Rotational Components Generation of Earthquake Ground Motion Using Translational Components

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SUMMARY:
The motion of a point is specified completely by its six components: three translations and three rotations. Three rotational components of ground motion are contained two rocking components and one torsional component. Translational components of ground motion are easily measurable by standard techniques, whereas the rotational components are not directly accessible. In this paper, rotational components of earthquake acceleration have been obtained using improved method from the corresponding available translational components based transversely isotropic elastic wave propagation and classical elasticity theories. With this improvement, it becomes possible to consider frequency dependent wave velocities on rotational components of the earthquake. For this purpose, three translational components of six earthquakes have been adopted to generate their relative rotational components based on SV and SH wave incidence by using the Fast Fourier transform. Rotational accelerations obtained from this improved approach are compared with other methods and results show that peak ground acceleration of rotational acceleration are near to real recorded data relative to other theoretical methods.

Keywords: Rotational Components, Rocking and Torsional Components, Frequency Dependent Wave Velocity

1. INTRODUCTION

Ideally, the kinematics of any point in the medium is three translational and three rotational components. The three rotational components of ground motion include the two rocking components and one torsional component. In the absence of direct measurements to record the rotational components of earthquake, these components have been estimated theoretically using classical elasticity theorem between rotation and translational motions and also wave propagation theorem in elastic half space.

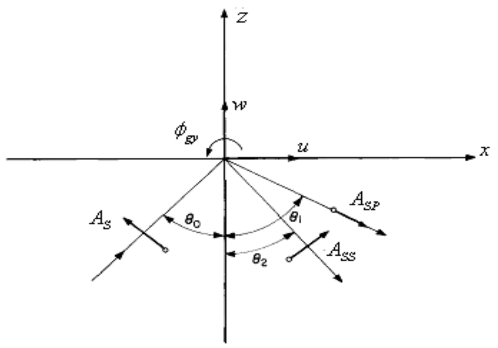
(Newmark, 1969) was perhaps the first to establish a relationship between the torsional and translational components of a ground motion based on constant velocity of wave propagation assumption. This method was used by several authors such as (Ghafory-Ashtiany and Singh, 1986) known as time derivation method. The assumption of constant plane wave velocity made for calculating the rotational components time histories from the translational components may be difficult to justify. Based on Newmark relationship, other researchers such as(Ghayamghamian, Nouri, Igel and Tobita, 2009) using data collected from the Chiba dense array, generated the torsional ground motion and analyzed several building models for different structural characteristics subjected to six correlated components of earthquake. (Nouri, Ghayamghamian, Hashemifard, 2010) made a comparison between different methods of torsional ground motion evaluation. They showed that torsional components obtained from time derivation methods with constant wave velocity used by (Ghafory-Ashtiany and Singh, 1986), are approximately 6 to 8 times larger than of dense array and other methods. The work based on direct empirical scaling of spectral amplitudes improved also the algorithms used for generation of synthetic strong motion accelerograms of translational components (Trifunac, 1971, 1976). More rational procedure have been developed by (Trifunac , 1982), (Lee and Trifunac, 1985, 1987) and (Castellani and Boffi, 1986, 1989), where the requirement of a constant plane wave velocity was relaxed and the dispersion and transient arrival times of waves in an elastic half-space were considered. In fact, the angle of incidence depends also upon the frequencies of the impinging harmonics of the ground motion. The dependence of the angle of incidence as well as the velocity of propagation of the wave on the frequency of the harmonics constituting the ground motion of the site must be considered when calculating the rotational components using the corresponding translational components. (Lee and Liang, 2008) have used (Lee and Trifunac, 1985, 1987) method to develop theories and algorithms for generating rotational motion from the corresponding available translational motions. They used translational accelerograms recorded at Pacoma dam in California, during the Feb 9, 1971 San Fernando earthquake. They generated their corresponding artificial torsional and rocking components and checked their results with others work. Some researchers (Hong-Nan Li, Li-Ye Sun and Su-Yan Wang, 2004) proposed an improved approach to obtain the rotational components of a seismic ground motion which included the effect of the relative contributions of the P , SV and SH waves to calculate time histories of rotational components. In their studies, the propagation velocity of different waves is considered and can be obtained from different relationships for each body and surface waves which depends on angle of incident waves in each frequency. In the past decade, rotational motions generated by large earthquakes in the far field have been successfully measured at sites in Germany, New Zealand and southern California (e.g., Igel, 2007) and observations agree well with classical elasticity theory (Suryanto et al., 2006). However resent rotational measurements in the near field of earthquakes in Japan (Takeo, 1998, 2009) and in Taiwan (Huang et al.,2003, 2006; Liu et al., 2009) indicate that rotational ground motion are 10 to 100 times larger than expected from the classical theory.

In this paper, rotational components of ground motion are obtained using both classical elasticity and elastic wave propagation theories based on SV and SH waves incidence of the earthquake. For this purpose, improvement of the approach proposed by (Hong-Nan Li, Li-Ye Sun and Su-Yan Wang, 2004), is applied. This improvement method permits that frequency dependent wave velocities and angle of each harmonic wave are taken into account.

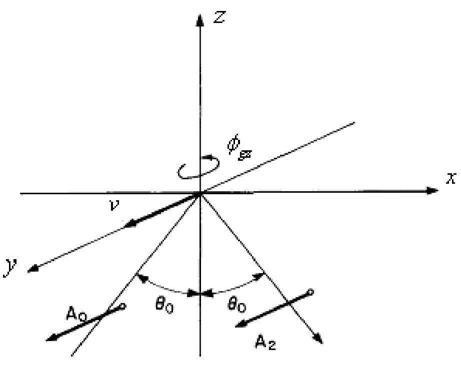
2. THEORY

2.1. Rotational component of SV and SH wave incident

Figures 1 shows the coordinate system, ground motion amplitudes u, v and w and the ray direction with the assumed positive displacement amplitudes of incident (A_S) and reflected P (A_{SS}) and SV (A_{SP}) waves. For the incident ray of SV waves in y=0 plane, the only non-zero components of motion are u, w and ϕ_{gy} . These characteristics are also defined for the plane x=0, where the only non-zero components of motion are v, w and ϕ_{gx} . For incident SH waves, there is no mode conversion and hence there is only one reflected SH wave with $\theta_2 = \theta_0$ and $A_2 = A_0$ according to figure (1-b).



(a)



(b)

Figure 1. Coordinate system for (a): incident SV wave, (b): incident SH wave (Datta, 2010).

Referred to figure (1-a), the angle of incidence, θ_0 , and reflected SV waves, θ_2 , are equal. The angle of reflected P wave is also denoted as θ_1 .

For harmonic waves of frequency ω, the potential functions are:

$$\psi_{SV} = A_S \exp i\omega\left(\frac{\sin\theta_0}{\beta}x - \frac{\cos\theta_0}{\beta}z - t\right) \tag{2.1}$$

$$\varphi_{SP} = A_{SP} \exp i\omega\left(\frac{\sin\theta_1}{\alpha}x + \frac{\cos\theta_1}{\alpha}z - t\right) \tag{2.2}$$

$$\psi_{SS} = A_{SS} \exp i\omega\left(\frac{\sin\theta_0}{\beta}x + \frac{\cos\theta_0}{\beta}z - t\right) \tag{2.3}$$

where α and β are the propagation velocities of P and S waves respectively(Datta, 2010).

The particle displacement u, w in the x, z directions are given by:

$$u = \frac{\partial\varphi_{SP}}{\partial x} + \frac{\partial(\psi_{SV} + \psi_{SS})}{\partial z} \tag{2.4}$$

$$w = \frac{\partial\varphi_{SP}}{\partial z} - \frac{\partial(\psi_{SV} + \psi_{SS})}{\partial x} \tag{2.5}$$

By imposing the free shear stress condition at the ground surface:

$$\tau_{xz}\Big|_{z=0} = \left[\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right]_{z=0} = 0 \tag{2.6}$$

The rocking component can be written as:

$$\phi_{gy} = \frac{1}{2}\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) \tag{2.7}$$

The resulting rocking component can be obtained from Eq. (1) to Eq. (6) as:

$$\begin{aligned}\phi_{gy} &= \frac{\partial w}{\partial x} = \frac{\partial^2\varphi_{SP}}{\partial z\partial x} - \frac{\partial^2(\psi_{SV} + \psi_{SS})}{\partial^2 x} \\ &= i\omega\frac{\cos\theta_1}{\alpha}i\omega\frac{\sin\theta_1}{\alpha}\varphi_{SP} - [(i\omega\frac{\sin\theta_0}{\beta})\psi_{SV} \\ &+ (i\omega\frac{\sin\theta_0}{\beta})^2\psi_{SS}]\end{aligned} \tag{2.8}$$

According to the Snell's law, $(\sin\theta_0)/\beta = (\sin\theta_1)/\alpha$, one can also obtain:

$$\varphi_{gy} = \frac{i\omega}{C_x}w \tag{2.9}$$

in which $C_x = \beta/\sin\theta_0$.

These equations are also applied for the other rocking component, φ_{gx} .

The Eq.(9) is rewritten as Eq.(10).

$$\varphi_{gy}(t) = \frac{i\omega}{C_x} w = (1e^{\frac{\pi}{2}i})(\frac{\omega}{C_x})(R_w . e^{i\theta_w}) = (\frac{\omega}{C_x} R_w)(e^{(\frac{\pi}{2}+\theta_w)i}) \quad (2.10)$$

In this equation, R_w is the vertical component Fourier amplitude and θ_w is its phase. This equation show that the rocking component Fourier amplitude is $\frac{\omega}{C_x} R_w$ and its phase is $\frac{\pi}{2} + \theta_w$.

For SH wave incident, the potential functions of incident and reflected waves are (Datta, 2010):

$$V_{SH} = A_0 \exp i\omega\left(\frac{\sin\theta_0}{\beta} x - \frac{\cos\theta_0}{\beta} z - t\right) \quad (2.11)$$

$$V_{SH'} = A_1 \exp i\omega\left(\frac{\sin\theta_0}{\beta} x + \frac{\cos\theta_0}{\beta} z - t\right) \quad (2.12)$$

The displacement field v caused by the incident and reflected waves in the y direction is:

$$v = 2V_{SH} = 2A_0 \exp i\omega\left(\frac{\sin\theta_0}{\beta} x - t\right) \quad (2.13)$$

Using Eq. (10), the torsion ϕ_{gz} is obtained by:

$$\begin{aligned}\phi_{gz} &= \frac{1}{2}\left(\frac{-\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{1}{2}\frac{\partial v}{\partial x}\bigg|_{z=0} = \frac{\partial V_{SH}}{\partial x} \\ &= i\omega\frac{\sin\theta_0}{\beta}\frac{v}{2} = \frac{i\omega}{2C_x} v\end{aligned} \quad (2.14)$$

in which $C_x = \beta / \sin\theta_0$.
This equation also can be rewritten as Eq.(10).
Assuming that the translational components u, v and w of the ground motion at the free surface can be measured, the rocking and torsional components of ground motion are obtained from Eqs. (9) and (14) respectively. In these equations, the frequency dependent angle of incident waves, $(\sin\theta_0)$ need to be obtained.
The improved approach developed by (Hong-Nan Li, Li-Ye Sun and Su-Yan Wang, 2004), is used to calculate the angle of incident waves. Using this approach and introducing $(x = \sin\theta_0)$ and based on Snell's law, Eqs. (15) and (16) are used to obtain the angle of incident SV and SH waves.

$$G = \frac{2x\sqrt{1-K^2x^2}}{K(1-2x^2)}, \qquad \theta_0 \prec \theta_C \quad (2.15)$$

$$G = -\frac{2x\sqrt{1-K^2x^2}}{iK(1-2x^2)}, \qquad \theta_0 \succ \theta_C \quad (2.16)$$

where $G = tg\bar{e} = w/u$ and $G = tg\bar{e} = w/v$ for rocking component in x-z and y-z plane due to SV waves; $G = tg\bar{e} = v/u$ for torsional component in x-y plane due to SH waves; $K = \alpha/\beta$ and $\theta_C = \arcsin(\beta/\alpha)$ the incident critical angle.

3. GENERATING ROTATIONAL COMPONENTS OF EARTHQUAKE

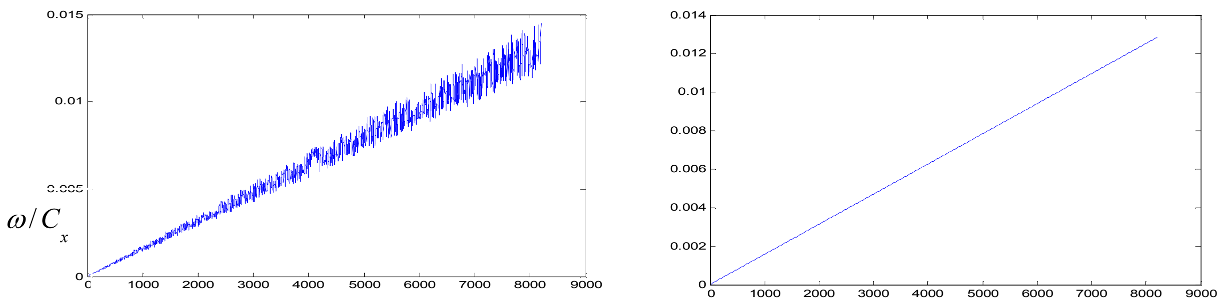
In this paper, six earthquakes are selected to study their relative rotational components. Characteristics of these six earthquakes are listed in table (1).

Table 1. Characteristics of earthquakes

Eartquake	Time of event	Station	Epicentre distance (km)	Recorded Component	PGA (g)	Shear wave velocity (m/s)
ImperialValley	1951/01/24	117 El Centro	28.24	Up-Down	0.013	213.4
				North-South	0.029	
				East-West	0.030	
San Fernando	1971/02/09	Pacoma Dam	11.86	Vertical	0.709	2016.1
				S74W	1.075	
				East-West	1.159	
Taft	1952/7/21	Lincoln School	35	Vertical	0.155	385.4
				S69E	0.179	
				East-West	0.170	
Tabas,Iran	1978/09/16	Boshrooyeh	74.66	Vertical	0.069	338.6
				Longitudinal	0.109	
				East-West	0.897	
Chi Chi Taiwan	1999/09/20	CWB9999917ALS	37.83	Vertical	0.074	553.4
				North-South	0.175	
				East-West	0.162	
Northridge	1994/01/17	000LA Dam	11.79	Vertical	0.424	629
				Longitudinal	0.511	
				East-West	0.346	

The Fourier amplitude and phase spectrum related to each component of earthquakes is obtained using Fast Fourier transform of their time histories. For each frequencies, the G and x parameters of Eqs. (15) and (16) are calculated. Then rocking and torsional components of earthquakes are obtained from Eqs. (9) and (14) respectively.

For verification of improved approach in this research, our results are compared with results of (Lee and Liang, 2008). In the mentioned work the San Fernando earthquake is considered for calculation. This earthquake is recorded on Feb 9, 1971at Pacoma dam station where its horizontal (S74W) and vertical components had a peak acceleration of 1055 and 696 cm/s2, respectively. The peak values of rocking and torsional accelerations are obtained 0.3725, -0.2480(rad/s/s) by (Lee and Liang, 2008) and -0.3833, -0.2545(rad/s/s) by our research, respectively. The differences between these results are about 3% that are due to different wave velocity and empirical scaling. In this research, the wave velocity is considered frequency dependent as shown in figure (2). For San Fernando earthquake the time history of rocking component obtained by (Lee and Liang, 2008) and this research are also shown in figure (3).



8192 discrete frequency
(a)

8192 discrete frequency
(b)

Figure2. The ratio of ω/C_x (a): frequency dependent (b): frequency independent

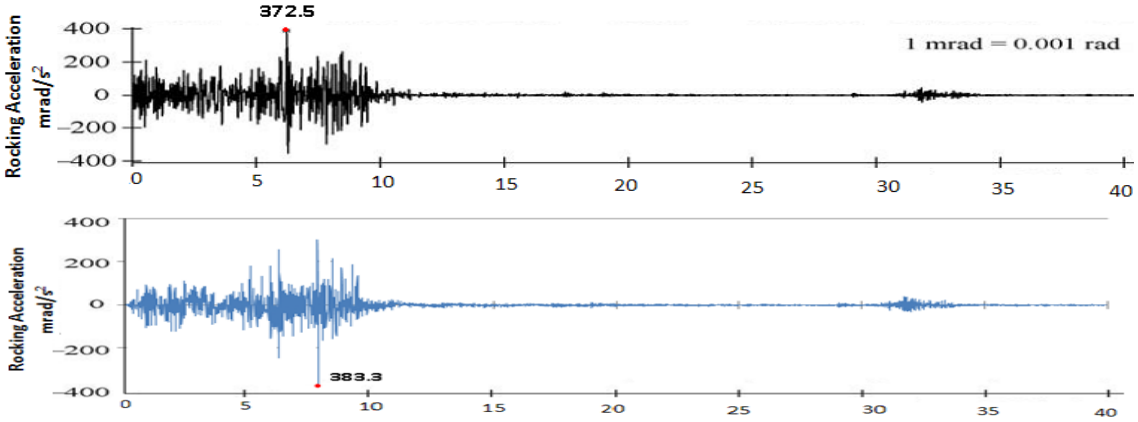


Figure 3. Rocking component acceleration for San Fernando earthquake (a): by (Lee and Liang, 2008) (b): in this research.

For each earthquake in table (1), the rocking Fourier amplitude spectrums are shown in figure (3). The rocking acceleration time history are obtained using inverse Fourier transform of these spectrums with phase difference of $\pi/2$ related to vertical components. In this research, the rocking and torsional acceleration and velocity of these six earthquakes are obtained and their peak values are listed in table (2).

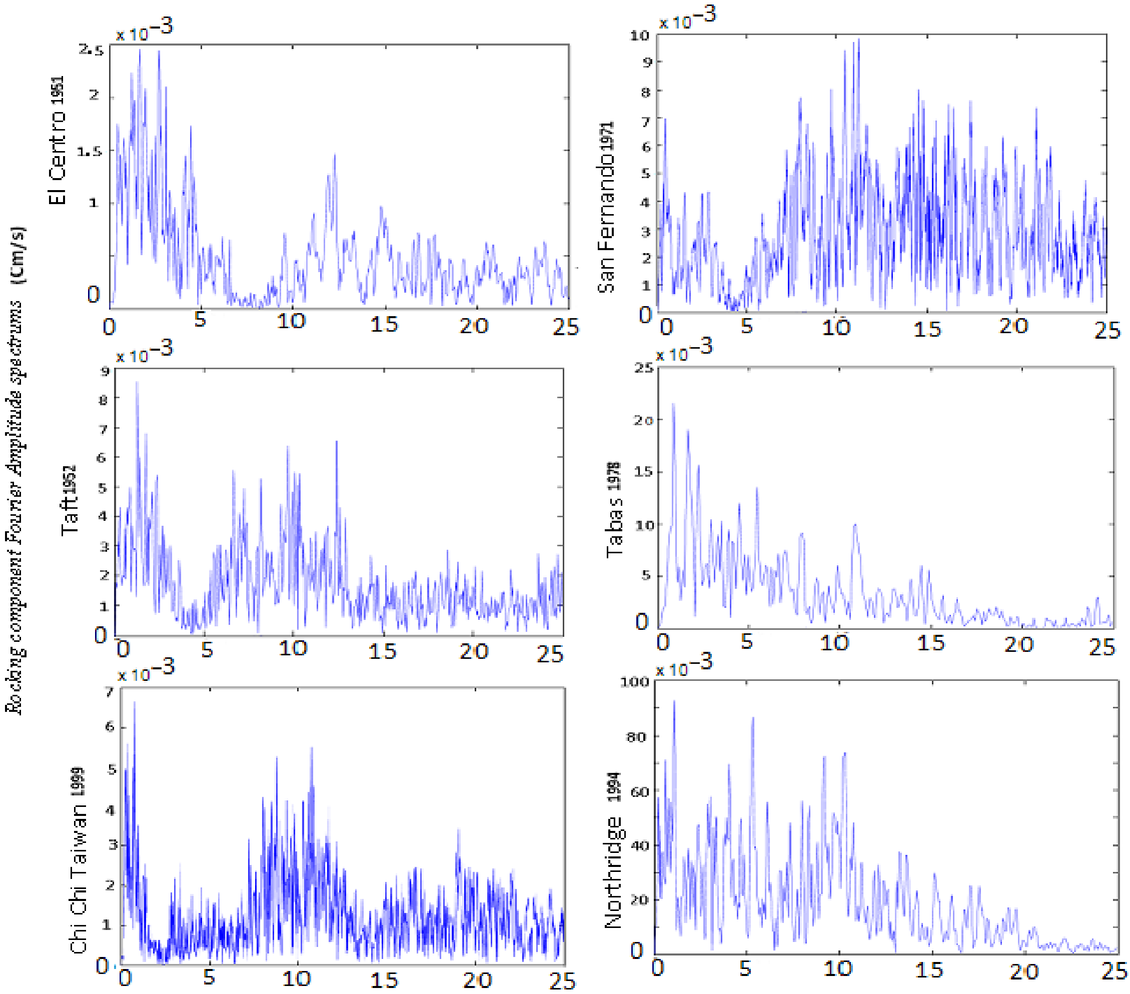


Figure2. Rocking component Fourier Amplitude spectrums of six earthquake

Table 2. The peak acceleration and velocity of rocking and torsional components of earthquakes

EARTHQUAKE		Imperial Valley	San Fernando	Taft	Tabas,Iran	Chi Chi Taiwan	Northridge
Rocking Comp.	$\ddot{\varphi}_{\max}(mrad/s^2)$	-6.17	-57	15.60	-108.5	11.5	447.4
Torsional Comp.	$\ddot{\varphi}_{\max}(mrad/s^2)$	3.73	-32.8	7.84	-58	-9.49	41

4.CONCLUSIONS

In this paper, improved approach is considered to generate rotational components of the earthquake. In absent of dense recorded data of translational components and real rotational data, this method can be applied to generate rotational components of the earthquake. This approach, which its frequency depends on wave velocity, has a good agreement with other theoretical methods and maximum rotational acceleration is much closer to the real data compared to result of elasticity theory

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