# Displacement-based seismic design of base-isolated bridges with viscous dampers

**F. Sanchez-Flores** Seismic Science, Mexico

**A. Igarashi** *Kyoto University, Japan* 



## SUMMARY:

A displacement-based design procedure is proposed for straight concrete bridges on seismic isolation devices with either linear or nonlinear viscous dampers. The isolators are modeled as bilinear hysteretic devices. The limit state considered is such that the piers and the deck remain elastic, while the isolated devices are allowed to behave inelastically. With the proposed methodology, the design forces on the piers are obtained directly, and the damping coefficient of the viscous dampers is straightforwardly calculated. This process avoids the iterative nonlinear calculations of the current force-based design, in such a way that the maximum displacement of the structural elements is kept under the target value. The design methodology is exemplified, verified by nonlinear time history analysis, and discussed to narrow down its range of applicability.

Keywords: Displacement-based design, bridge design, viscous dampers

# **1. INTRODUCTION**

Through the years, extensive research has been done to enhance the seismic performance of structures with isolators and new seismic energy dissipating devices or damping systems. Currently, these devices are regulated by some of the most well-known major design codes (AASHTO 1999, BSSC 1997, BSSC 2000, JRA 2002). In parallel with research on new technology, new design methodologies were also developed to fulfill the requirements of the performance-based seismic engineering (SEAOC, 1995). One of the most promising methods is based on the displacement control, namely displacement-based design (DBD) (Priestley *et al.* 2007).

Conceptually, DBD uses displacement as a parameter to control structural damage. Like other design methods, DBD is an iterative process for new structures since the final cross-sections of the elements are unknown. The application of displacement-based concepts for assessment or evaluation of existing structures is more straightforward since characteristics of the structure, such as material properties and elements geometry, are known.

Even though the issued DBD methodologies cover most of the structures currently in use, there is not yet a design methodology for isolated bridges with supplemental dampers.

Viscous dampers have shown to be reliable, efficient, and economic devices that protect structures subjected to large earthquakes. Viscous dampers are typically used in the design of new bridges or in the retrofit of existing ones when the isolators are not capable of restricting large displacements. Due to the lack of stiffness, viscous dampers do not significantly affect the period of vibration of the structure.

Although there is a well-established force-based routine for the design of base-isolated structures with viscous dampers, the characteristics of the dampers are initially assumed. If the displacements exceed the allowable value the characteristics of the dampers are modified. Then, the conventional practice of carrying out a series of trial and error process for design of supplemental dampers requires a lot of computation time due to inelastic time history analyses.

In this study, a novel displacement-based design procedure for bridges with seismic isolation and viscous dampers is developed, exemplified and validated.

The preliminary steps of the method are based on the approach of Priestley *et al.* (2007). The rest of the method was developed to straightforwardly calculate the damping coefficient of viscous dampers in order to maintain the displacement of the structural elements under a predefined target value. In the proposed method, the bridge is assumed to be fully supported by bilinear hysteretic isolators (IS) with either linear or nonlinear viscous dampers (VD) in all pier-girder locations. The target displacement is defined by the maximum displacement of the bearing isolators and the elastic pier deformation.

## 2. DISPLACEMENT-BASED DESIGN

## 2.1. Numerical Model for continuous and multi-span simply supported deck bridge

The proposed design procedure addresses both continuous and simply supported multi-span deck bridges. It is based on an equivalent linear single-degree-of-freedom (SDOF) model of the bridge obtained from a simplified two-degree-of-freedom (2DOF) model of each pier-IS-VD-deck system derived as follows (see Fig. 1):

The tributary mass of the pier  $(m_p)$ , located on the top of the pier, is computed as the sum of its tributary mass and the mass of the pier cap. The tributary mass of the girder-deck system  $(m_g)$  is set equal to the mass of half span on the left and half span on the right for continuous bridges. For multi-span bridges in the transverse direction, the tributary mass is similarly computed due to the displacement restrictions imposed by the joints. In the longitudinal direction, two independent IS-girder systems supported by the same pier with different tributary girder-deck masses are considered. The 2DOF system includes the equivalent damping and stiffness of the pier (denoted by  $\xi_p$  and  $k_p$  respectively), and the equivalent damping and stiffness of the isolators IS (denoted by  $\xi_p$  is assumed as 5%.

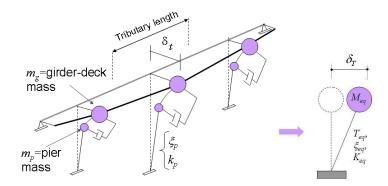


Figure 1. Deformation of the base-isolated bridge and equivalent SDOF model

In both the transverse and longitudinal directions, the maximum displacement of the pier-IS-VD-deck system at the peak seismic response is given by the sum of the corresponding elastic displacement of the pier ( $\delta_{y,p}$ ) and maximum inelastic displacement of the IS ( $\delta_{max,b}$ ). If the displacement of the ground ( $\delta_{gr}$ ) is considered, the total displacement of the *i*<sup>th</sup> pier-IS-VD-deck system is given by Eqn. 2.1.

$$(\delta_t)_i = (\delta_{y,p} + \delta_{\max,b} + \delta_{gr})_i$$
(2.1)

Although the relative displacements of the piers and the isolators may change in each pier-IS-VD-deck system, their sum remains constant and equal to  $\delta_t$  (see Fig. 2).

Due to the objectives of this research, the displacements of the foundation and the ground shall not be considered hereafter.

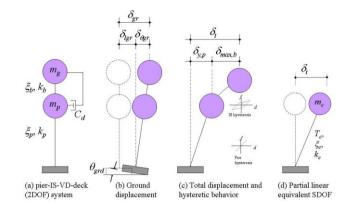


Figure 2. Displacements of the 2DOF and partial equivalent SDOF

# **2.2. Design performance objectives**

The design philosophy of the proposed methodology is based on the general requirement that the full serviceability of the bridge should be maintained after the design earthquake. Thus, the performance levels of immediate operation (IO) and life safety (LS) should be satisfied. For the former, the system satisfies the structural performance criteria if the piers remain elastic and the isolators develop incipient inelastic behavior. For the latter, the piers must remain elastic. In either case, the isolators must develop the expected inelastic behavior with the displacement restrictions imposed by the viscous dampers. Therefore, the performance levels can be achieved if the design satisfies the following criteria:

- 1. The seismic response of the superstructure (girder, deck, movement joints, etc.) and substructure (piers, abutments and foundations) must essentially remain elastic for new bridges (Cardone *et al.*, 2009). In this study, existing bridges which have piers that behave inelastically are not considered.
- 2. The isolator must be able to sustain the designated maximum design displacement. Adequate clearance satisfying the maximum design displacement should be provided in both the longitudinal and transverse directions, to allow for the movements of joints and to avoid impacts between structural elements. The design displacement of the IS in each direction is assigned based on its physical characteristics.
- 3. The resulting maximum force and stroke of the viscous damper during the design earthquake must be lower than the maximum allowable values.

The following assumptions are made: a) the input ground motion is uniform at the base of all piers, b) the center of stiffness of the pier-IS system coincides with the center of mass of the girder (the torsional effects and relative rotations at the joints are neglected), and c) effects of higher modes are not significant. Then, the displaced configuration of each pier-IS-VD-deck system results in uniform displacements in the transverse direction characterized by a rigid translation of the deck (see Fig. 1).

# 2.3. Methodology

i.

Although some of the steps may follow a different order than the one suggested here, the proposed methodology consists of the following steps:

## Selection of the preliminary pier cross section

The geometry of the bridge is limited by either physical restrictions or architectural design. With this information the cross section of the columns is proposed by gravitational analysis. For simplicity, in this study the *P*- $\Delta$  effects are taken into account since the gravitational analysis so that the second order effects can be neglected during the design process. The yielding displacement of the piers ( $\delta_{y,p}$ ) can be calculated with the equations from Priestley (2007).

With the obtained cross section, the stiffness of the pier,  $k_p$ , and the period of the structure without isolation,  $T_0$ , are calculated. To take advantage of the supplemental damping, the equivalent period of the isolated bridge with viscous dampers  $T_e$  will be set to approximately

$$T_e = \psi T_0 \tag{2.2}$$

where  $\psi$  is a constant typically set to 2 as suggested by the Japanese Road Association (2002).

#### *ii.* Selection of the isolator bearings $\delta_b$

The dimensions of the IS bearings can be estimated from preliminary gravitational and seismic static analysis. The damping coefficient is obtained with Eqn. 2.3, and the total secant stiffness of the *j* bearings on each pier is given by  $k_b = \Sigma k_{b,i}$ .

$$\xi_b = \frac{W}{2\pi k_b (\delta_{\max,b})^2} \tag{2.3}$$

Alternatively,  $\xi_b$  can be calculated with formulations explicitly derived by bilinear isolators such as the following: (Jara and Casas, 2006)

$$\xi_b = 0.05 + 0.05 \ln(\mu_b) \tag{2.4}$$

*iii.* Calculation of the period of the 2DOF system The equivalent period of the bridge is given by the following equation:

$$T_{eq} = 2\pi \sqrt{\frac{M_{eq}}{K_{eq}}}$$
(2.5)

where  $M_{eq}$  is the sum of the first-mode participating mass of each pier-IS-VD-deck system,  $m_i$ , and the corresponding mass of the abutments  $m_{ab}$ . The equivalent stiffness  $K_{eq}$  is given by summing each stiffness  $k_e$  and the abutment stiffness  $k_{ab}$  as

$$K_{eq} = \sum_{i}^{n} (k_{e})_{i} + \sum k_{ab}$$
(2.6)

where  $k_e$  is the equivalent stiffness obtained by summing, in parallel, the individual stiffness of the contributing components ( $k_p$  and  $k_b$ ) of each 2DOF system as

$$k_e = \frac{k_b k_p}{k_b + k_p} \tag{2.7}$$

The stiffness of the isolators in the abutments can be calculated either with the procedure proposed by Priestley (1996) or by matching the value of the equivalent period in regular bridges

The periods  $T_e$  and  $T_{eq}$  are then compared. If they differ more than a predefined tolerance  $\varepsilon$  (in this study equal to 0.01) the selection of the isolators is reviewed until the convergence is achieved.

#### iv. Target displacement of the pier-IS-deck system

The target displacement of the whole bridge  $(\delta_T)$  is given by the following equation:

$$\delta_{T} = \frac{\sum_{i=1}^{n} m_{i} \delta_{t,i}^{2}}{\sum_{i=1}^{n} m_{i} \delta_{t,i}}$$
(2.8)

where  $m_i$  is the contributing effective mass for the first mode, the subscript *i* represents the *i*<sup>th</sup> span, and  $\delta_t$  represents the design target displacement of each pier-IS-VD-deck.

*v.* Equivalent damping of each the pier-IS system The damping provided by each pier-IS system is calculated with known parameters as

$$\xi_{IS} = \frac{\left(\xi_b + \xi_p \frac{k_b}{k_p}\right)}{\left(1 + \frac{k_b}{k_p}\right)}$$
(2.9)

A suitable combination of the ratio  $k_b/k_p$  and  $\xi_b$  should be chosen if the viscous dampers will be the main source of damping. In the present study, values of  $k_b/k_p \le 1$  and  $\xi_b \le 15\%$  are adopted so the equivalent damping of the pier-IS system without dampers ( $\xi_{IS}$ ) contributes with a small portion of the maximum damping of the system ( $\xi_{max}$ ).

#### vi. Equivalent damping from each pier-IS-VD system

Since  $T_{eq}$  is known, the equivalent damping  $\xi_{eq}$  that satisfies the specified target displacement is calculated from the displacement spectra. First, it is verified that  $T_{eq \min} < T_{eq} \leq T_{eq \max}$ , where the periods  $T_{eq \min}$  and  $T_{eq \max}$  are given by the values which satisfy  $\delta_T = S_D(T_{eq \min}, \xi = \xi_{IS})$  and  $\delta_T = S_D(T_{eq}, \xi_{eq})$ . Secondly, the equivalent damping of the system,  $\xi_{eq}$ , is obtained from the spectra as the value which satisfies  $\delta_T = S_D(T_{eq}, \xi_{eq})$  as shown in Fig. 3. In this study the maximum damping is set as 60%.

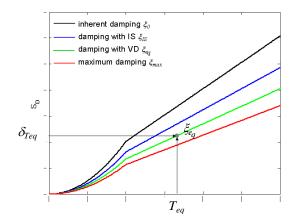


Figure 3. Equivalent spectral damping

#### vii. Coefficient of viscous dampers

In the velocity exponent model, the force developed by a viscous damper is defined based on the velocity of the system according to the following relationship:

$$F_d = C_d \left| \dot{u} \right|^{\alpha} \operatorname{sgn}(\dot{u}) \tag{2.10}$$

where  $C_d$  is the damping coefficient,  $\dot{u}$  is the relative velocity between the ends of the damper,  $\alpha$  represents the damping exponent and *sgn* is the signum function, which satisfy  $sgn(\dot{u}) = 1$  if  $\dot{u} \ge 0$ and  $sgn(\dot{u}) = -1$  if  $\dot{u} < 0$ . The constants  $C_d$  and  $\alpha$  are specific properties for each damper. When  $\alpha=1$ , the relationship given by Eqn. 2.10 is linear, otherwise, the relationship is nonlinear.

Once the damping  $\xi_{eq}$  is obtained, the parameter  $C_d$  of the viscous dampers can be calculated. To this end, it is convenient to define the following ratios:  $\alpha_l = \omega_l^2 / \omega_b^2$ ,  $\omega_b^2 = k_b / m_b$ ,  $\gamma = m_p / (m_p + m_b)$ ,  $R_p = k_b / k_p$ , and the parameter  $D = \delta_b - \delta_p$ . The frequencies  $\omega_{l,2}$  are defined as

$$\omega_{1,2}^{2} = \frac{(1-\gamma)}{R_{p}} \frac{\left[1 + R_{p} / (1-\gamma)\right] \pm \left\{\left[1 + R_{p} / (1-\gamma)\right]^{2} - 4\gamma R_{p} / (1-\gamma)\right\}^{1/2}}{2\gamma} \omega_{b}^{2}$$
(2.11)

Then, the constant  $C_d$  for linear or nonlinear dampers is calculated with the following equations (Jenn-Shin and Yi-Shane, 2005)

For linear dampers:

$$C_{b} = 2m_{b}\omega_{b}\left(\frac{\xi_{eq}\sqrt{\alpha_{1}}\left[(1-\gamma)+(1-\alpha_{1})^{2}\gamma\right]-\xi_{p}\sqrt{(1-\gamma)/R_{p}}(1-\alpha_{1})^{2}}{\alpha_{1}^{2}(1-\gamma)}-\xi_{b}\right)$$
(2.12)

For nonlinear dampers:

$$C_{b} = \frac{2\pi m_{b}(1+R_{p})^{\alpha-1}}{\omega_{b}^{\alpha-1}D^{\alpha-1}\lambda} \left( \frac{\xi_{eq}\sqrt{\alpha_{1}} \left[ (1-\gamma) + (1-\alpha_{1})^{2}\gamma \right] - \xi_{p}\sqrt{(1-\gamma)/R_{p}}(1-\alpha_{1})^{2}}{\alpha_{1}^{2}(1-\gamma)} - \xi_{b} \right)$$
(2.13)

### viii. Verification of the displacements

The resulting maximum displacements ( $\delta_g$ ,  $\delta_p$ , and  $\delta_b$ ) are verified and must satisfy the following restrictions:  $\delta_g \leq \delta_t$ ,  $\delta_p \leq \delta_{y,p}$ , and  $\delta_b \leq \delta_{max,b}$ . If these relationships are not satisfied, the damping provided by the VD can be approximated once more with the displacement ratios.

#### *ix. Capacity design of the structural elements*

In this step the piers and foundations are designed by capacity with the following forces:

$$(V_{dg})_i = (k_e \delta_g)_i \tag{2.14}$$

$$(M_{dg})_i = (V_{dg}H)_i$$
(2.15)

where  $V_{dg}$  and  $M_{dg}$  are the design shear and moment, respectively.

Additionally, the mechanical characteristics of the IS and VD are fully specified based on the obtained parameters ( $C_d$ ,  $\xi_e$ ) and those assumed at the beginning of the analysis ( $k_b$ ,  $\mu_b$ ).

#### 2.4. Accuracy of the displacement's estimation

To evaluate the accuracy of this method, the procedures outlined in this study were applied to a bridge with an input consisting of 30 artificial earthquakes. These earthquakes are compatible with the displacement spectra given by the JRA (2002) for Type 1 earthquakes (EQ  $S_{IO}$ ) (See Fig. 4).

The accuracy of the method is then evaluated by the mean error between the target displacements and the displacements obtained with the nonlinear time history analysis ( $\delta_{THA}$ ) with

$$E_{mean} = \frac{1}{K} \sum_{k=1}^{N} \frac{(\delta_{THA})_k}{(\delta_t)_k}$$
(2.16)

where *K* is the total number of earthquakes, and *k* is the  $k^{th}$  earthquake.

If  $E_{mean}=1$ , the estimation of the target displacement is optimum. If  $E_{mean}>1$  the resulting displacements exceed the target values and a refinement of the design must be done. If  $E_{mean}<1$  the target displacement is not exceeded and, a judgment for a rational  $E_{mean}$  should be made.

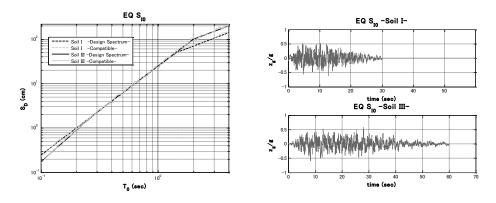


Figure 4. Artificial earthquakes compatible with the displacement spectra

## **3. APPLICATION**

The bridge considered in the example is a modified version of one used in previous studies (Priestley *et al.* 1996, Cardone et al. 2008). The geometry is kept the same as the original example by Priestley, although modifications on the size of the elements are done in order to illustrate the proposed methodology. The bridge consists of four 50 m spans, a deck with a box cross section of 5.25 m<sup>2</sup>, a moment of inertia for bending around the vertical axis of 74 m<sup>4</sup> and a weight per unit of length of 200 kN/m. The piers are characterized by rectangular hollow cross sections with the sizes shown in Fig. 5. All the piers have a cap of 500kN weight. Unlike the example presented by Priestley *et al.* (1996), the bridge in this study is assumed to be entirely made of concrete with compressive strength equal to 30 MPa. The period of the bridge without IS and VD is calculated as  $T_0$ =0.70. The equivalent period (coincident with the base-isolated period) is set to  $T_e \approx 2T_0 \approx 1.41$ .

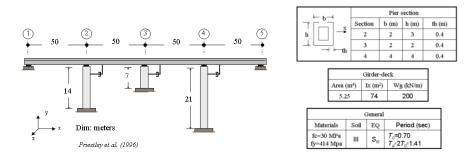


Figure 5. Geometry, properties and materials of the bridge

To illustrate the method, the VD's coefficients for all the piers will be calculated, then the resulting information will provide the design parameters for both, the structural elements and the energy dissipation devices:

The preliminary stiffness and damping of the pier, and bearing isolators are calculated to match  $T_{eq}=1.41$  sec., which is the period calculated with the preliminary sections. Then, the maximum

displacements of the piers, girder and isolators are calculated as shown in the column  $\delta_{lS}$  of Table 3.1. The target displacements of the girder and the bearing at each pier location are set as 0.30 and 0.25 m, respectively as typical allowable displacement values. Excepting piers, the displacements from structural elements are greater than their corresponding target ones. Therefore, in order to limit the displacements, additional dampers are required.

Due to the mass and deck deformation regularity, the target displacement calculated with Eqn. 2.8 coincides with the target displacement at each pier location. Moreover, the contributing damping at each pier location is equal to the damping of the system.

From the displacement spectra, the total amount of damping ( $\xi_e$ ) required to achieve the target displacement of the system at the selected period is obtained. Then, the coefficients of the dampers are calculated for linear (LIN) and nonlinear (NL) dampers ( $\alpha$ =0.3) (see Table 3.1).

The displacements are verified by calculating the exact displacements with a finite element model. Table 3.1 shows the averaged results obtained by analysis with 30 compatible spectra earthquakes. The displacement profiles and the error  $E_{mean}$  show the method gives conservative but accurate predictions (see Fig. 6).

Finally, the design forces in the pier can then be calculated with Eqn. 2.14 and Eqn. 2.15.

Section	Element	k <sub>e</sub> (kN/m)	Damping	$\delta_{jS}(\mathbf{m})$	$\delta_{\tau}(\mathbf{m})$	δ <sub>πια</sub> (m) NL	E <sub>meen</sub> NL	δ <sub>πял</sub> (m) LIN	E <sub>meen</sub> LIN
1	IS	10250	ξ <sub>6</sub> =16%		δ <sub>ζδ</sub> =0.250	0.25	1.0	0.240	0.96
2	Pier	111738	ξ <sub>ρ</sub> =5%	0.039	8 <sub>,10</sub> =0.07	0.051	0.73	0.051	0.73
	Girder		ξ <sub>g</sub> =5%	0.398	δ <sub>i</sub> =0.30	0.281	0.94	0.279	0.93
	IS	25154	$\xi_{b} = 14\%$	0.359	δ <sub>46</sub> =0.25	0.230	0.92	0.228	0.91
	VD linear		Cd=3316 kN*s/m		Stroke max=0.25			0.228	0.91
	VD nonlinear		Cd <sub>al</sub> =2281 kN*s/m		Stroke max=0.25	0.230	0.92		
3	Pier	277981	ξ <sub>ρ</sub> =5%	0.014	8 <sub>7.0</sub> =0.04	0.04	1.00	0.038	0.95
	Girder		ξ <sub>g</sub> =5%	0.355	δ <sub>i</sub> =0.30	0.29	0.97	0.288	0.96
	IS	21935	ξ <sub>6</sub> =14%	0.341	δ <sub>46</sub> =0.25	0.25	1.00	0.250	1.0
	VD linear		Cd=2520 kN*s/m		Stroke max=0.25			0.250	1.0
	VD nonlinear		Cd <sub>nl</sub> =1724 kN*s/m		Stroke max=0.25	0.25	1.00		
4	Pier	277981	ξ <sub>ρ</sub> =5%	0.037	δ <sub>,e</sub> =0.10	0.051	0.51	0.053	0.53
	Girder		ξ <sub>g</sub> =5%	0.380	δ <sub>i</sub> =0.30	0.283	0.94	0.272	0.91
	IS	21935	ξ <sub>6</sub> =14%	0.343	δ <sub>εδ</sub> =0.25	0.232	0.93	0.219	0.88
	VD linear		Cd=3664 kN*s/m		Stroke max=0.25			0.219	0.88
	VD nonlinear		Cd <sub>al</sub> =2515 kN*s/m		Stroke max=0.25	0.232	0.93		
5	IS	10250	ξ <sub>6</sub> =16%		δ <sub>ιο</sub> =0.250	0.251	1.00	0.231	0.92

Table 3.1. Application -Results of the DBD methodology-

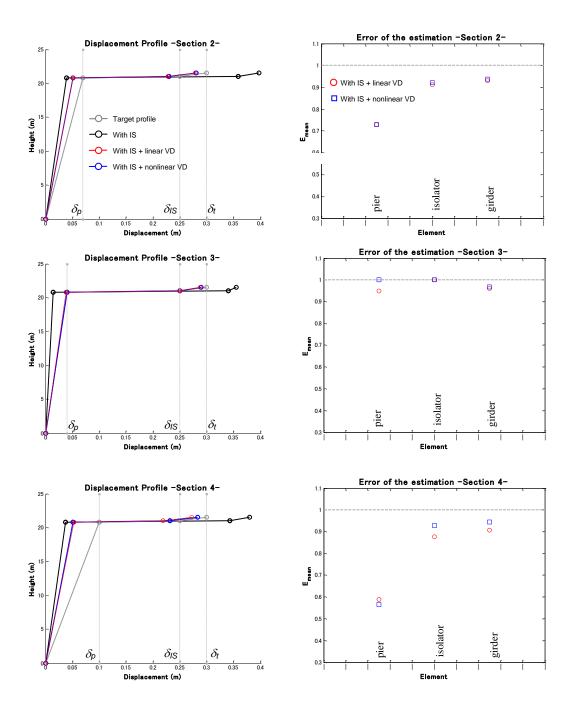


Figure 6. Maximum displacement profile and Error of the estimation

# 4. CONCLUSIONS

A new Displacement-Based Design (DBD) methodology for continuous and multi-span bridges with base isolation and viscous dampers was presented. The performance level of the bridge was defined by the elastic behavior of the piers and deck and the inelastic behavior of the isolators characterized by a bilinear model.

The design procedure was applied to a 4-span continuous simply supported bridge, characterized by an irregular layout of pier heights. The predictions of the displacement-based design method were compared with the results obtained by nonlinear time-history analyses. To this end, a set of 30 spectrum compatible accelerograms were used. From this comparison, it was verified that the bridges

and piers designed with the proposed DBD method satisfied the desired performance level by achieving the target displacements of the structural elements (girder, piers, isolators, and dampers) while the piers remain elastic. Based on the results, the method was proved to be efficient and accurate so it can be used for preliminary design to avoid the iterative nonlinear analysis required in the current force-based design to calculate the viscous damper coefficients.

Safety factors were omitted so the methodology is general enough and compatible with any design Code. In the current form, the presented methodology can be straightforwardly implemented in any computational program.

The proposed design procedure, however, is not intended to cover all the aspects related to the design of bridges with seismic isolation and viscous dampers. It establishes the fundamental steps under a rational methodology for the effective calculation of the viscous damper coefficients to control displacements under severe earthquakes. It is also important to mention that in the presented methodology, the optimum size of the pier cannot be directly obtained. However, if some iterations are done the pier's size can be optimized as shown in Fig. 6 for Section 3.

Finally, it is important to mention that although the method can be applied independently at two bridge directions (transverse and longitudinal); the use of a suitable combination which consider the bidirectional effects of the earthquake is strongly recommended.

## REFERENCES

- American Association of State Highway and Transportation Officials (AASHTO). (1999). Guide Specifications for Seismic Isolation Design, Washington, D.C.
- Building Seismic Safety Council (BSSC). (1997). NEHRP Guidelines for the Seismic Rehabilitation of Buildings. Report FEMA 273/274, Federal Emergency Management Agency. Washington, D.C.
- BSSC, (2000). Prestandard and Commentary for the Seismic Rehabilitation of Buildings. Federal Emergency management Agency. Washington, D.C.
- Cardone D., Dolce M. and Palermo G. (2009). Direct displacement-based design of seismically isolated bridges. *Bulletin of Earthquake Engineering*. **7:2**, 391–410
- Japan Road Association (JRA). (2002). Seismic Design Specifications for Highway Bridges (in Japanese).
- Jara M. and Casas J. R. (2006). A direct displacement-based method for the seismic design of bridges on bi-linear isolation devices. *Engineering Structures*. **28:6**, 869–879.
- Jenn-Shin and Yi-Shane. (2005). Design formulations for supplemental viscous dampers to highway bridges. *Earthquake Engineering & Structural Dynamics*. **34:13**, 1627-1642.
- Priestley M. J. N., Calvi G. M. and Kowalsky M. J. (2007). Displacement-based seismic design of structures, IUSS Press, Pavia, Italy.
- Priestley M. J. N., Seible F. and Calvi G. M. (1996). Seismic design and retrofit of bridges, John Wiley & Sons, New York, USA.
- SEAOC. (1995). Performance-based seismic engineering. Vision 2000 Committee, Structural Engineers Association of California, Sacramento, CA.