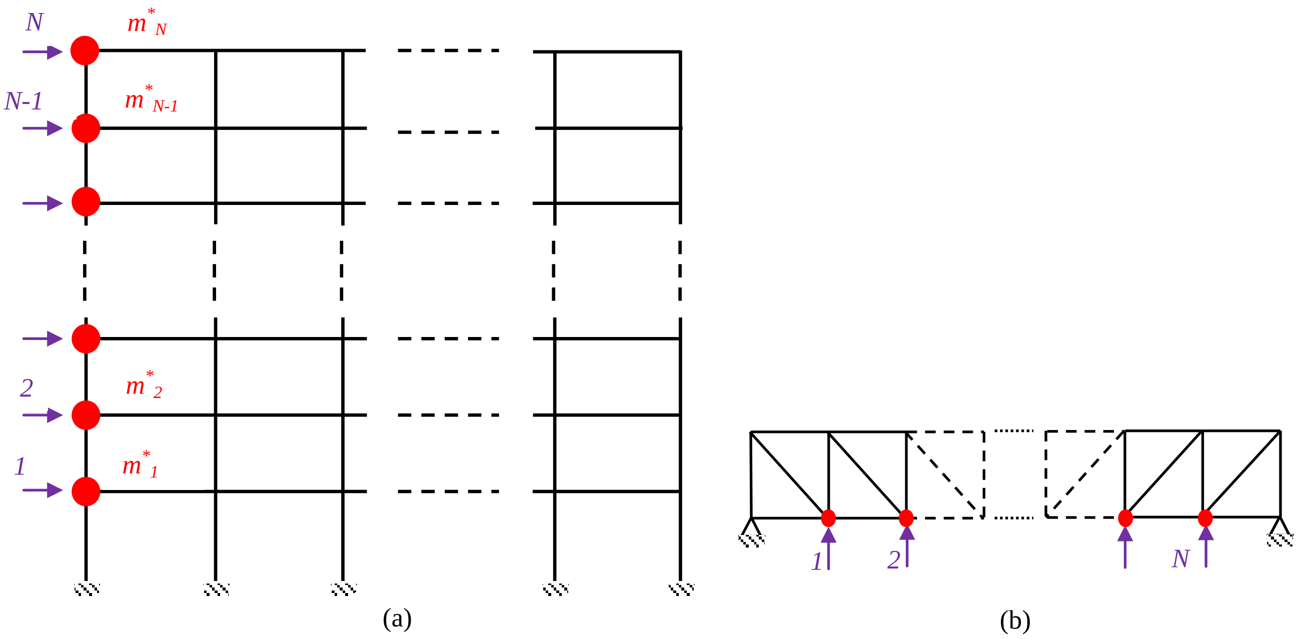

Figure 2.5. selected DOFs for the system identification process, and the concentrated masses for flexural structure (a) and Pratt truss (b)

$$
u_{Et} = \begin{bmatrix}
u_{1t} & u_{2t} & u_{3t} & u_{4t} & \dots & u_{Nt} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & \dots 0 & 0 & 0 \\
0 & u_{1t} & 0 & 0 & \dots & 0 & u_{2t} & u_{3t} & u_{4t} & \dots & u_{Nt} & 0 & 0 & 0 & 0 & 0 & \dots & 0 & \dots 0 & 0 & 0 \\
0 & 0 & u_{1t} & 0 & \dots & 0 & 0 & u_{2t} & 0 & \dots & 0 & u_{3t} & u_{4t} & \dots & u_{Nt} & 0 & \dots & 0 & \dots 0 & 0 & 0 \\
0 & 0 & 0 & u_{1t} & 0\dots & 0 & 0 & 0 & u_{2t} & 0\dots & 0 & 0 & u_{3t} & 0\dots & 0 & u_{4t} & \dots & u_{Nt} & \dots 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \dots & \dots u_{N-1,t} & u_{Nt} & 0 \\
0 & 0 & 0 & 0 & \dots & u_{1t} & 0 & 0 & 0 & 0 & u_{2t} & 0 & 0 & 0 & u_{3t} & 0 & \dots 0 & u_{4t} & \dots 0 & u_{N-1,t} & u_{Nt}
\end{bmatrix}
\tag{2.6}
$$

Similar definition is applied to $\dot{u}_{Et}$ and $\ddot{u}_{Et}$. So, Eqn. 2.4 in terms of introduced matrices can be re-written as:

$$
\ddot{u}_{Et}.M_E^* + \dot{u}_{Et}.C_E^* + u_{Et}.K_E^* = f_t(t) \tag{2.7}
$$

Assuming that the “tree-coefficient-matrices” of $\ddot{u}_{Et}$, $\dot{u}_{Et}$, and u_{Et} can be accumulated in the rectangular matrix *R*, Eqn. 2.7 can be rewritten for N_{0s} time steps as:

$$
\underbrace{[\ddot{u}_{Et} \quad \dot{u}_{Et} \quad u_{Et}]_{(NN_{0s})\times\left[3.\frac{N(N+1)}{2}\right]}}_{R} \begin{Bmatrix} M_E^* \\ C_E^* \\ K_E^* \end{Bmatrix}_{\left[3.\frac{N(N+1)}{2}\right]\times 1} = \underbrace{\{f(\mathrm{t})\}_{NN_{0s}\times 1}}_{F} \tag{2.8}
$$

Where, *F* is the column vector of single input in N_{0s} time steps. The three characteristic vectors of M^*_E, C^*_E, and K^*_E, can be identified from the inverse solution of Eq. 2.8 as:

$$
\begin{Bmatrix} M_E^* \\ C_E^* \\ K_E^* \end{Bmatrix} = (R^{-1}).F \tag{2.9}
$$

Considering that the number of time steps is relatively high, this simply means that the number of rows of the accumulated matrix *R* is more than its columns ($NN_{0s} \rangle \left[3.\frac{N(N+1)}{2}\right]$) and thus R^{-1} is Pseudo inverse of the accumulated matrix *R*.

Using the elements of the identified characteristic vectors, the characteristic matrices of the structure can be identified. The point of paramount importance here is that if the single excitation is applied at the first storey of a flexural structure, the identification results will bear the least possible errors. This is because the energy of this type of loading is distributed between all the storeys of the structure resulting in the lower amount of the identification errors. If the single excitation is applied at the last storey, the first mode’s contribution to the results will become more apparent than the higher modes, In contrast, when the force is applied at the first storey, the real contribution of higher modes will be preserved more.

3. NUMERICAL RESULTS OF THE PROPOSED METHOD

The proposed method has been applied to identify a six-storey two-span flexural linear structure with 4-meter spans, and 3-meter storeys height as shown in Fig. 3.1, and with the properties shown in Table 3.1. The following assumptions were also made in the numerical studies: a) Concentrated mass nodes equal to 2 *ton*, are considered for each node of the model; b) The system properties have been identified along the lateral (1 to 6) DOFs; c) A periodic impulsive force with amplitude of 981 *kgf*, and period of 3 *sec* is applied along the 1st DOF as shown in Fig. 3.1; d) $\Delta t=0.005$ *sec*; e) $N_0=6000$; f) 1% and 2% RMS ambient random white noise has been added to the response and force.

Eqns. 3.1 to 3.3 show the exact value and identification errors for the obtained mass (kgf.sec^2/cm), stiffness (kgf/cm), and damping (kgf.sec/cm) matrices in the cases of 1% and 2% noises. Each element of the shown matrices includes: $\left\{\begin{array}{c} \text{Exact value} \\ (\%\ \ \text{Identification error in the case of 1\% noises}) \\ (\%\ \ \text{Identification error in the case of 2\% noises}) \end{array}\right\}$.

The values of the elements of obtained characteristic matrices decrease, as they get away from diagonal elements. Further, the identification errors of the elements bearing very small values with respect to the diagonal elements are not important and are shown as NC. Eqns. 3.1 to 3.3 show the results for the flexural structure. As indicated in these equations, the maximum error observed was 2.95%.

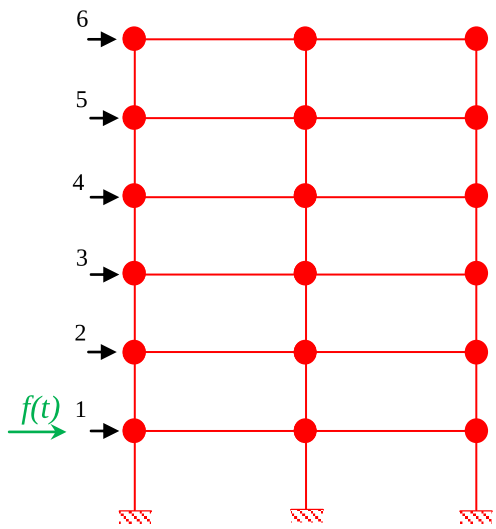

Fig. 3.1. The assumed model of flexural structure

Table 3.1. Properties of Members for the Models in the Numerical Analysis

Member property / Member location	Section height (*cm*)	Section Area (*cm*^{2})	Moment of Inertia (*cm*^{4})	Elasticity Modulus (*kgf/cm*^{2})	Weight per length (*kgf/cm*)
Storeys 1 to 3	14	43	1510	2.039×10^6	0.337
Storeys 4 to 6	10	26	450	2.039×10^6	0.204

$$M=\begin{bmatrix}
\begin{Bmatrix}5.834\\(0.002)\\(-0.02)\end{Bmatrix} & \begin{Bmatrix}0.128\\(0.226)\\(0.191)\end{Bmatrix} & \begin{Bmatrix}0.028\\(-0.419)\\(-2.314)\end{Bmatrix} & \begin{Bmatrix}-0.008\\(0.564)\\(-1.108)\end{Bmatrix} & NC & NC\\
 & \begin{Bmatrix}5.815\\(0.032)\\(0.023)\end{Bmatrix} & \begin{Bmatrix}0.11\\(1.282)\\(-2.894)\end{Bmatrix} & NC & NC & NC\\
 & & \begin{Bmatrix}5.919\\(0.047)\\(-0.087)\end{Bmatrix} & NC & NC & NC\\
 & Symm. & & \begin{Bmatrix}5.965\\(-0.021)\\(-0.2)\end{Bmatrix} & NC & NC\\
 & & & & \begin{Bmatrix}5.953\\(-0.132)\\(-0.575)\end{Bmatrix} & NC\\
 & & & & & \begin{Bmatrix}6.056\\(-0.29)\\(-0.53)\end{Bmatrix}
\end{bmatrix} \tag{3.1}$$

$$K=\begin{bmatrix}
\begin{Bmatrix}6977\\(0.03)\\(-0.05)\end{Bmatrix} & \begin{Bmatrix}-3865\\(0.05)\\(0.05)\end{Bmatrix} & \begin{Bmatrix}753\\(-0.03)\\(0.27)\end{Bmatrix} & \begin{Bmatrix}-48\\(-2.34)\\(-0.44)\end{Bmatrix} & NC & NC\\
 & \begin{Bmatrix}5885\\(0.02)\\(0.01)\end{Bmatrix} & \begin{Bmatrix}-3131\\(0.02)\\(0.06)\end{Bmatrix} & \begin{Bmatrix}318\\(0.48)\\(0.48)\end{Bmatrix} & NC & NC\\
 & & \begin{Bmatrix}3576\\(0.08)\\(0.05)\end{Bmatrix} & \begin{Bmatrix}-1339\\(0.22)\\(0.21)\end{Bmatrix} & \begin{Bmatrix}283\\(0.63)\\(1.94)\end{Bmatrix} & \begin{Bmatrix}-36\\(0.85)\\(2.95)\end{Bmatrix}\\
 & Symm. & & \begin{Bmatrix}2000\\(0.06)\\(-0.07)\end{Bmatrix} & \begin{Bmatrix}-1149\\(0.05)\\(-0.18)\end{Bmatrix} & \begin{Bmatrix}210\\(0.14)\\(-0.49)\end{Bmatrix}\\
 & & & & \begin{Bmatrix}1732\\(0.04)\\(-0.48)\end{Bmatrix} & \begin{Bmatrix}-824\\(0.06)\\(-0.56)\end{Bmatrix}\\
 & & & & & \begin{Bmatrix}642\\(-0.02)\\(-0.52)\end{Bmatrix}
\end{bmatrix} \tag{3.2}$$

$$C=\begin{bmatrix}
\begin{Bmatrix}41.97\\(0.04)\\(-0.35)\end{Bmatrix} & \begin{Bmatrix}-13.98\\(0.15)\\(-0.45)\end{Bmatrix} & \begin{Bmatrix}-9.44\\(-0.33)\\(-0.85)\end{Bmatrix} & \begin{Bmatrix}0.98\\(-0.91)\\(2.01)\end{Bmatrix} & NC & NC\\
 & \begin{Bmatrix}53.09\\(0.05)\\(0.19)\end{Bmatrix} & \begin{Bmatrix}-17.55\\(-0.19)\\(0.03)\end{Bmatrix} & \begin{Bmatrix}-4.23\\(1.14)\\(-0.35)\end{Bmatrix} & NC & NC\\
 & & \begin{Bmatrix}34.99\\(0.1)\\(0.51)\end{Bmatrix} & \begin{Bmatrix}-6.13\\(0.7)\\(1.3)\end{Bmatrix} & \begin{Bmatrix}-4.45\\(-0.37)\\(-0.8)\end{Bmatrix} & \begin{Bmatrix}0.91\\(1.02)\\(2.27)\end{Bmatrix}\\
 & Symm. & & \begin{Bmatrix}21.58\\(0.19)\\(0.3)\end{Bmatrix} & \begin{Bmatrix}-6.72\\(-0.07)\\(0.58)\end{Bmatrix} & \begin{Bmatrix}-3.75\\(0.14)\\(-1.11)\end{Bmatrix}\\
 & & & & \begin{Bmatrix}25.63\\(0.04)\\(-0.31)\end{Bmatrix} & \begin{Bmatrix}-7.48\\(-0.18)\\(-0.5)\end{Bmatrix}\\
 & & & & & \begin{Bmatrix}10.24\\(0.07)\\(-0.28)\end{Bmatrix}
\end{bmatrix} \tag{3.3}$$

As Eqn. 3.1 shows, the identification results for obtained mass matrix of the flexural structure are satisfactory, and the rate of the identification errors of diagonal values falls within 0.6% at the maximum. Assuming that total system mass matrix of the flexural structure is diagonal, the identification errors of off-diagonal elements of the obtained mass matrix are not important, because these elements are very small with respect to the diagonal elements. As observed in Eqns. 3.2 and 3.3, the identification results for obtained stiffness and damping matrices are excellent, and the identification errors of diagonal values do not exceed 0.52%.

At the fifth and sixth storeys, the identification errors of the obtained mass and stiffness matrices are slightly increased compared to the lower storeys. The identificatin errors of diagonal elements of the obtained damping matrix are relatively uniformly distributed. Generally, the identification errors are within an acceptable range. The most identification errors of diagonal values in 2% noise level for mass, stiffness, and damping are -0.575%, -0.52%, and 0.51% respectively.

4. THE EFFECT OF THE PROPOSED METHOD ON THE SYSTEM MODAL PROPERTIES

To further evaluate the validity of the proposed method, the effect of identified system properties on the system modal characteristics of the six-storey flexural building with 5% noise level was studied. For this purpose, the obtained mass, stiffness, and damping matrices were used to calculate the frequencies and mode shapes subjected to the Cape Mendocino earthquake (California, 1992) with a 7.2-magnitude main shock. Table 4.1 shows that the identified frequencies and mode shapes are in the excellent agreement with the actual values, especially for the first few modes.

Table 4.1. Error in the Frequency and Mode Shapes of the Six-Storey Flexural Building (5% Noise)

Frequency						
Mode No.	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}
Ident. Frequency (rad/s) (% Error)	3.088 (0.28)	8.279 (0.11)	14.391 (-0.06)	22.216 (-0.2)	28.789 (-0.1)	44.636 (-0.04)
Mode shape						
Storey/DOF	Real (% Error)	Real (% Error)	Real (% Error)	Real (% Error)	Real (% Error)	Real (% Error)
1	1 (0.00)	1 (0.00)	1 (0.00)	1 (0.00)	1 (0.00)	1 (0.00)
2	2.5 (0.44)	2.23 (-0.23)	1.73 (0.29)	0.97 (-0.41)	0.34 (-0.66)	-1.04 (0.15)
3	4.01 (0.34)	2.88 (-0.27)	1.17 (0.66)	-0.5 (2.78)	-1.05 (0.43)	0.53 (0.2)
4	7.08 (0.78)	2.16 (-0.07)	-1.86 (0.62)	-1.54 (0.77)	0.68 (1.05)	-0.12 (0.03)
5	10.12 (0.93)	-0.27 (-0.19)	-1.83 (1.4)	1.91 (1.36)	-0.38 (2.13)	0.03 (-4.88)
6	11.94 (1.05)	-2.59 (-0.12)	1.81 (0.77)	-0.8 (1.99)	0.11 (2.93)	-0.005 (NC)

5. CONCLUSIONS

In this paper, a general method was presented for the system identification of any kind of structures such as shear, flexural (with or without bracing), and truss under single excitation in which the mathematical problems of matrix-assisted inverse solutions, due to the input force applied along one of the DOFs of the structure, have been well solved, using the data from the measured input and output response. The proposed method can be drawn on to identify the matrices of mass, stiffness, and damping along the lateral or vertical DOFs of the structure. Also, an effective technique of averaging the segments of input and output data for noise reduction in the time domain was introduced.

Based on the findings, the presented method is effective, and can achieve relatively precise results, despite the presence of systemic noises. By increasing the noise levels and structure DOFs, the identification errors will also increase. Similarly the identification error for the estimated damping increases with an increase in structural stiffness. It was also observed that if the single force is applied at the first storey of the flexural structures, the identification results will bear the lowest possible errors.

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