# Viscous Damping Properties of Small Scaled Structures during Dynamic Experiments

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## SUMMARY:

Viscous damping properties have been required in order to predict the accurate earthquake response of building structures. Static loading tests, free vibration tests, and shaking table tests were executed with several small scaled structures as specimens, and an evaluation formula for the viscous damping force was proposed in proportional to the response acceleration. A pseudo-dynamic test of the identical specimen was carried out utilizing the proposed formula in order to confirm the accuracy of simulated dynamic response.

Keywords: viscous damping, dynamic response, free vibration, shaking table, pseudo-dynamic test

# 1. INTRODUCTION

Viscous damping is one of the most effective factors which affect dynamic responses of building structures during earthquakes. The mechanism of viscous damping has not been clarified numerically, although several phenomena, such as the inner friction of materials, backlashes of member's connection, the interaction of the building and the ground, and so on, are supposed as concerning reasons [1]. When dynamic earthquake response analyses of buildings are carried out with computer programs, viscous damping force proportional to response velocity has been supposed in the equation of motion generally. Validity of the approach is not certain mechanically. Detailed information of viscous damping is required for the accurate earthquake response analyses of building structures.

Several experiments using identical specimens were executed to identify the viscous damping properties corresponding to the shaking attitude of structures. The specimen was a small scaled structure consisted of flexible four aluminium columns with a top steel plate and a steel basement. Static loading tests, free vibration tests, and shaking table tests were carried out with identical specimens. Discussing those tests results, an estimation method for viscous damping was proposed in this paper. A pseudo-dynamic test of the specimen was carried out utilizing the proposed method for viscous damping in order to confirm the accuracy of simulated dynamic response.

## 2. SPECIMEN AND PARAMETER

For the experiments, small scaled structures which had flexible four aluminium columns were built as specimens. As shown in **Figure 1**, the four columns were connected stiffly using fixing blocks to a rigid H-section steel as basement and a rigid steel plate as top mass, in order to consider the specimen as one degree of freedom mass system. Supposed inverse symmetry deflections of those columns were confirmed by static loading tests of the specimen.

Test parameters were column section, column height, and mass as shown in **Table 1**. Naming of each specimen according to the test parameter was also shown in the table. Eight types of specimens were constructed totally for each experiment.

In this study, aluminium specified JIS (Japanese Industrial Standards) A6063 was adopted as column material with considering flexibility and workability. Yield strength, tensile strength and

Young's modulus of used aluminium are shown in **Table 2** as mechanical properties. Those properties were obtained by tensile tests of aluminium test-pieces.

Calculated stiffness and natural frequency of each specimen are shown in Table 3. Those values were calculated with size and properties of aluminium columns and mass of specimen, considering the specimen as one degree of freedom mass system.

# **3. STATIC LOADING TEST**

Static loading tests of specimens were carried out using the testing apparatus shown in Figure 2. A shaking table locked with stoppers was used as a reaction floor. The specimen was loaded laterally by an actuator (electric motor type). A counter weight in the figure was appended in order to cancel the weight of the actuator. The restoring force of the specimen was measured by a load cell installed at the tip of the actuator, while the lateral displacement at the top of the specimen was measured by a laser transducer set to a rigid frame. The loading program is shown in Figure 3. As shown in the figure, lateral displacement amplitude was gradually increased from 10 mm to 90 mm, repeating plus and minus. The plus direction is the case of pushing by the actuator, while the minus direction is the case of pulling. The elastic stiffness of each specimen was confirmed to agree with the calculated value shown in the Table 3 mostly.

Table I. Paramete	Column width		
Column section (mm)	Column height (mm)	Mass (kg)	$\overline{)}$
2 x 20	300	$8.17 \rightarrow M1$	
2 x 30	350	$12.74 \rightarrow M2$	Colur

1

(	20-300-M1
(	Column height

Mass

Table 2. Mechanical properties of aluminium for column

Yield strength	Tensile strength	Young's modulus	
(N/mm <sup>2</sup> )	(N/mm <sup>2</sup> )	$(N/mm^2)$	
188	209	$6.30 \ge 10^4$	

Specimen	stiffness (N/mm)	Natural frequency (Hz)	Specimen	stiffness (N/mm)	Natural frequency (Hz)
20-300-M1	0.99	1.75	20-300-M2	0.86	1.31
30-300-M1	1.75	2.32	30-300-M2	1.63	1.79
20-350-M1	0.56	1.31	20-350-M2	0.43	0.92
30-350-M1	1.05	1.79	30-350-M2	0.93	1.35

Table 3. Calculated stiffness and natural frequency of specimen



Figure 1. Small scaled structure as specimen

As an example of test results, hysteresis loops of < 20-300-M1 > are shown in **Figure 4** (a). Those loops of all tests were stable spindle-shaped like the figure. The hysteresis loop was simulated by Menegotto-Pinto Model [2]. As shown in the figure, "Model", it was simulated loop by Menegotto-Pinto Model, was fitted to "Test", it was the test result. The equivalent damping factor,  $h_{eq}$ , was calculated on each loop. About the case of < 20-300-M1 >, the comparison of "Test" and "Model" was shown in **Figure 4** (b). The ratio of  $h_{eq}$  was in agreement over 90% while the displacement amplitude was more than 50mm. It was confirmed that Menegotto-Pinto Model was suitable in order to estimate the restoring force of this specimen in the elasto-plastic region.



Figure 2. Testing apparatus for static loading test



Figure 3. Loading program for static loading test



Figure 4. Hysteresis behaviour of static loading test and Menegotto-Pinto Model < 20-300-M1 >

### **4. FREE VIBRATION TEST**

Free vibration tests of specimens were carried out on the locked shaking table shown in the **Figure 2** as the first dynamic experiment. The initial displacement was given at the top of the specimen in order to vibrate the specimen in the elastic region. The lateral displacement at the top of the specimen was measured by the laser transducer during the test.

The equation of motion for this free vibrating specimen is expressed as Eqn. 4.1. From this equation, the viscous damping force  $(D_F)$  could be obtained from the inertia force  $(m\ddot{x})$  and restoring force (kx) as shown in Eqn. 4.2.

$$m\ddot{x} + D_F + k_X = 0 \tag{4.1}$$

where,  $\ddot{x}$ ; response acceleration, x; response displacement,  $D_F$ ; viscous damping force, m; mass, k; elastic stiffness

$$D_F = -(m\ddot{x} + k_X) \tag{4.2}$$

As an example, waveforms of the inertia force, the restoring force and the viscous damping force during the case of < 20-300-M1 > are shown in **Figure 5**. It was found that the obtained damping force was proportional to the response acceleration as same as the inertia force, instead of response velocity. An estimation method for the viscous damping force is proposed in Eqn. 4.3.

$$D_F' = 2h'\sqrt{mk} \quad \ddot{x} \tag{4.3}$$

where,  $D_F$ '; estimated viscous damping force, h'; supposed damping factor

Supposed damping factor, h', was calculated by the obtained damping force of each test, and analyzed as a relation with the elastic stiffness, k. As shown in the **Figure 6**, the supposed damping factor, h', has the linear relation with  $1/k^2$ . In this study, the supposed damping factor, h', of the specimen could be calculated by Eqn. 4.4.

$$h' = \frac{0.2}{k^2} \quad (\%) \tag{4.4}$$







**Figure 6.** Relationship of supposed damping factor; *h* and stiffness; *k* 

## 5. SHAKING TABLE TEST

Shaking table tests of specimens were performed utilizing the shaking table shown in the **Figure 2**. The table was vibrated by an electromagnetic dynamic jack in the lateral direction. The specimen was set to the middle of the table, while the actuator system was taken off from the table. The lateral displacement at the top of the specimen, the lateral displacement of the table and the lateral acceleration on the table were measured during the test. The inputted ground motions were sine waves which amplitude was increased gradually, while the frequency and the maximum acceleration of them were varied in several tests.

As an example of test results, waveforms of the response displacement and the ground acceleration of the case of < 20-300-M1 > are shown in **Figure 7**. The response displacement intended the relative lateral displacement at the top of the specimen, while the ground acceleration was the measured lateral acceleration on the shaking table.

About each test result, the viscous damping force was extracted by Eqn. 4.2. In the formula, the Menegotto-Pinto Model was useful in order to estimate the restoring force, because the specimen behaved not only in the elastic region but also the plastic region. The obtained viscous damping force of the case of < 20-300-M1 > was plotted in **Figure 8** as "Measured". "Calculated" in the figure was the simulated viscous damping force calculated using Eqn. 4.3 and Eqn. 4.4. Hence the waveform had close agreement with "Measured", it was confirmed that the proposed estimation method for viscous damping was effective for dynamic response of the specimen.



Figure 7. Waveforms of response displacement and ground acceleration during shaking table test < 20-300-M1 >



Figure 8. Measured and calculated damping force during shaking table test < 20-300-M1 >

### 6. PSEUDO-DYNAMIC TEST

A pseudo-dynamic test of the specimen < 20-300-M1 > was performed in order to compare with the dynamic response during a shaking table test of the identical specimen. The testing apparatus for the pseudo-dynamic test was same to the static loading test shown in the **Figure 2**. The testing component with the flow of data is shown in **Figure 9**. In the PC, the specimen was supposed as one degree of freedom mass model, and the equation of motion for the mass model was expressed as Eqn. 6.1. In the equation, proposed damping force,  $D_F$ , which was proportional to the response acceleration as shown in Eqn. 6.2, was employed with the supposed damping factor, h'=0.203 (%) as shown in Eqn. 6.3. The restoring force, Q, was obtained from the loading actuator, while the recorded lateral acceleration on the table during the shaking table test was adopted as the ground acceleration,  $\ddot{x}_0$ .

$$m\ddot{x} + D_F' + Q = -m\ddot{x}_0 \tag{6.1}$$

$$D_F' = 2h' \sqrt{mk} \quad \ddot{x} \tag{6.2}$$

$$h' = \frac{0.2}{k^2} = 0.203 \,(\%) \tag{6.3}$$

where,  $\ddot{x}$ ; response acceleration,  $\ddot{x}_0$ ; ground acceleration,

*m*; mass, *k*; elastic stiffness, *Q*; restoring force,

 $D_F$ '; estimated viscous damping force, h'; supposed damping factor

For the numerical computing in the PC, the operator splitting method [3] was utilized as the time integration technique, considering manageability and stability. The time increment was set to 0.01 sec, according to the data recording period during the shaking table test. This pseudo-dynamic test simulated the dynamic response of the specimen during 0 - 6 sec of the shaking table test.

When the actuator was loading the specimen to the target displacement, the overshooting was only allowed as the displacement control error [4] while the undershooting was rejected. The precise restoring force at the target displacement of the specimen was estimated by the linearly interpolation method [4] utilizing the measured restoring force.

The comparison of displacement waveforms of the pseudo-dynamic (PSD) test and the shaking table (SHK) test are shown in **Figure 10**. Those waveforms were overlapped mostly, although the slight gap of cycles was found partially. The difference of displacement amplitude of them was settled to less than 5 % in almost all portions as shown in **Figure 11**. Comparing cyclic frequencies of both tests in **Figure 12**, the close proximity of them is found after the response time of 1.5 sec.



Figure 9. Component for pseudo-dynamic test

The quarter cycle periods of both tests are compared in **Figure 13**, dividing the case of loading and unloading. The distribution of them had the similar tendency in each case. It was verified that the pseudo-dynamic test simulated the shaking table test of the identical specimen in high accuracy.



Figure 10. Displacement waveforms during pseudo-dynamic (PSD) test and shaking table (SHK) test



Figure 11. Ratio of displacement amplitude; pseudo-dynamic (PSD) test / shaking table (SHK) test



Figure 12. Cyclic frequency during pseudo-dynamic (PSD) test and shaking table (SHK) test



Figure 13. Quarter cycle period during pseudo-dynamic (PSD) test and shaking table (SHK) test

## 7. CONCLUSIONS

In order to discuss about detailed information of viscous damping, several experimental works were carried out in this study. Specimens for the experiments were small scaled structures which had flexible aluminium columns, while test parameters were column section, column height, and mass of the specimen. The specimen was regarded as one degree of freedom mass model because the columns were deflected in inverse symmetry when the specimen was loaded laterally.

Static loading tests and free vibration tests were executed using the identical specimens. From those test results, the viscous damping force of the specimen vibrating in the elastic region was extracted from the inertia force and the restoring force. Considering the extracted damping force, an estimation formula for the viscous damping force was proposed in proportion to the response acceleration while the supposed damping factor was in inverse proportion to square of the elastic stiffness.

Shaking table tests of specimens were executed, and the dynamic responses of specimens were obtained in the elasto-plastic region. Extracting the viscous damping force from test results, it was confirmed that the proposed estimation method for viscous damping was effective for dynamic response of the specimen in this study.

A pseudo-dynamic test of the specimen was performed in order to compare with the dynamic response during a shaking table test of the identical specimen. Proposed viscous damping force was employed with the supposed damping factor in the equation of motion solved by the numerical computing. Comparing the response displacement and the cyclic frequency, it was verified that the pseudo-dynamic test simulated the shaking table test of the identical specimen in high accuracy.

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