# Proposal of a simplified method for estimating seismic risk of structures

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#### SUMMARY

In seismic risk analysis of structures, information on seismic hazard and seismic fragility are required. Both hazard and fragility functions (or curves) are convolved numerically, and seismic risk (or annual probability of failure) of the structure is evaluated. In this paper, a simplified method for seismic risk estimation is proposed, where information on seismic hazard curve and the capacity of the structure are used and numerical integration is not required, and the seismic risk can be obtained graphically from the hazard curve. The proposed method is demonstrated comparing with seismic risk estimated by numerical integration using the real hazard curves. The method is investigated in comparison with theoretical approach from the closed form risk equation.

Keywords: seismic risk, seismic hazard, fragility, seismic margin, risk equation

# **1. INTRODUCTION**

In the process of seismic risk evaluation of structures, seismic hazard analysis and fragility analysis are conducted, and then both hazard and fragility are convolved to estimate the seismic risk. In case specific functions of both seismic hazard and fragility are assumed, it is known that seismic risk is given as closed-form function (Kennedy 1999; McGuire 2004), which is called +risk equationg but in general cases numerical integration is required. For important and/or critical structures, from the seismic hazard estimated specific at the site and the fragility of the structure, numerical calculation (numerical integration) are conducted to estimate seismic risk. However, for ordinary buildings and civil structures distributing in a wide area it is difficult to take the same steps from the financial and technical reasons as ordinary structural engineers are not necessarily familiar with the estimation method of seismic risk. Of late, it is proposed that risk information should be applied to seismic design, rehabilitation planning of structures, and planning for seismic disaster mitigation, etc., and information on seismic hazard has been prepared by public organizations and the hazard information tends to be available. But even in these cases, numerical calculation is needed for risk estimation, which could sometimes be an obstacle for non-specialist to use risk information effectively. In this paper, a simplified method for estimating seismic risk is proposed through the concept of risk diagramø proposed by the authors (Nakajima et al. 2011) and numerical calculation is not required. The method is examined and compared with the results from the risk equation.

# 2. APPROACH BY RISK EQUATION

#### 2.1. Risk equation

Seismic risk (or annual probability of failure)  $P_F$  of a structure is estimated from seismic hazard evaluating annual exceedance of probability (in some cases  $\exists$  frequencyøis used instead of  $\exists$  probabilityø, but for small value of probability both can be considered the same) of ground motion intensity and fragility analysis evaluating conditional probability of failure for given ground motion intensity are conducted as,

$$P_F = \int_0^\infty H(a) \cdot (dF(a, A_{Fm}, \beta) / da) da$$
(2.1)

where H(a) is the seismic hazard for ground motion intensity  $\exists a \beta F(a, A_{Fm}, \beta)$  is the fragility,  $A_{Fm}$  is the median capacity of the structure expressed in the scale of  $\exists a \beta$  and  $\beta$  is the parameter denoting uncertainty of the capacity  $A_F$ , which is logarithmic standard deviation in case log-normal distribution is assumed. In general, seismic risk is calculated from Eqn. (2.1) conducting numerical integration, but it is known that under the assumption that the hazard is given as a power function and the fragility is given as lognormal distribution function, then Eqn. (2.1) is given as a closed-form (e.g., Kennedy 1999; McGuire 2004). In this case hazard function is assumed as a power function as

$$H(a) = K_I \cdot a^{-K}_H \tag{2.2}$$

where  $K_I$  is an appropriate constant,  $K_H$  is the slope of the hazard curve on a log-log scale. The slope is expressed using another parameter  $A_R$  related to the parameter  $K_H$  as  $K_H = 1/\log_{10} A_R$ . And fragility function is assumed to be given as

$$F(a) = ([\ln (a) - \ln (A_{Fm})]/\beta)$$
(2.3)

where  $\beta$  is calculated from logarithmic standard deviations  $\beta_S$  and  $\beta_R$  with respect to response and resistance respectively as  $\beta = \sqrt{\beta_S^2 + \beta_R^2}$ . Then,  $P_F$  in Eqn. (2.1) is given by the following equation which is called risk equation  $\beta$ 

$$P_F = K_I \cdot (A_{Fm})^{-\kappa} + \exp\left[(1/2) \left(K_H \cdot \beta\right)^2\right]$$
(2.4)

Removing  $K_I$  from Eqn. (2.2), Eqn. (2.4) becomes

$$P_{F} = H(a) \left(A_{Fm}/a\right)^{-K}_{-H} \cdot \exp\left[\left(1/2\right) \left(K_{H} \cdot \beta\right)^{2}\right]$$
(2.5)

## 2.2. Estimation of risk by risk equation

Kennedy (Kennedy (1) 1999) proposed the concept -Simplified Hybrid Methodøin which seismic risk can approximately be estimated as

$$P_{\rm F} \approx 0.5 H \left( A_{F10\%} \right)$$
 (2.6)

where  $H(A_{F10\%})$  (notation is changed from the original paper) is the hazard for the ground motion intensity corresponding to failure probability of 10% which is given from the fragility curve. Herein, we follow the process in which Eqn. (2.6) is derived with ground motion intensity corresponding to different failure probability level from 10%.  $A_{Fm}$  in Eqn. (2.5) is the median of the capacity, and equal to the ground motion intensity which gives 50% probability of failure in the fragility curve and denoted by  $A_{F50\%}$ . Likewise ground motions corresponding to 1%, 5%, 10%, 15% failure probability are respectively denoted by  $A_{F1\%}$ ,  $A_{F5\%}$ ,  $A_{F10\%}$ ,  $A_{F15\%}$ . Assuming fragility function is given by Eqn. (2.3) i.e.,  $A_F$  is lognormally distributed,  $A_{Fn\%}$  is related to  $A_{Fm}$  as

$$A_{Fn\%} = A_{Fm} \cdot \exp\left(-q\beta\right) \tag{2.7}$$

where q = 2.33, 1.65, 1.28, 1.04, 0 for n = 1, 5, 10, 15, 50 for example.  $A_{F1\%}=A_{Fm} \cdot \exp(-2.33\beta)$  is called HCLPF (High Confidence Low Probability of Failure) capacity (e.g., ASCE 1999) in terms of ground motion intensity. From Eqns. (2.5) and (2.6), the ratio of risk  $P_F$  to hazard at  $A_{Fn\%}$  is given as

$$P_{F}/H(A_{Fn\,\%}) = \exp\left[(1/2)(K_{H}\beta)^{2} - q\beta K_{H}\right]$$
(2.8)

As far as the risk equation is used, this ratio is determined from  $K_H$  and  $\beta$  irrespective of the median

capacity  $A_{Fm}$ . For actual seismic hazard, assumption used in the derivation of the risk equation does not hold, and the ratio is affected by  $A_{Fm}$  and the effect is discussed in chapter **3.2**. In Fig. 2.1 relationships between  $P_F/H$  ( $A_{Fn\%}$ ) and  $\beta$  are shown with the slope parameter  $A_R$  (=10<sup>1/KH</sup>) of the hazard curve as a parameter. From these figures followings can be said;

(1) Within the range of the parametric survey in most cases the ratio  $P_{F}/H$  ( $A_{Fn\%}$ ) becomes less than 1.0 except for the case of  $P_{F}/H$  ( $A_{F50\%}$ ), and in some cases the ratio becomes more than 1.0 when  $A_{R}=1.5$ , i.e., when the slope of the hazard curve is steep. As a general trend the ratio decreases with  $\beta$ , but this trend is affected by the value of  $A_{R}$ , and when  $A_{R}=1.5$  this trend does not hold. One can see in Eqn. (2.8) that the ratio takes extreme value when  $\beta = q/K_{H}$ , and this trend can be explained. In the case of  $A_{F50\%}$  where Eqn. (2.8) does not have extreme value the ratio monotonically increases with  $\beta$ .

(2) As for  $P_F/H(A_{F10\%})$  and  $P_F/H(A_{F15\%})$ , change of the ratio is gradual and the ratios are located nearly between 0.5 and 1.0 (shaded zone in Fgs. 3(c) and 3(d)) with the exception of the case of  $A_R$ =1.5. In usual case uncertainty parameter  $\beta$  is considered larger than 0.3, Eqn. (2.6) gives good approximation except for the case of  $A_R$ =1.5. If one does not like the exception of  $A_R$ =1.5, from the result of  $P_F/H$  ( $A_{F5\%}$ ) one may use  $A_{F5\%}$  instead of  $A_{F10\%}$  in Eqn. (2.6). Furthermore, if one allows conservative estimate,  $P_F \approx H(A_{F5\%})$  may be used.

## **3. APPLICATION OF RISK DIAGRAM**

## 3.1. Concept of risk-diagram

As mentioned in **2.2** approximate formula for risk estimation is proposed based on the risk equation. The risk equation is derived using two assumptions, i.e., lognormal distribution function for fragility curve and power function for seismic hazard curve. The former one is based on the assumption of



**Figure 2.1.**  $P_F/H(A_{Fn\%})$  vs.  $\beta$  for different  $\exists n\% \phi$  value

**Figure 3.1.** Risk diagram  $(P_F \text{ vs. } A_{Fm} \text{ with } \beta \text{ as a parameter})$ 

lognormal distribution for the capacity (strength) and the response of the structure, which is widely accepted and used in the practice of seismic risk analysis or PRA (Probabilistic Risk Assessment) for nuclear power plants and may be used without loss of generality. However, seismic hazard curve is site specific and generally not linear on log-log scale, so the latter assumption may not hold for actual case. The authors (Nakajima, Ootori and Hirata 2011) proposed the concept of risk diagram, where seismic risk is calculated by numerical integration of Eqn. (2.1) for site specific hazard curve for given values of  $\beta$  and  $A_{Fm}$  specifying fragility, then seismic risk is expressed graphically for the specific site. Once the risk diagram is obtained for a specific site, one can estimate the seismic risk of structures and equipments at the site with different capacities and uncertainties from this diagram. Fig. 3.1 shows an example of the risk diagram, from which one can estimate seismic risk of the structure from the median  $A_{Fm}$  and the uncertainty  $\beta$  of the capacity  $A_F$ . The capacity is given in terms of ground motion intensity (e.g., PGA, spectral acceleration  $S_A(T, h)$ ). When there is no uncertainty in capacity (i.e.,  $\beta = 0$ ), fragility function becomes Heaviside¢s step function, and its derivative with respect to  $\pm \alpha \beta$  becomes Dirac¢s  $\delta$  function, then Eqn. (2.1) becomes

$$P_F = \int_0^\infty H(a) \cdot \delta(a - A_{Fm}) da = H(A_{Fm})$$
(3.1)

and the curve in the diagram with  $\beta = 0$  becomes identical with the seismic hazard curve. Once the risk diagram is obtained at the site, one can estimate the risk of the structure with its median and logarithmic standard deviation from the diagram. However, to obtain risk diagram for a certain site with the specific hazard curve, many cases of numerical integration of Eqn. (2.1) for different values of  $A_{Fm}$  and  $\beta$  has to be conducted. In the next chapter a simplified method to estimate the seismic risk using the site-specific hazard information is shown.

## 3.2. Estimation of risk using site specific hazard curve

Consider a site (site-1) where site specific seismic hazard curve is obtained. As shown in Fig. 3.1 risk



**Figure 3.2.**  $P_F$  vs.  $A_{Fn\%}$  for different  $\div n\%$  øvalue

Figure 3.3. Concept of estimating risk from hazard curve

diagram shows the relationship between  $A_{Fm}$  (= $A_{F50\%}$ ) and  $P_F$ . Here, consider the coordinate transformation of the curves with respect to abscissa (x-axis), i.e., using Eqn. (2.7) the relationship  $A_{Fm}$ -  $P_F$  is converted to that of  $A_{Fn\%}$  -  $P_F$ , where points on the original curve ( $A_{Fm}$ ,  $P_F$  ( $A_{Fm}$ )) is transformed to the ( $A_{Fn\%}$ ,  $P_F$  ( $A_{Fm}$ )) on the converted curve depending on  $\beta$ . As mentioned about Eqn. (3.1), seismic hazard curve in original risk diagram (before coordinate transformation) is identical to  $P_F$  with  $\beta = 0$ , seismic hazard curve is not affected by coordinate transformation. Figs. 3.2 show  $P_F$  -  $A_{Fn\%}$ relationships (n=1, 5, 10, 15, 50), and in Figs. 3(b) and 3(d),  $P_F$  -  $A_{Fn\%}$  relationships on another site (site-2) are also shown. From these figures the followings are observed;

(1) In the case  $A_{Fm}$  is transformed to  $A_{F5\%}$ ,  $A_{F10\%}$ ,  $A_{F15\%}$  risk diagram curves tend to converge to one curve. In these cases,  $\div$ convergenceøis most apparent for the case of  $A_{F10\%}$ .

(2) Seismic hazard curve overlaps these converged curves ( $A_{F10\%}$ ,  $A_{F15\%}$ ), or enveloping them ( $A_{F5\%}$ ).

(3) In the original risk diagram, seismic hazard curve gives the lower bound of the risk diagram curves. With the decrease of n of n%, converted risk diagram curves become smaller compared to seismic hazard curve.

(4) What is pointed out in (2) indicates that  $P_F$  can be estimated from seismic hazard curve and appropriately converted  $A_{Fm}$ . From the investigation here,  $A_{F10\%}$  or conservatively  $A_{F5\%}$  is recommended.

From these, a procedure to estimate seismic risk of structures is proposed as follows (see Fig. 3.3).

(I) Estimate median capacity of structure  $A_{Fm}$  (median ground motion intensity at which failure of the structure occurs) and its uncertainty  $\beta$ .

(II) Estimate ground motion intensity  $A_{Fn\%}$  from Eqn. (2.7), which correspond to failure probability of n% in the fragility curve. As a value of n% 10% is recommended. If conservative estimate is required 5% may be used.

(III) Seismic hazard for  $A_{Fn\%}$  i.e.,  $H(A_{Fn\%})$  gives estimate of the risk. With the recommended value of 10% or 5% for  $A_{Fn\%}$ , seismic hazard for  $A_{Fn\%}$  i.e.,  $H(A_{Fn\%})$  gives estimate of the risk as





**Figure 3.4.**  $P_F/H(A_{Fn\%})$  vs.  $A_{Fm}$  for different -n%øvalue

$$P_{\rm F} \approx H \left( A_{F10\%} \right)$$
  
or  
$$P_{\rm F} \approx H \left( A_{F5\%} \right) \quad \text{(for conservative estimate)} \tag{3.2}$$

As a result, above formula for seismic risk estimation is similar to Eqn. (2.6). Thus far in the process of estimating seismic risk based on the risk diagram, seismic hazard curve with respect to PGA is used. For the case of seismic hazard curve with respect to other ground motion intensities such as, peak ground velocity PGV, spectral acceleration Sa (T, h), etc., which is considered appropriate to use for seismic fragility analysis, this approach is considered effective, as the behaviour of the hazard curve is similar to that for PGA.

In Figs. 3.4 relationships between  $P_F/H$  ( $A_{F10\%}$ ) and  $A_{Fm}$  and  $P_F/H$  ( $A_{F5\%}$ ) and  $A_{Fm}$  are shown for two sites (site-1 and site-2). Relationship between  $P_F/H$  ( $A_{Fn\%}$ ) and  $A_{Fm}$  obtained using actual hazard curve is dependent on  $A_{Fm}$ , whereas  $P_F/H$  ( $A_{Fn\%}$ ) obtained from risk equation is not dependent on  $A_{Fm}$ , due to the assumption of constant slope of seismic hazard curve in log-log scale. From these figures the followings can be observed.  $P_F/H$  ( $A_{Fn\%}$ ) increase with the increase of  $A_{Fm}$ , in the cases of  $\beta = 0.4, 0.5$ , but when  $\beta$  becomes smaller the trend of  $P_F/H$  ( $A_{Fn\%}$ ) with  $A_{Fm}$  becomes flat or decreases with  $A_{Fm}$ . Although  $P_F/H$  ( $A_{F10\%}$ ) becomes larger than 1.0 as mentioned about Fig. 3.2, considering the accuracy required in risk analysis, this underestimate will be accepted.  $P_F/H$  ( $A_{F5\%}$ ) becomes smaller than  $P_F/H$  ( $A_{F10\%}$ ) and less than 1.0 in all cases. Depending on judgement of those who use the result of seismic risk, estimate can be made selecting factor  $\alpha$  in the equation below and  $\alpha$ is recommended to select between 0.5 and 1.0

$$P_{\rm F} \approx \alpha H \left( A_{F10\%} \right)$$
  
or  
$$P_{\rm F} \approx \alpha H \left( A_{F5\%} \right)$$
(3.3)

#### 4. CONCLUSIONS

In this paper a simplified method to estimate seismic risk of structures was proposed based on the seismic risk diagram proposed by the authors. The method uses information on the seismic hazard and the capacity of the structure, and numerical integration is not required. This method is consistent with the preceding researches by Kennedy based on the risk equation, but is different in that in the proposed method actual seismic hazard curve is made use of and the effects of non-constant slope of the seismic hazard curve is taken into account.

The proposed method is easy to use for those who are not familiar with seismic risk analysis and considered to be useful when risk information is required as in the seismic design of structures, rehabilitation planning of structures, et al.

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