Inelastic Response and Ductility Demand of Structures

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SUMMARY
Current seismic codes require from the seismically designed structures to be capable to withstand inelastic deformations. Many studies dealt with the development of different inelastic spectra with the aim to simplify the evaluation of inelastic deformation and performance of structures. Recently, the concept of inelastic spectra has been adopted in the global scheme of the performance-based seismic design through capacity-spectrum methods. In this paper, the median of the ductility demand ratio for 80 ground motions is presented for different levels of normalized yield strength, defined as the yield strength coefficient divided by the peak ground acceleration (PGA). The influence of the post-to-preyield stiffness ratio on the ductility demand is investigated. Determined by regression analysis of the data, two design equations have been developed; one for the ductility demand as function of period, post-to-preyield stiffness ratio, and normalized yield strength, and the other for the inelastic deformation as function of period and peak ground acceleration valid for periods longer than 0.6 seconds. The equations are useful in estimating the ductility and inelastic deformation demands for structures in the preliminary design. It was found that the post-to-preyield stiffness has a negligible effect on the ductility factor if the yield strength coefficient is greater than the PGA of the design ground motion normalized by gravity.

Keywords: Ductility, Inelastic, Deformation, Capacity-Spectrum Method, Ground Motion

1. INTRODUCTION

Since the development of the Capacity-Spectrum Method (CSM) (Freeman *et al.* 1975), many response spectra have been proposed to replace the conventional elastic spectra in order to achieve accurate evaluation of the inelastic response of structures. Newmark and Hall (1973) proposed inelastic response spectra of 5% damped single degree of freedom (SDOF) system based on the elastic spectra.

Sheng and Biggs (1980) constructed inelastic response spectra for different ductility ratios and damping coefficients. Iwan (1980) and Kowalsky (1994) developed empirical equations to define the period shift and equivalent viscous damping ratio to estimate the maximum displacement demand of inelastic SDOF system from its linear representation. Ridell *et al.* (1989) introduced reduction factors for constructing inelastic design response spectra from elastic spectrum. Similarly, Krawinkler and Nassar (1990) studied strength reductions by using 33 ground motions recorded during the 1989 Whittier Narrow, California earthquake. Later (1991), they evaluated the average inelastic spectra of bilinear and stiffness degrading systems subjected to 15 ground motions and proposed a functional form of the reduction factor with respect to ductility factor, natural period and post-to-preyield stiffness ratio. Similarly, Miranda (1992) used 124 ground motions recorded during various earthquake events. Emphasis is given to the influence of soil conditions on the inelastic strength and deformation demands of SDOF systems.

The use of inelastic design spectrum in the context of capacity-spectrum methods was suggested by

Bertero (1995), and introduced by Reinhorn (1997) and Fajfar (1999). Recently, Miranda (2000) proposed procedures based on displacement modification factors in which the maximum inelastic displacement demand of multi-degree of freedom (MDOF) system is estimated by applying certain displacement modification factors to maximum deformation of equivalent SDOF system having the same lateral stiffness and damping coefficient as that of MDOF system. In another study, Miranda and Ruiz-Garcia (2002) evaluated six possible alternative methods to estimate the maximum inelastic deformations of SDOF systems. The evaluated methods estimate the maximum inelastic deformation using functions of displacement ductility factor to compute equivalent periods and equivalent damping ratios (Rosenblueth and Herrera 1964; Gulkan and Sozen 1974; Iwan 1980; Kowalsky 1994), or to compute displacement modification factors (Newmark and Hall 1982; Miranda 2000). The study aimed at evaluating the accuracy of approximate methods for the preliminary design of structures. Later, Akkar and Miranda (2002) conducted a statistical evaluation of five of the above methods. The study showed that for periods longer than $1.0\ s$ all methods produce relatively good results. In the short period region, equivalent linear methods proposed by Iwan and Guyader and the one proposed by Kowalsky tend to overestimate deformation demands. The errors produced by any of the evaluated approximate methods can be relatively larger, particularly for lateral strength ratios larger than four.

In this paper, we first present the development of the ductility demand response spectrum. This is followed by the development of the response spectra for four selected data-sets of ground motion records, each containing 20 records representing short to long magnitude and short to long distances from fault. The influence of the post-to-preyield stiffness ratio on the ductility demand is investigated. Next, nonlinear regression analysis of the data is conducted, two design equations are developed; one for the ductility demand as function of period, post-to-preyield stiffness ratio, and normalized yield strength, and the other for the inelastic deformation as function of period and peak ground acceleration valid for periods larger than 0.6 seconds. The equations are useful in estimating the ductility and inelastic displacement demands for structures in the preliminary design.

2. DUCTILITY DEMAND RESPONSE SPECTRUM

2.1. Equation of Motion in Terms of Ductility

Considering an inelastic single-degree of freedom system, its motion when subjected to an earthquake ground motion is governed by the following equation:

$$m\ddot{x} + c\dot{x} + f(x,\dot{x}) = -m\ddot{u}_g(t) \qquad (2.1)$$

where, m, c, and f represent the mass, damping, and the resisting force of the system, respectively, $\ddot{u}_g(t)$ denotes the earthquake acceleration. The resisting force f is defined as the sum of a linear part and a hysteretic part:

$$f = k_P x + Qz \qquad (2.2)$$

In the above, k_P is the postyield stiffness, Q is the yield strength, and z represents the dimensionless variable that characterizes the Bouc-Wen model of hysteresis (Bouc, 1971; Wen, 1976), it is given by:

$$\dot{z} = \frac{\dot{x}}{x_y}[A - |z|^n(\gamma\ sgn(\dot{x}z) + \beta)] \qquad (2.3)$$

In the above equation, x_y is the yield displacement, and A, n, γ, and β are parameters that control the shape of the hysteresis loop. Material degrading is not considered in this study. However, to accommodate material degradation in the hysteresis model, the Baber-Noori (Baber and Noori, 1985) version of the Bouc-Wen model may be used.

Substituting Eqn. 2.2 into Eqn. 2.1 and dividing by m yields:

$$\ddot{x} + 2\xi\omega\dot{x} + \frac{k_P}{m}x + \frac{Q}{m}z = -\ddot{u}_g(t) \tag{2.4}$$

Or simply:

$$\ddot{x} + 2\xi\omega\dot{x} + \alpha\omega^2 x + qgz = -\ddot{u}_g(t) \tag{2.5}$$

In which ξ, ω, α, and q represent the damping ratio, circular frequency, post-to-preyield stiffness ratio, and the yield strength coefficient (defined as yield strength divided by the system weight W: $W = mg$, g stands for the gravity), respectively.

Next, Eqn. 2.5 is rewritten in terms of displacement ductility factor, μ. Substituting: $x = x_y\mu$, $\dot{x} = x_y\dot{\mu}$, and $\ddot{x} = x_y\ddot{\mu}$, and in Eqn. 2.5 and dividing by x_y gives:

$$\ddot{\mu} + 2\xi\omega\dot{\mu} + \alpha\omega^2\mu + \frac{qgz}{x_y} = -\frac{1}{x_y}\ddot{u}_g(t) \tag{2.6}$$

By doing the same to the dimensionless variable z, Eqn. 2.3 may be expressed in terms of $\dot{\mu}$ as:

$$\dot{z} = \dot{\mu}[A - |z|^n(\gamma\, sgn(\dot{\mu}z) + \beta)] \tag{2.7}$$

The term $\frac{qg}{x_y}$ in Eqn. 2.6 is rewritten as:

$$\frac{qg}{x_y} = \omega^2(1-\alpha) \tag{2.8}$$

Solving Eqn. 2.8 for x_y yields:

$$x_y = \frac{qg}{\omega^2(1-\alpha)} \tag{2.9}$$

Substituting Eqn. 2.8 and Eqn. 2.9 into Eqn. 2.6 gives:

$$\ddot{\mu} + 2\xi\omega\dot{\mu} + \alpha\omega^2\mu + \omega^2(1-\alpha)z = -\frac{\omega^2(1-\alpha)}{qg}\ddot{u}_g(t) \tag{2.10}$$

We observe from Eqn. 2.10 that for a given ground acceleration, $\mu(t)$ depends on ξ, ω, α, and q.

2.2. System Controlling Parameters and Normalization

To obtain meaningful system response to an ensemble of ground motions, the system yield strength coefficient has to be defined relative to the intensity of individual ground motions. Using the parameter η introduced by Mahin and Lin (1983) as:

$$\eta = \frac{qg}{PGA} \tag{2.11}$$

where, PGA stands for the Peak Ground Acceleration. Incorporating η into Eqn. 2.10 results:

$$\ddot{\mu} + 2\xi\omega\dot{\mu} + \alpha\omega^2\mu + \omega^2(1-\alpha)z = -\frac{\omega^2(1-\alpha)}{\eta}\overline{\ddot{u}}_g(t) \tag{2.12}$$

In which, $\overline{\ddot{u}}_g$represents the ground acceleration normalized with respect to the PGA:

$$\overline{\ddot{u}}_g(t) = \frac{\ddot{u}_g(t)}{PGA} \tag{2.13}$$

The ground acceleration has been normalized such that its value varies from -1 to 1. Eqn. 2.12 implies that for a given inelastic system, if α and η are fixed, the intensity of the ground motion has no effect on the peak normalized deformation, μ. This permits the construction of the ductility response spectrum for an ensemble of ground motions with common frequency content but variable intensity.

2.3. Constant$-\eta$ Ductility Response Spectrum

The procedure to construct the ductility response spectrum for inelastic systems corresponding to specified levels of normalized yield strength η, is summarized in the following steps:

1. Define the ground motion $\ddot{u}_g(t)$;

2. Select and fix the damping ratio ξ and the post-to-preyield stiffness ratio α (α = 0 for elastoplastic system) for which the spectrum is to be plotted;

3. Specify a value for η;

4. Select a value for elastic period T;

5. Determine the ductility response $\mu(t)$ of the system with T, ξ, and α equal to the values selected by solving Eqn. 2.12 along with Eqn. 2.7. From $\mu(t)$ determine the peak ductility factor μ;

6. Repeat steps 4 and 5 for a range of T, resulting in the spectrum values for the η value specified in step 3;

7. Repeat steps 3 to 6 for several values of η

Given the excitation and the properties T, ξ, α, and η of an inelastic SDOF system, it is desired to determine the peak deformation, x_m. The yield displacement of the system is derived from Eqn. 2.9 and Eqn. 2.11 as follows:

$$x_y = \frac{\eta}{\omega^2(1-\alpha)} PGA \qquad (2.14)$$

The value of the ductility factor is read from the spectrum developed by the above procedure and multiplied by x_y to obtain the peak deformation, x_m.

3. STATISTICAL ANALYSIS

Based on statistical analysis of response data, the nonlinear spectra are developed next. Four ensembles of ground motions, each with 20 records, are included in this study. The ensembles, denoted by LMSR, LMLR, SMSR, and SMLR represent four combinations of large ($M = 6.6 - 6.9$) or small ($M = 5.8 - 6.5$) magnitude and short ($R = 13 - 30\ km$) or long ($R = 30 - 60\ km$) epicentral distance. These motions were obtained from PEER Strong Motion Database, first used by H. Krawinkler; their parameters are available in (Chopra *et al.* 2003).

3.1. Median Ductility Demand

Median normalized spectra were computed for each ground motion of the four ensembles. Median ductility-demand spectra for LMSR ensemble are shown in Fig. 3.1 and Fig. 3.2 as a function of T for fixed damping ratio ξ = 5%; all results presented in this paper are for this damping ratio. The spectrum is divided logically into three period regions according to the procedure described in Chopra (2001), where T_b marks the transition from the acceleration-sensitive region to the velocity-sensitive region which ends at $T\sim$ 3s. The acceleration-sensitive region is divided judgmentally into two regions at

$T = T_a$, at this point the spectrum would be easily idealized by a series of straight lines.

The results for fixed post-to-preyield stiffness ratio (Fig. 3.1) permit the following observations on how the normalized yield strength influences the inelastic action indicated by μ in various spectral regions. For systems having a normalized yield strength $\eta < 1$, μ decreases rapidly with an increase of the normalized yield strength. For systems having $\eta \geq 1$, in the acceleration-sensitive region, starting at $\mu = 1$ at $T = T_a = 0.1\,s$, μ increases for shorter periods where μ is affected little by η. For periods between T_a and $T_b = 0.6\,s$, the ductility demand is essentially constant and affected little by η. For periods larger than T_b, the ductility demand decreases in a manner similar to systems with $\eta < 1$.

From Fig. 3.1 for fixed ductility factor, and consistent with previous studies (e.g., Newmark and Hall 1982, Bozorgnia et al. 2010, among others) it is clear that the yield strength can be substantially reduced if a moderate level of ductility (e.g., $\mu = 2$) is sustained by the structure. Larger available ductility can only result in a moderate reduction in the strength demand, especially for periods larger than 0.5 s. The results for constant$-\eta$ plots in Fig. 3.2 indicate that the post-to-preyield stiffness ratio has no effect on the median ductility demand for systems having $\eta \geq 1$. For systems with $\eta < 1$ the Post-to-preyield stiffness ratio reduces the ductility demand only for periods shorter than T_b, and for periods greater than T_b, α has a small effect on the ductility factor.

Computed from the data of Fig. 3.1 and Fig. 3.2, the ratio of median μ for bilinear and elastoplastic systems is plotted in Fig. 3.3 against T for fixed α and a range of η values. Fig. 3.3 indicates that the amount of reduction in ductility factor due to post-to-preyield stiffness ratio α is roughly constant for periods greater than T_b, increases for shorter periods ($T < T_b$), and disappears for all range of periods for $\eta \geq 1$. This observation implied that ignoring post-to-preyield stiffness ratio in estimating the ductility demand is too conservative for seismic evaluation of existing structures with periods in the acceleration-sensitive region; this is similar to what Chopra and Chintanapakdee (2004) concluded. However, the post-to-preyield stiffness ratio may be ignored in estimating the ductility demand for structures with short periods ($T < T_b$) if they have a normalized yield strength greater than 1. In other words, for a particular ground motion characterized by *PGA*, one may ignore the effect of the post-to-preyield stiffness ratio in evaluating structures with periods shorter than 0.6 s (low rise buildings) if their yield strength coefficients are greater than the *PGA* normalized by g.
The results for constant$-\eta$ plotted in Fig. 3.2 indicate that regardless the post-to-preyield stiffness ratio; the response of systems is elastic in the following cases:

- $\eta = 0.25$ and $T \geq 3\,s$
- $\eta = 0.5$ and $T \geq 2$ s
- $\eta = 0.75$ and $T \geq 1.45\,s$
- $\eta = 1$ and $T \geq 1\,s$
- $\eta = 1.5$ and $T \geq 0.75\,s$
- $\eta > 1.5$ at all periods.

Study of the dispersion of the ductility demands was conducted by computing the coefficients of variation (COV) for each ensemble of ground motions. Fig. 3.4(a) shows the dispersion of μ as a function of natural period for $\alpha = 3\%$ and different levels of normalized yield strength, while Fig. 3.4(b) shows the dispersion of μ for $\eta = 1$ and different post-to-preyield stiffness ratios for LMSR ensemble of ground motions. It can be noted that, in general, dispersion of ductility demands is not constant over the whole spectral regions and it depends on the natural period *T* and the level of normalized yield strength η. In general, dispersion increases as period of vibration increases, this is the case because the *PGA* has been used in the normalization. However, this study concerns systems having periods in the acceleration and till velocity-sensitive region. The post-to-preyield stiffness ratio has little effect on the dispersion, especially for periods larger than 1 s.

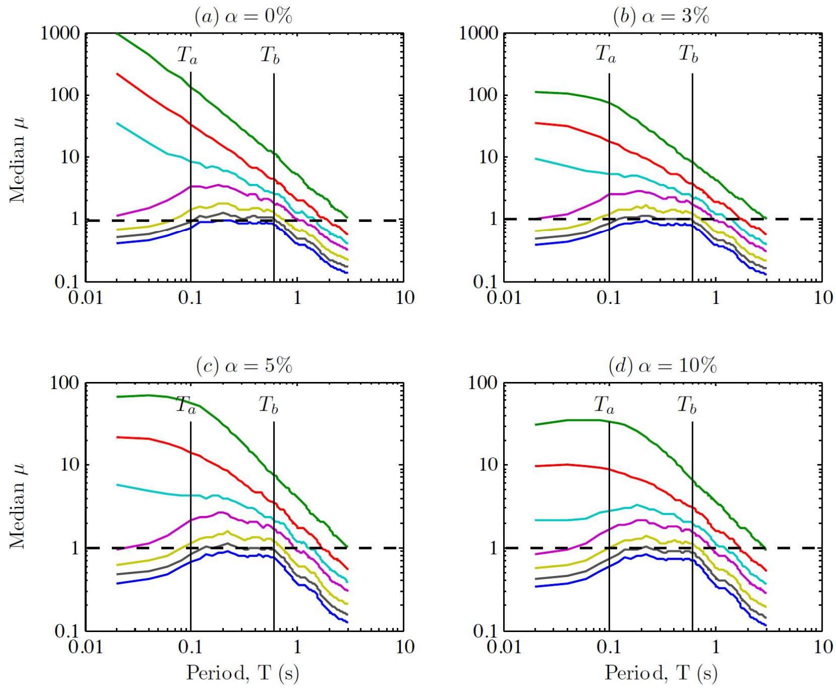

Figure 3.1 Median ductility demand for inelastic systems computed for LMSR ensemble of ground motions (η = 0.25, 0.5, 0.75, 1, 1.5, 2, 2.5 from top line to bottom line).

3.2. Effect of Earthquake Magnitude and Distance

The computations that led to the preceding subsection were repeated for the LMSR, LMLR, SMSR, and SMLR of ground motion ensembles. The median μ versus T functions are quite similar, this is demonstrated in the plots for the four ensembles shown in Fig. 4.1. This similarity has been observed by Chopra and Chintanapakdee (2004) in the plots of C_μ and C_R coefficients versus T.

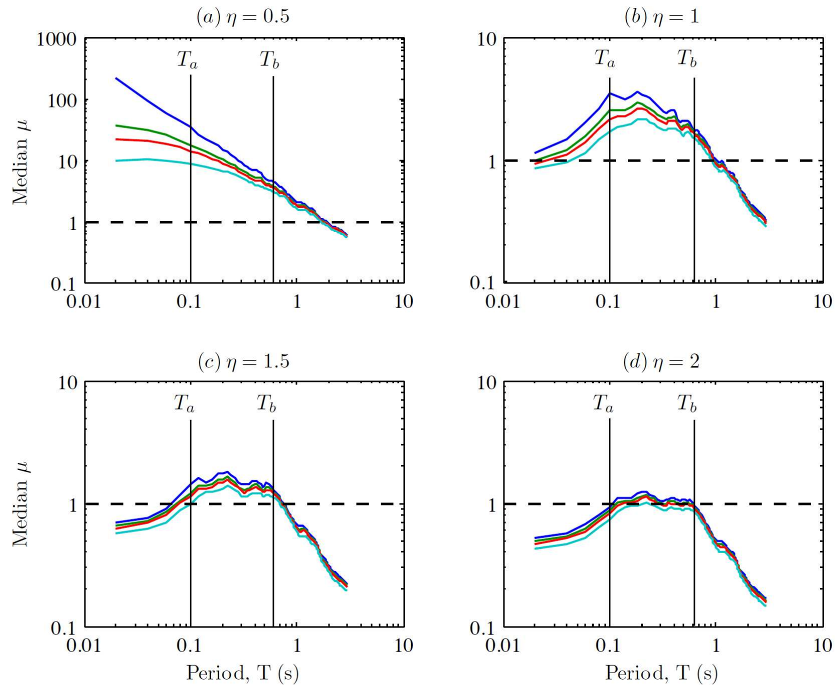

Figure 3.2 Influence of post-to-preyield stiffness ratio α (α = 0%, 3%, 5%, 10% from top line to bottom line) on the median of ductility demand μ for inelastic systems subjected to LMSR ensemble of ground motions, presented for fixed values of normalized yield strength η = 0.5, 1, 1.5, and 2.

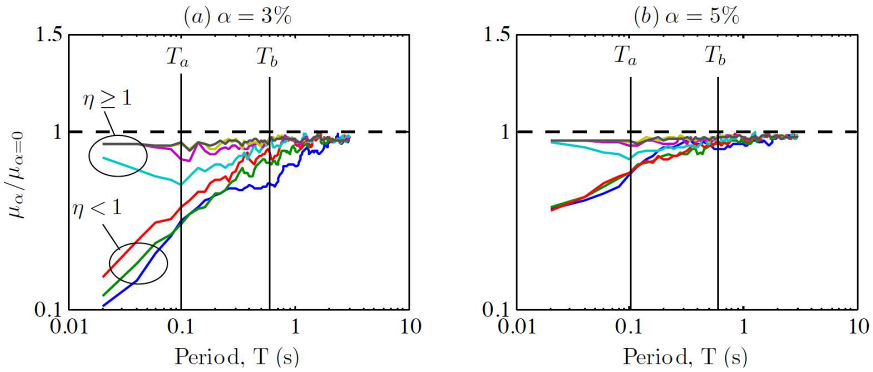

Figure 3.3 Ratio of median ductility demand μ for bilinear and elastoplastic systems: (a) influence of η for α =3% and (b) influence of η for α =5%.

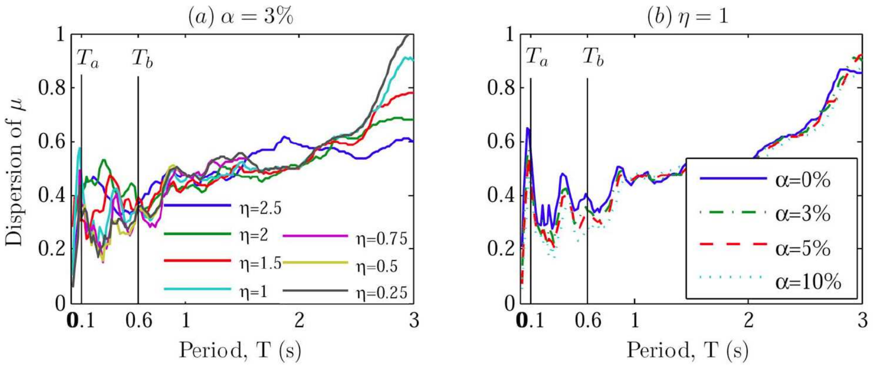

Figure 3.4 Dispersion of ductility demand μ for inelastic systems subjected to the LMSR ensemble of ground motions: (a) influence of normalized yield strength η at post-to-preyield stiffness ratio α =3% and (b) influence of α for η =1.

4. ESTIMATING OF DUCTILITY AND INELASTIC DEFORMATION DEMANDS

Presented next is an equation that fits the median μ data for any of the four ensembles of ground motions, starting from T_a. Such an equation for μ has been derived in terms of the normalized yield strength η, post-to-preyield stiffness ratio α, and period of vibration T:

$$\mu = \frac{a(b)^{1/T}(T)^{\eta-c}}{\eta} \tag{4.1}$$

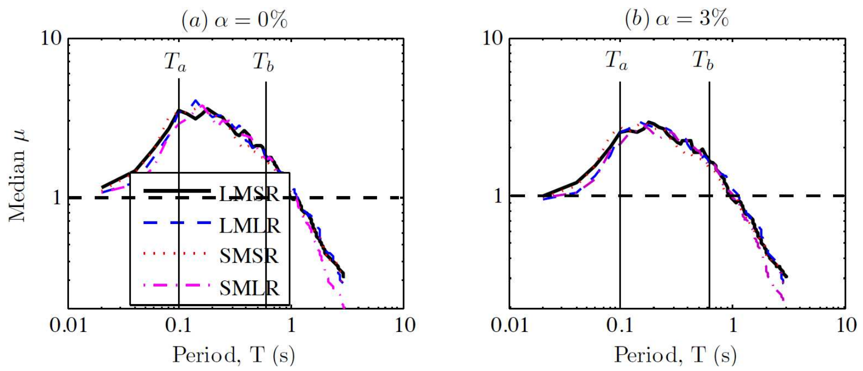

Figure 4.1 Comparison of ductility demand μ for LMSR, LMLR, SMSR, and SMLR ensembles of ground motions: (a) μ for α =0% and η =1; (b) α =3% and η =1.

Table 4.1 Parameters in Eqn. 4.1 and 4.2 for each value of η and α

Parameters	$\eta < 1$				$\eta \geq 1$ (all values of α)
	α (%)				
	0	3	5	10	
a	1.24	1.12	1.08	1.04	1.23
b	0.98	0.94	0.92	0.88	0.85
c	1.69	1.65	1.68	1.68	1.21+η

The numerical parameters a, b, and c were determined from the response data by nonlinear regression analysis using Datafit software (Oakdale Engineering) and they are tabulated in Table 4.1 for different ranges of η and α. However, for values of $\eta \geq 1$ and periods larger than T_b, the post-to-preyield stiffness ratio is ignored since it has little influence on the ductility factor. For $\eta \geq 1$ and for periods between T_a and T_b, ductility is observed to be nearly independent of period *T*, depending on the value of η and α (see Table 4.2). Figure 4.2 shows that Eqn. 4.1 agrees well with median μ computed for the LMSR ground motions. The inelastic deformation x_m can be determined by multiplying the ductility factor computed from Eqn. 4.2 by the yield displacement x_y determined from Eqn. 2.14:

$$x_m = \frac{PGA}{4\pi^2}\left[\frac{a(b)^{1/T}(T)^{\eta+2-c}}{(1-\alpha)}\right] \tag{4.2}$$

Table 4.2 Values of ductility μ for $T_a \leq T < T_b$

η	α (%)			
	0	3	5	10
1	-	2.5	2	1.65
1.5	1.5	1.4	1.3	1.3

T_a= 0.1s, T_b= 0.6 s

Eqn. 4.1 and Eqn. 4.2 show an advantage over the coefficients like C_μ and C_R that require the determination of the elastic response first. In addition, for periods longer than $T_b = 0.6\,s$ a unified equation is found to fit the median ductility demand:

$$\mu = \frac{\omega^2(1-\alpha)}{\eta}0.027T^{0.84} \tag{4.3}$$

The median inelastic deformation x_m is then determined using Eqn. 2.14 for the yield displacement:

$$x_m = 0.027\,T^{0.84}PGA \tag{4.4}$$

This equation provides the deformation for a given median *PGA* that reflects the intensity of the design ground motions, regardless the yield strength and post-to-preyield stiffness ratio.

5. CONCLUSIONS

The main goal of this study was to estimate the displacement ductility and inelastic deformation demands for structures. For this purpose, the ductility demand μ was defined. The following conclusions can be drawn from the results of this study:

1. For systems with normalized yield strength smaller than one, the ductility factor decreases rapidly; this factor is very sensitive to the yield strength.

2. For systems with a normalized yield strength greater or equal to one, μ decreases in the acceleration sensitive region and it is affected little by the value of η.

3. For systems with periods between 0.1 s and 0.6 s and normalized yield strength greater or equal to

one, the ductility demand is essentially constant.

4. For systems with η> 2, the response is elastic regardless the period of the system.

5. The post-to-preyield stiffness ratio has no effect on the median ductility demand for systems with a normalized yield strength greater than one. For systems with normalized yield strength smaller than one, the post-to-preyield stiffness ratio reduces the ductility demand only for periods longer than $0.6\ s$, and has essentially little effect on the ductility demand for longer periods.

6. Consistent with other studies, the yield strength can be substantially reduced if a moderate level of ductility is sustained by the structure (say μ = 2). Larger available ductility can only result in a moderate reduction in the strength demand, especially for periods larger than $0.5\ s$.

7. Ignoring post-to-preyield stiffness ratio in estimating the ductility demand is too conservative for seismic evaluation of structures with periods in the acceleration-sensitive region. However, the post-to-preyield stiffness ratio may be ignored when the normalized yield strength is greater than one.

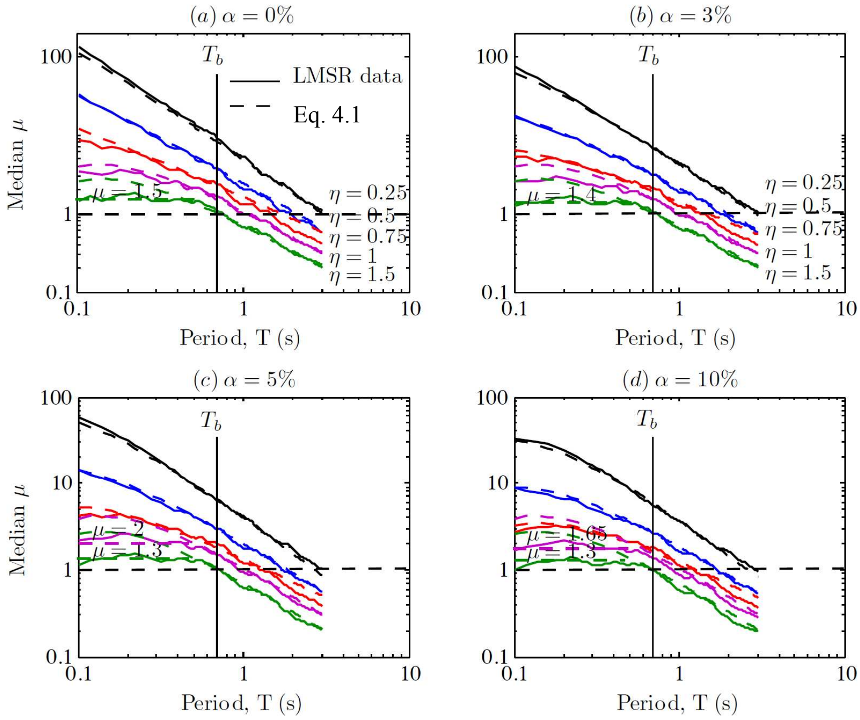

Figure 4.2 Comparison of ductility demand μ estimated by Eqn. 4.1 with computed data for LMSR ensemble of ground motions for: (a) elastoplastic systems (α =0%) and (b, c, d) bilinear systems (α =3%, 5%, 10%).

8. For the selected accelerograms, it was shown that the ductility demands are not affected by the earthquake magnitude nor by the epicentral distance.

9. The dispersion of μ is not constant over the whole period range; it depends on T and η.

10. Simplified equations for ductility and inelastic deformation demands for inelastic systems with periods shorter than $3\ s$ have been developed. These equations are simple and provide a good estimation of median ductility demand and inelastic displacement of new or rehabilitated structures when the yield strength is known.

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