# **Response Modification Factor for the Design** of Seismically Isolated Buildings

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#### SUMMARY:

Seismic design codes in the U.S. regulate the response modification factor ( $R_I$ ) that reduces minimum lateral forces when designing seismically isolated buildings. However, a factor of this kind has never been introduced in the Japanese code. Therefore, in Japan, structural engineers determine minimum lateral forces independently and the values they use may have limited rationality. This paper investigates the applicability of the response modification factor to the Japanese code in order to design seismically isolated buildings with more rational minimum lateral forces. First, the method of designing seismically isolated buildings using  $R_I$  is examined. Then, the seismic performance of a minimally code-compliant 6-story medium-rise reinforced concrete building is evaluated through design using  $R_I$  and a number of time-history analyses. Synthesizing these studies has led to some key findings for using  $R_I$  in the design of seismically isolated buildings.

Keywords: Minimum lateral force, Equivalent linearization, Seismic isolation, Incremental dynamic analysis

# **1. INTRODUCTION**

Seismic isolation is the most effective technology for protecting structures from the damaging effects of earthquakes. It has been extensively used worldwide over the past three decades. The widespread use of seismic isolation has necessitated better establishment and understanding of seismic design codes for designing new seismically isolated buildings and retrofitting old ones.

In the U.S., a lateral force reduction factor, called the 'response modification factor',  $R_I$ , directly impacts the ability of the lateral system of a superstructure on an isolation system to resist damaging effects of earthquakes. Lateral force is divided by this factor,  $R_I$ , to reduce the minimum lateral force for the isolation system and so determine the minimum lateral force for the superstructure above the isolation system. The factor is restricted to values between 1.0 and 2.0, depending on the type of superstructure. For example,  $R_I$  for an ordinary reinforced concrete moment frame is 1.125. The restricted range of  $R_I$  values is based on the performance objective for isolated buildings, implied in the U.S. codes, to essentially preclude yielding action and damage in isolated buildings.

In Japan, the situation is rather different. According to a review of 537 isolated buildings conducted by the Building Center of Japan (BCJ) from 1983-1999, 75 % had a design base shear coefficient less than 0.15 (2006). There are even some seismically isolated buildings with design base shear coefficients less than 0.1, and the trend is toward lower base shear coefficients. In Japan, there is no factor like  $R_I$  involved in the decision of the base shear coefficient, so structural engineers determine the minimum lateral forces independently, and the methods they use to arrive at those values may have limited rationality.

Although the current code-prescribed values for  $R_I$  have been selected through a process relying on judgment as well as on quantitative evaluation of limited data, the method of deciding the minimum lateral force using  $R_I$  can help determine a rational value for the design base shear of a superstructure if the value of  $R_I$  is reasonably stipulated. In this paper, in order to design seismically isolated

buildings with more rational minimum lateral forces, we investigate the applicability of the response modification factor to the Japanese code by conducting a number of analyses.

# 2. DESIGN METHOD FOR SEISMICALLY ISOLATED BUILDINGS USING THE RESPONSE MODIFICATION FACTOR

The design method for seismically isolated buildings using  $R_I$  evaluated herein is described in codes and specifications that are currently used widely in the U.S. (e.g. ASCE (2010) and NEHRP (2009)). The method is simple and uncomplicated, so it is possible to design seismically isolated buildings even with hand calculations. One distinctive trait of this design method is its emphasis on the isolation system's displacement. It is based on the following steps: (a) set a design response spectrum; (b) represent the isolated building with a single-degree-of-freedom system; (c) assume the peak isolator displacement,  $D_D$ ; (d) construct the isolation system force-displacement loop at  $D_D$ ; (e) calculate the effective stiffness,  $K_D$ , effective period,  $T_D$ , and effective damping,  $\beta_D$ , on the basis of the constructed loop and Eqn. 2.1,

$$K_D = \frac{F_D}{D_D}, \quad T_D = 2\pi \sqrt{\frac{M}{K_D}}, \quad \beta_D = \frac{E_D}{2\pi K_D D_D^2}$$
(2.1)

where  $F_D$  is the shear force at displacement  $D_D$ , M is the mass above the isolation system, and  $E_D$  is the energy dissipated in a cycle of the constructed loop; (f) calculate the spectral acceleration,  $S_a$ , from the design response spectrum for the period corresponding to  $T_D$ ; (g) calculate the spectral displacement based on Eqn. 2.2,

$$D'_{D} = \frac{1}{B_{D}} \frac{gS_{a}}{\omega_{D}^{2}} \qquad (\omega_{D} = \frac{2\pi}{T_{D}})$$

$$(2.2)$$

where  $D'_D$  is the displacement demand,  $B_D$  is the damping factor corresponding to  $\beta_D$ , g is the acceleration due to gravity, and  $\omega_D$  is the natural circular frequency; (h) repeat steps (c) through (f) until the assumed displacement,  $D_D$ , and the calculated displacement,  $D'_D$ , are sufficiently close; (i) upon calculation of the displacement demand, the design isolation system force,  $V_b$ , is obtained from the force-displacement loop and Eqn. 2.3;

$$V_b = K_D D_D \tag{2.3}$$

(j) calculate the minimum lateral force for the structural elements above the isolation system,  $V_s$ , using Eqn. 2.4.

$$V_s = \frac{V_b}{R_I} \tag{2.4}$$

Note that ASCE/SEI 7-10 (2010) prescribes that "the  $R_I$  factor shall be based on the type of seismic force-resisting system used for the structure above the isolation system and shall be three-eighths of the value of R given for conventional, fixed base buildings, with a maximum value not greater than 2.0 and a minimum value not less than 1.0."

#### **3. ANALYTICAL SIMULATIONS**

#### **3.1. Design response spectrum**

The design response spectrum for use is the 5 % damped acceleration spectrum, which is specified in the Japanese building code (2000). The acceleration spectrum on the surface of the site can be obtained by Eqn. 3.1.

$$S_a(T) = Z G_S(T) S_0(T)$$
(3.1)

where *T* is the fundamental period, *Z* is the seismic hazard zone factor, which varies between 0.7 and 1.0, based on the seismicity.  $G_S(T)$  is a soil amplification factor dependent on soil profile.  $G_S(T)$  is calculated based on the soil properties above engineering bedrock either by the simplified method where soil is classified as one of three types, or by the precise method calculated using a wave propagation procedure that considers the non-linearity of the soil profile.  $S_0(T)$  is the design spectral acceleration at engineering bedrock (the shear wave velocity is larger than 400 m/s).  $S_0(T)$  is shown in Eqn. 3.2 for a level 2 input (approximately a 500-year return period).

$$S_0(T) = \begin{cases} 3.2 + 30T & (T \le 0.16) \\ 8.0 & (0.16 < T \le 0.64) & [\text{cm}/\text{sec}^2] \\ 5.12/T & (0.64 < T) \end{cases}$$
(3.2)

In this study, the values of Z and  $G_s(T)$  are both 1.0, representing the construction of a seismically isolated building on engineering bedrock.

#### **3.2. Ground motions**

The value of  $R_I$  is evaluated based on the results of non-linear response history analyses. These analyses are performed for 8 ground motions (5 random phases, the 1995 Kobe earthquake JMA Kobe NS phase, the 2011 Tohoku earthquake Furudono EW phase, and the 2011 Tohoku earthquake Iwanuma EW phase). These ground motions are fitted to the design response spectrum in the frequency domain.

Following design practice in Japan (1994), the degree of compatibility of the synthetic input motion with the design spectrum is defined by the following four parameters: (a) the ratio of the input motion response spectrum to the design spectrum (herein,  $\varepsilon$ ) should not be less than 0.85, (b) the coefficient of variation of  $\varepsilon$  should be less than 0.05, (c) the total average value of  $\varepsilon$  should be within the range of  $1\pm$  0.02. Fig. 3.1 presents an example of synthetic motions, based on the 8 ground motions, scaled to meet the design spectrum.



Figure 3.1 Design response spectrum and 8 fitted ground motion spectra

#### 3.3. Superstructure design

We used a 6-story, reinsforced concrete moment resisting frame building as shown in Fig. 3.2(a). The building is  $1 \times 3$  bays in plan, each measuring  $10 \text{ m} \times 10 \text{ m}$ . The isolation system is located at foundation level. For dynamic response analysis, the building is modelled as a nonlinear shear type system with two degrees of freedom (2-DOF) as detailed in Fig. 3.2(b). The fundamental period is 0.36 s for the fixed-base superstructure. Stiffness-proportional viscous damping of 2.0 % is applied to the superstructure. The skeleton curve for the story shear force versus the story drift relationship is idealized as a tri-linear curve using the Takeda hysteresis model (1970). The characteristic values used to define the skeleton curve are shown in Fig 3.2(c). The first, second and third breaks represent cracking, yielding and unloading, respectively.

### 3.4. Isolation system design

The isolation system is designed according to the method described in section 2. The location and type of isolation system used are shown in Figs. 3.3(a) and (b). The isolation system consists of 8 lead rubber bearings. The total thickness of the rubber is 200 mm (both  $\phi$  800,  $\phi$  700). We examine a total of 9 cases of isolation systems whose values of horizontal stiffness,  $K_{H}$ , and intercept yield strength,  $Q_d$ , vary, considering differences in material properties that occur during the manufacture of the devices. All the cases and their properties are listed on Table 3.1. In the table, Case 202 is the standard case, and the production tolerance is applied to the other 8 cases listed.  $D_D$  and  $V_D/W$  are also listed for each case, where W is the weight of the structure above the isolation system. The hysteresis model developed by Kikuchi and Aiken (1997) was used for the shear hysteresis properties of the lead rubber bearings. Fig. 3.3(c) shows the hysteretic loop of the model. This model is capable of accurately predicting the force-displacement relationship of lead rubber bearings up to large shear strain levels.



Figure 3.2 (a) Reinforced concrete moment resisting frame building, (b) 2-DOF model and (c) Takeda model



Figure 3.3 (a) Location of isolators, (b) types of isolators and (c) hysteresis model for lead rubber bearings

Case	KH (%)	<i>Qd</i> (%)	DD(m)	Vb/W
101	+15	+15	0.264	0.124
102	+15	0	0.288	0.125
103	+15	-15	0.314	0.128
201	0	+15	0.269	0.114
202 (standard)	0	0	0.293	0.115
203	0	-15	0.322	0.117
301	-15	+15	0.276	0.105
302	-15	0	0.300	0.105
303	-15	-15	0.332	0.107

Table 3.1. Examined cases and their properties

#### **3.5.** Seismic response analyses

A series of nonlinear time history analyses was conducted on the 2-DOF system considered previously. The design base shear,  $V_s$ , is determined by dividing the value of  $V_b$  by an arbitrary  $R_I$ , so the relationship between the value of  $R_I$  and the response of a seismically isolated building is identified. In this section, the results of the analysis of Case 202 are presented as an example.

Fig. 3.4(a) shows the relationship between  $R_I$  and the ductility ratio,  $\mu$ , of the superstructure. The value of  $\mu$  begins to increase much more sharply in relation to  $R_I$  as the value of  $R_I$  increases past a certain value. This means that, when designing superstructures with large values of  $R_I$ , the ductile response of seismically isolated buildings increases rapidly for the design earthquake. In such case, adopting a large value of  $R_I$  is extremely dangerous.

Fig. 3.4(b) shows the relationship between  $R_I$  and the energy dissipation ratio,  $E_a$ .  $E_a$  is the ratio of energy dissipated by the isolation system to the total energy dissipated by the superstructure and the isolation system. The value of  $E_a$  decreases as the value of  $R_I$  increases past a certain value. This means that when designing superstructures with large values of  $R_I$ , the energy dissipated by the isolation system decreases rapidly for the design earthquake. In such cases, using large values of  $R_I$  in the design goes against the basic objective of isolating buildings: preventing damage of structural components by allowing the isolation system to dissipate input energy.

Fig. 3.4(c) shows the relationship between  $\mu$  and  $E_a$ . The value of  $E_a$  decreases as the value of  $\mu$  increases. This means that the input earthquake energy is dissipated constantly over the range of small values of  $\mu$  (such as  $\mu \leq 1$ ), however, as the superstructure becomes ductile, the ratio of energy dissipated by the isolation system becomes relatively smaller.



Figure 3.4 Analysis results (a)  $\mu$  versus *RI*, (b) *Ea* versus *RI* and (c) *Ea* versus  $\mu$ 

#### 4. EVALUATION OF RESPONSE MODIFICATION FACTOR

The results in the last chapter indicate that while a large value of  $R_I$  can cause catastrophic failure in a design level earthquake,  $R_I$  is reasonably safe if the value of  $R_I$  is not large enough to let buildings go ductile. In the U.S. seismic design codes, although the value of  $R_I$  is determined for each type of seismic force-resisting system above the isolation system, their values were empirically decided and might lack scientific verification. Therefore, we believe that the value of  $R_I$  can be increased until the superstructure starts to yield, or behave in a ductile response. Here, the response modification factor,  $\rho$ , is set as in Eqn. 4.1.

$$\rho = R_I$$
 when  $\mu = 1$  (4.1)

The values of  $\rho$  differ depending on which of the 8 ground motions is used, thus 8 values of  $\rho$  are obtained in each case, and the median value of  $\rho$  is set as  $\rho_{50}$ . We investigated the values of  $\rho$  and  $\rho_{50}$  for various types of seismically isolated buildings by conducting non-linear response analyses. Fig. 4.1(a) shows  $\rho$  and  $\rho_{50}$  for the cases that are listed in Table 3.1. The gray plots represent  $\rho$  and the red plots represent  $\rho_{50}$ . The cases that have smaller values of intercept yield strength,  $Q_d$ , have larger values of  $\rho$  and  $\rho_{50}$  (compare Cases 101, 201, and 301 with Cases 103, 203, and 303, respectively). The cases with larger horizontal stiffness ( $K_H$ ) values have larger values of  $\rho$  and  $\rho_{50}$  (compare Cases 101, 302, and 303, respectively). The values of  $\rho$  of the standard case (Case 202) range between 1.282 and 1.563, with  $\rho_{50}$  being 1.418.

Fig. 4.1(b) shows the values of  $\rho$  and  $\rho_{50}$  of the models for which the fundamental periods,  $T_0$ , are 0.24, 0.27, 0.30, 0.33, 0.36, 0.39, 0.42, 0.45, and 0.48 s for the fixed-base superstructure. Models with smaller values of  $T_0$  represent buildings with shorter distances between floors, while those with larger values of  $T_0$  represent buildings with larger heights per floor. The values of  $\rho$  and  $\rho_{50}$  vary depending upon the value of  $T_0$ , and the models with smaller values of  $T_0$  have larger values of  $\rho$  and  $\rho_{50}$ .

Fig. 4.1(c) shows  $\rho$  and  $\rho_{50}$  for those models in which the values of the stiffness-proportional viscous damping,  $\xi$ , for the superstructure are 0.01, 0.015, 0.02, 0.025, 0.03, 0.035, 0.04, 0.045, and 0.05. The values of  $\rho$  and  $\rho_{50}$  vary depending on the value of  $\xi$ , and the models with larger values of  $\xi$  have larger values of  $\rho$  and  $\rho_{50}$ .



**Figure 4.1** Results of analyses: (a)  $\rho$  and  $\rho_{50}$  by Case, (b)  $\rho$  and  $\rho_{50}$  versus  $T_0$ , and (c)  $\rho$  and  $\rho_{50}$  versus  $\xi$ 

# **5. RESPONSE OF SEISMICALLY ISOLATED BULDINGS TO EARTHQUAKES STRONGER THAN THE DESIGN LEVEL**

# 5.1. Procedure

FEMA P695 (2009) introduces a methodology to provide a rational basis for determining building system performance and response parameters that will result in equivalent safety against collapse in an earthquake for buildings with different seismic force-resisting systems. The methodology is based on a probabilistic safety assessment against building collapse that utilized many ground motions as well as Incremental Dynamic Analysis (2002), in which individual ground motions are scaled to increasing intensities until the structure reaches a collapse point.

We use the methodology developed by FEMA P695 in this study. The values of  $\rho_{50}$  obtained in chapter 4 were used for  $R_I$  to decide the values of  $V_s$ . We define the design response spectrum and the scaled ground motions used in the previous chapters as ground motion amplification (hereafter, GMA) 1.0. The GMA is gradually increased from 0.05 until the superstructure reaches a collapse point. The point at which each ground motion induces collapse is judged directly from the results of dynamic response analyses. Collapse is deemed to have occurred when there is excessive lateral displacement, defined herein as the point where the ductility ratio of the superstructure,  $\mu$ , exceeds 4 (2011). Next, the median collapse capacity is defined as the median GMA obtained from the 8 ground motions at their collapse intensities. This median GMA at the collapse point is herein defined as the collapse margin ratio (hereafter, CMR). For instance, if the CMR is 3, the superstructure does not collapse until it experiences an earthquake 3 times more intense than the design earthquake.

# **5.2. Evaluation of response**

The results of the analysis of Case 202 (standard conditions, see Table 3.1) are illustrated in Fig. 5.1(a), where each point in the figure corresponds to the result of one nonlinear dynamic response analysis of the model subjected to one ground motion record scaled to a particular intensity level. The result of each analysis is plotted with GMA on the vertical axis versus the ductility ratio,  $\mu$ , recorded in the analysis on the horizontal axis. Each line in this figure connects results for a given ground motion scaled to increasing spectral intensities. Differences among the lines reflect differences in the response when subjected to different ground motions with different phase characteristics. The slopes of the curves are steep for small values of  $\mu$ , and flatten at larger values of  $\mu$ . The number typed on Fig. 5.1(a) indicates the CMR of this model. In this case, the CMR is 1.39; therefore, the isolated building does not collapse until an earthquake 1.39 times larger than the base quake strikes.

Fig. 5.1(b) shows the results of each analysis plotted as shear strain of the isolation system on the vertical axis versus  $\mu$  recorded in the analysis on the horizontal axis, which corresponds to Fig. 5.1(a). The slopes of the curves are steep for small values of  $\mu$ , and flatten for larger values of  $\mu$ . The number typed on Fig. 5.1(b) indicates the shear strain of this model at the collapse point. In this case, the shear strain reaches 138.2 % at the point when the superstructure collapsed due to a stronger ground motion.

Table 5.1(a) shows the CMR and shear strain of the isolators at the collapse points for all the cases listed in Table 3.1. The cases with larger values of  $Q_d$  have larger CMRs than those with smaller values of  $Q_d$ . Differences in the values of CMR obtained using the value of  $K_H$  are very slight, but the cases with larger values of  $K_H$  have larger CMR values. The values of CMR for the 9 cases range between 1.31 and 1.51, and the values of shear strain at the collapse point range between 129.7 % and 152.8 %.

Table 5.1(b) shows the CMR and shear strain of the isolator at the collapse points, of models with different  $T_0$  values. The models with larger  $T_0$  have larger values of CMR. Similarly, models with larger values of  $T_0$  strain more than those with smaller values of  $T_0$ . The values of CMR range between 1.17 and 1.60, and the values of shear strain at the collapse point range between 107.4 % and 152.9 % among the 9 cases.

Table 5.1(c) shows the CMR and shear strain of the isolator at the collapse point, of models with different values of  $\xi$ . The models with smaller values of  $\xi$  have larger values of CMR. Likewise, models with smaller values of  $\zeta$  strain more than those with smaller values of  $\zeta$ . The values of CMR range between 1.35 and 1.41, and the shear strains at the collapse point range between 130.1 % and 142.7 % among the 9 cases.

Table 5.1(d) shows the CMR and shear strain of the isolator at the collapse point of models with different post-yield stiffness to elastic stiffness ratios,  $b_y$ , (= $K_3/K_1$  in Fig. 3.2(c)). Models with larger values of  $b_y$  have larger CMRs. Likewise, models with larger values of  $b_y$  deform more than those with smaller values of  $b_{v}$ . The values of CMR range between 1.36 and 1.81, and the values of shear strain at the collapse point range between 130.4 % and 211.8 % among the 9 cases.



**Figure 5.1** Analysis results (a) GMA versus  $\mu$  and (b)  $\gamma$  versus  $\mu$ 

CMI	R and shear strain	in of the iso	plators at the c	coll	apse of supers	tructure	
	(a) CMR and shear strain by Case				(b) CMR and shear strain by $T_0$		
		CMR	Strain (%)			CMR	Strain (%)
	Case 101	1.51	129.7		$T_0 = 0.24$	1.17	107.4
	Case 102	1.44	135.6		$T_0 = 0.27$	1.23	115.9
	Case 103	1.31	131.3		$T_0 = 0.30$	1.30	126.8
	Case 201	1.49	141.3		$T_0 = 0.33$	1.36	134.7
	Case 202	1.39	138.2		$T_0 = 0.36$	1.39	138.2
	Case 203	1.33	139.9		$T_0 = 0.39$	1.54	144.4
	Case 301	1.45	138.4		$T_0 = 0.42$	1.50	141.4
	Case 302	1.40	140.5		$T_0 = 0.45$	1.50	142.2
	Case 303	1.43	152.8		$T_0 = 0.48$	1.60	152.9
	(c) CMR and shear strain by $\xi$		]	(d) CMR and shear strain by $b_v$			
		CMR	Strain (%)			CMR	Strain (%)
	$\xi = 0.010$	1.41	142.7		$b_y = 0.0010$	1.36	130.4
	$\xi = 0.015$	1.40	140.1		$b_y = 0.0025$	1.36	131.0
	$\xi = 0.020$	1.39	138.2		$b_y = 0.0050$	1.37	132.6
	$\xi = 0.025$	1.39	138.0		$b_y = 0.0075$	1.38	135.9

<b>Table 5.1.</b> CMR and shear strain of the isolators at the collapse of superstruc	tur
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CMR and	(d)		
	CMR	Strain (%)	
0.010	1.41	142.7	$b_y =$
0.015	1.40	140.1	$b_y =$
0.020	1.39	138.2	$b_y =$
0.025	1.39	138.0	$b_y =$
0.030	1.39	136.9	$b_y =$
0.035	1.38	136.2	$b_y =$
0.040	1.37	133.5	$b_y =$
0.045	1.36	132.5	$b_y =$
0.050	1.35	130.1	$b_y =$

(d) CMR and shear strain by $b_y$					
	CMR	Strain (%)			
$b_y = 0.0010$	1.36	130.4			
$b_y = 0.0025$	1.36	131.0			
$b_y = 0.0050$	1.37	132.6			
$b_y = 0.0075$	1.38	135.9			
$b_y = 0.0100$	1.39	138.2			
$b_y = 0.0200$	1.43	150.1			
$b_y = 0.0300$	1.52	162.8			
$b_y = 0.0400$	1.66	182.1			
$h_{\rm v} = 0.0500$	1.81	211.8			

#### 6. CONCLUSIONS

We investigated the method for designing seismically isolated buildings that is currently practiced widely in the U.S., which uses the response modification factor,  $R_I$ . We also examined the seismic performance of a minimally code-compliant 6-story medium-rise reinforced concrete building. Synthesis of these studies has led to the following conclusions:

- i) In the U.S. seismic design codes, the value of  $R_I$  is determined based on the type of superstructure, but this study indicates that differences in the properties of the seismic isolation devices used influences the value of  $R_I$  to an extent, which cannot be ignored.
- ii)  $R_I$  can be set at 1.42 for a standard case model (Case 202 in Table 3.1) of the design of seismically isolated buildings.
- iii) A building model (Case 202) having its design shear coefficient reduced by  $\rho_{50}$  had a collapse margin ratio (CMR) of 1.39.
- iv) The shear strain of the isolator when the superstructure collapsed due to a rare earthquake stronger than the design ground motion reached 138.2 (Case 202). This amount of strain is not enough to break the seismic isolation device. However, it is worth paying attention to potential problems that may arise in long-period ground motions or when construction of buildings on small sites is considered.

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