THEORETICAL ANALYSIS ON DYNAMIC RESPONSES OF SINGLE SURFACE SLOPE DURING EARTHQUAKE

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SUMMARY:
In generally, input S-wave outcomes P-wave and S-wave due to wave mode conversion on the free air surface, and then the superposition of waves results in dynamic responses. In this paper, seismic wave was regarded as elastic wave to research the regularity of slope dynamic responses. Three categories influential factors were taken into consideration, such as material parameters, seismic wave parameters and slope geometry shape. The analytical results reveal that the distribution regularity of amplification factors depends on wave frequency, lower frequency induces more remarkable dynamic responses; the maximum of amplification factor relies on Poisson's ratio and wave input angle, larger Poisson's ratio with smaller input angle causes larger value of maximum amplification factor; slope angle, dynamic Young's modulus and density determine the thickness of the salient region affected by earthquake. Therefore, conclusion is drawn that seismic wave frequency, Poisson's ratio and input angle are the key parameters to the stability of single surface slope.

Keywords: single surface slope, dynamic responses, wave superposition

1. INTRODUCTION

It is often reported after destructive earthquakes in hilly areas that infrastructure at the slope top section suffered more intensive damage than those located at the base. Davis *et al*. observed earthquake acceleration at the top of the slope was bigger than that at foot during aftershock of the San Fernando earthquake(Davis and West, 1973). Takano monitored the slope's earthquake responses and pointed out that seismic intensity at the top of slope was about one degree bigger than that at the foot(Qi, S.W., 2002.). Seismic record data also suggest that surface topography plays an important role in ground motion (Rogers *et al*., 1974; Griffiths and Bollinger, 1979; Tucker *et al*., 1984). Lots of researches have already discussed slope dynamic responses by different ways. Qi applied dimension analysis method to analyze the influential factors on the elevation amplification effect (Qi, S.W.,2002, 2006.). Jiang et al. made use of model test to research slope's earthquake responses (Jiang, L.W. *et al*., 2010), and numerical model is generally applied to analyze slope's dynamic responses. (He, Y.L.and Chen, S.Y., 1998; Paolucci, R. 2002; Qi, S.W., *et al*., 2003; Bouckovalas, G.D. and Papadimitriou, A.G.,2005; Lenti, L. and Martino, S., 2012).

Slope is divided into single surface slope and double surface slope, based on the number of the inclined free air surface. This article just discusses the dynamic responses of the single surface slope. As a result of dynamic responses depend on the parameters of slope material, seismic wave and slope geometry shape, moreover, different slope parts appear different responses. In the following, this paper will narrowly discuss three main influential factors on dynamic responses of single surface slope by using theoretical deduction. The first factor is slope material, such as Poisson's ratio, dynamic Young's modulus and density. The second factor is seismic wave parameters, such as wave frequency and input angle with regard to the perpendicular line of slope inclined surface. For single surface slope, its geometry shape is decided by slope angle, therefore, the third factor is slope angle.

2. WAVE MODE CONVERSION ON THE FREE AIR SURFACE

Earthquake shaking energy is transmitted by P-wave and S-wave. In generally, horizontal earthquake loading is applied when analyzing slope dynamic responses, hence, the following will regard S-wave as source wave which acts on the slope. When seismic S-wave inputs into the slope and encounters the inclined surface (free air surface), S-wave will be reflected, meanwhile, generating P-wave, as shown in **Figure 2.1**. In which, XOY is the global coordinate system, $lo'm$ is the local coordinate system. α denotes S-wave input angle and its reflected S-wave angle related to the perpendicular line to slope inclined surface, β refers to the angle of reflected P-wave transmitting direction related to the perpendicular line to slope inclined surface. θ represents slope angle.

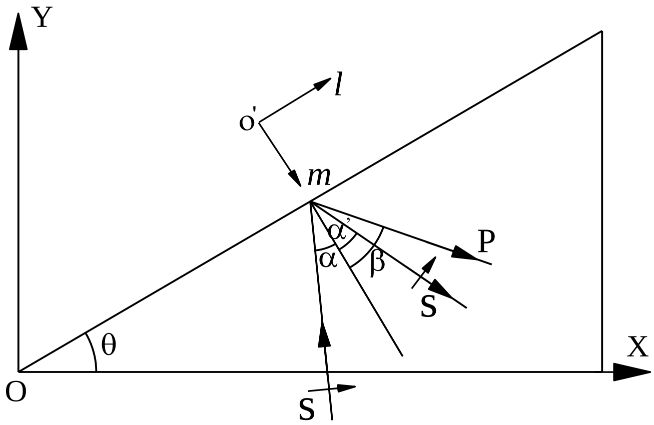

Figure 2.1. Sketch of S-wave mode conversion on the inclined free air surface of single slope

The displacement formulae of seismic wave can be expressed as follow, with respect to the local coordinate system.

Input S-wave: $S_d = Ae^{i(k_l l + k_m m - \omega t)}$, in which $k_l = \frac{\omega}{v_s}\sin\alpha$, $k_m = \frac{\omega}{v_s}\cos\alpha$

Reflected S-wave: $S_d^{'} = A^{'}e^{i(k_l^{'} l + k_m^{'} m - \omega^{'} t)}$, in which $k_l^{'} = \frac{\omega^{'}}{v_s}\sin\alpha^{'}$, $k_m^{'} = \frac{\omega^{'}}{v_s}\cos\alpha^{'}$

Reflected P-wave: $S_d^{''} = Be^{i(k_l^{''} l + k_m^{''} m - \omega^{''} t)}$, in which $k_l^{''} = \frac{\omega^{''}}{v_p}\sin\beta$, $k_m^{''} = \frac{\omega^{''}}{v_p}\cos\beta$

In above all formulae, S_d, $S_d^{'}$, $S_d^{''}$ mean seismic wave displacement; k, $k^{'}$, $k^{''}$ represent wave number; ω, $\omega^{'}$, $\omega^{''}$, refer to wave circular frequency; v_s, v_p denote velocity of S-wave and P-wave, respectively. According to elastic wave theory, all of the aforementioned three formulae should obey wave equation (Hu, J.X. et al., 2006), and the boundary conditions on the inclined free air surface are shown as Eqn.2.1:

$$\omega = \omega^{'} = \omega^{''} \quad \text{and} \quad k_l = k_l^{'} = k_l^{''} \tag{2.1}$$

Therefore, deriving from wave equation combined with boundary conditions, Eqn.2.2 can be obtained:

$$\alpha = \alpha^{'} \quad \text{and} \quad \frac{\sin\alpha}{v_s} = \frac{\sin\beta}{v_p} \tag{2.2}$$

Meanwhile, the displacement reflection coefficients of P-wave and S-wave are shown as following Eqn.2.3 and Eqn.2.4, respectively,

$$\eta_{pd} = \frac{B}{A} = \frac{-D^2 \sin 4\alpha}{D^2 \cos^2 2\alpha + \sin 2\beta \sin 2\alpha} \tag{2.3}$$

$$\eta_{sd} = \frac{A'}{A} = \frac{\sin 2\alpha \sin 2\beta - D^2 \cos^2 2\alpha}{D^2 \cos^2 2\alpha + \sin 2\beta \sin 2\alpha} \tag{2.4}$$

In which, $D = \frac{v_p}{v_s} = \sqrt{\frac{2-2\upsilon}{1-2\upsilon}}$. η_{pd}, η_{sd} refer to amplitude ratio of reflected P-wave and reflected S-wave to input S-wave, respectively. They are named as displacement reflection coefficients. Subscript *p* and *s* refer to P-wave and S-wave, respectively. Subscript *d* refers to displacement. Anlysis of Eqn.(2.3) and Eqn.(2.4) reveals P-wave reflection coefficient and S-wave reflection coefficient are both just related to Possion's ratio and input angle.

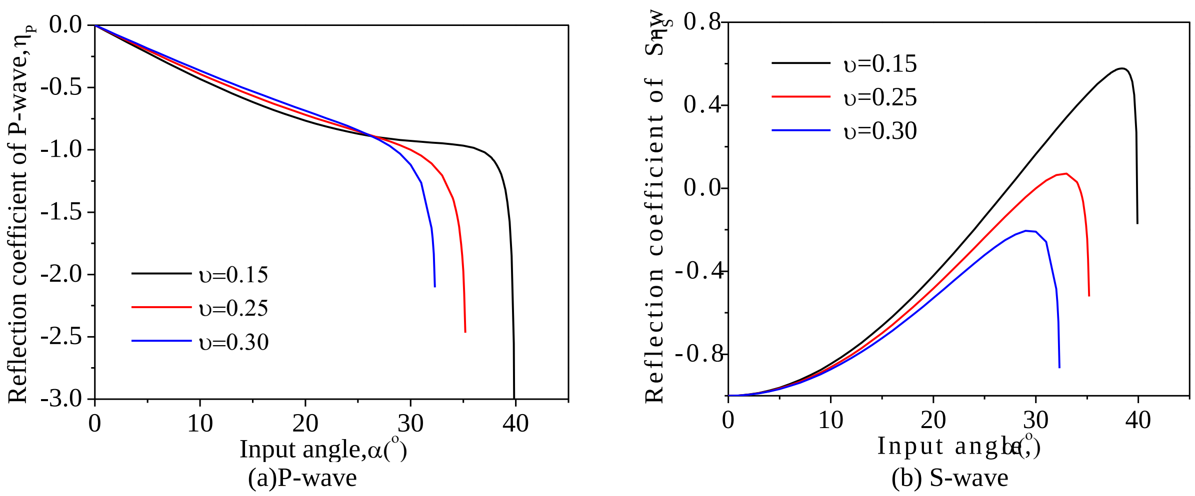

Figure 2.2 P-wave and S-wave reflection coefficients related to Poisson's ratio and input angle

The Eqn.2.2 is the Snell law, it can be used to deduce the input critical angle, expressed as $arcsin(v_s sin\beta/v_p)$. It means when S-wave inputs into slope, and then will be reflected on the slope inclined free air surface, only if input angle $\alpha < arcsin(v_s sin\beta/v_p)$, P-wave will be generated. Therefore, Eqn.2.3 and Eqn.2.4 are available when input angle smaller than the input critical angle.

According to the elastic wave hypothesis, the displacement formula of seismic wave (S_d) was taken first and second order derivation of time, velocity and acceleration formulae were obtained, as following Eqn.2.5,respectively:

$$S_v = -i\omega S_d \quad \text{and} \quad S_a = -i\omega^2 S_d \tag{2.5}$$

Due to ω is a constant value, hence, velocity and acceleration reflection coefficients of P-wave or S-wave are equal to their corresponding displacement reflection coefficient, shown as following Eqn.2.6:

$$\eta_{pd} = \eta_{pv} = \eta_{pa} \quad \text{and} \quad \eta_{sd} = \eta_{sv} = \eta_{sa} \tag{2.6}$$

η_{pv}, η_{sv} refer to velocity reflection coefficients; η_{pa}, η_{sa} refer to acceleration reflection coefficients.

3. SEISMIC WAVE SUPERPOSITION

From the view point of wave transmission, seismic waves reflect and appear wave mode conversion on the free air surface, generating a complicated wave field in the slope. Different parts of slope show different dynamic responses as a result of seismic wave superposition, such as deformation, crack, even failure.

According to the theory of wave transmission, there are three wave-bundles encountering at point D, shown as **Figure 3.1,** that is, AO_1D, BO_2D, and CD. The vibrating directions of each wave are shown at point D. Superposition of displacement vector is as following Eqn.3.1:

$$\boldsymbol{S}_D=\boldsymbol{S}_d + \boldsymbol{S}'_d + \boldsymbol{S}''_d \tag{3.1}$$

In which, $\boldsymbol{S}_d$, $\boldsymbol{S}'_d$, $\boldsymbol{S}''_d$ represent displacement vector of input S-wave, reflected S-wave and reflected P-wave, respectively.

Supposed displacement function of input wave is $f(t)$, then each wave at point D can be expressed as Eqn.3.2:

$$S_d = f(t+t_s) \qquad S'_d = \eta_{sd} f(t+t_{ss}) \qquad S''_d = \eta_{pd} f(t+t_{sp}) \tag{3.2}$$

In which t_s denotes duration of input S-wave from point C to point D； t_{ss} denotes duration from B pass point O_2 to point D; t_{sp} stands for duration from A pass point O_1 to point D. η_{pd} , η_{sd} are the S-wave and P-wave displacement reflection coefficients, respectively calculated by Eqn.2.3 and Eqn.2.4.

As **Figure 3.1.** shown, the global coordinate of point D is (x , y), then all of the durations can be calculated by Eqn.3.3.

$$\begin{cases} t_s = \dfrac{y}{v_s \cos(\alpha-\theta)} \\ t_{ss} = \dfrac{y\cos\alpha + (x\sin\theta - y\cos\theta)\cos(\alpha+\theta)}{v_s\cos(\alpha-\theta)\cos\alpha} + \dfrac{x\sin\theta - y\cos\theta}{v_s\cos\alpha} \\ t_{sp} = \dfrac{y\cos\beta + (x\sin\theta - y\cos\theta)\cos(\beta+\theta)}{v_s\cos(\alpha-\theta)\cos\beta} + \dfrac{x\sin\theta - y\cos\theta}{v_p\cos\beta} \end{cases} \tag{3.3}$$

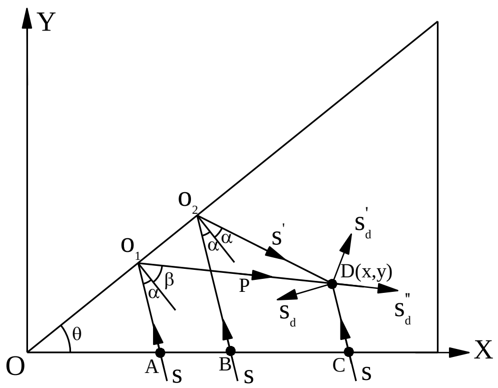

Figure 3.1. Sketch of seismic wave superposition

According to the principle of vector superposition, horizontal and vertical displacement at point D can be expressed as Eqn.3.4:

$$\begin{cases} S_{hor} = S_d^{'} \cos(\alpha+\theta) + S_d^{''} \sin(\beta+\theta) - S_d \cos(\theta-\alpha) \\ S_{ver} = S_d^{'} \sin(\alpha+\theta) - S_d^{''} \cos(\beta+\theta) - S_d \sin(\theta-\alpha) \end{cases} \tag{3.4}$$

S_{hor} refers to horizontal component of displacement; S_{ver} refers to vertical component of displacement.

Therefore, displacement amplification factor at point D is calculated by Eqn.3.5:

$$\xi_d = \frac{\sqrt{\max\left|S_{hor}^2 + S_{ver}^2\right|}}{\max\left|S_d\right|} \tag{3.5}$$

If seismic wave was considered as a simple harmonic wave, not taking damping into consideration, the velocity (ξ_v) and acceleration amplification factor (ξ_a) are the same as the displacement amplification factor, that is, $\xi_d = \xi_v = \xi_a$. Although this theoretical solution of slope dynamic responses isn't absolutely consistent with the actual, however, the regularity of slope dynamic responses and key influential parameters to slope stability can be revealed by making use of this theoretical solution.

4. ANALYSIS OF INFLUENTIAL FACTORS TO SLOPE DYNAMIC RESPONSES

Slope dynamic responses are affected by slope material, seismic wave and slope geometry shape. Dynamic Young's modulus(E_d), Poisson's ratio (υ) and density (ρ) were chosen as slope material parameters. Seismic wave parameters are comprised of wave amplitude (A), frequency (f) and transmitting duration (t) in the slope. According to Eqn.3.3, wave transmitting duration to each point is decided by input angle (α) when slope material and slope shape are constant, meanwhile, wave amplitude don't affect displacement amplification factor based on Eqn.3.5.Therefore, frequency (f) and input angle (α) were used to stand for seismic wave parameters. The geometry shape of single slope can be described by slope angle (θ). Hence, displacement amplification factors of dynamic responses can be expressed as following Eqn.4.1:

$$\xi_d = F(E_d, \upsilon, \rho, f, \alpha, \theta) \tag{4.1}$$

Table 4.1. Comparison table with different influential parameters

Case	Slope angle $\theta(^{O})$	Slope material			Seismic wave		Max(η_d)
		Young's modulus E_d (GPa)	Poisson's ratio υ	Density ρ(Kg/m3)	Frequency f (Hz)	Input angle $\alpha(^{O})$	
(a)	20	6	0.15	2700	10	20	1.94
(b)	20	3	0.15	2700	10	20	1.94
(c)	20	6	0.15	2000	10	20	1.94
(d)	20	6	0.25	2700	10	20	1.97
(e)	20	6	0.25	2700	20	20	1.97
(f)	20	6	0.25	2700	50	20	1.97
(g)	30	6	0.15	2700	10	10	1.99
(h)	30	6	0.15	2700	10	20	1.94
(i)	30	6	0.15	2700	10	30	1.86

*comparing the cases with the same shading color

In order to analyze each parameter how to affect the displacement amplification factor, Eqn.3.5 was used to calculate 144 cases with different values of each parameter, such as θ=20°,30°; E_d=3GPa,6GPa; υ=0.15,0.25; ρ=2000 Kg/m^3,2700 Kg/m^3; f=10Hz,20Hz,50Hz; α=10°,20°,30°. Nine typical cases out of 144 cases are listed in the **Table 4.1.**, and the corresponding distribution regularity figures of displacement amplification factors are shown in **Figure 4.1.**.

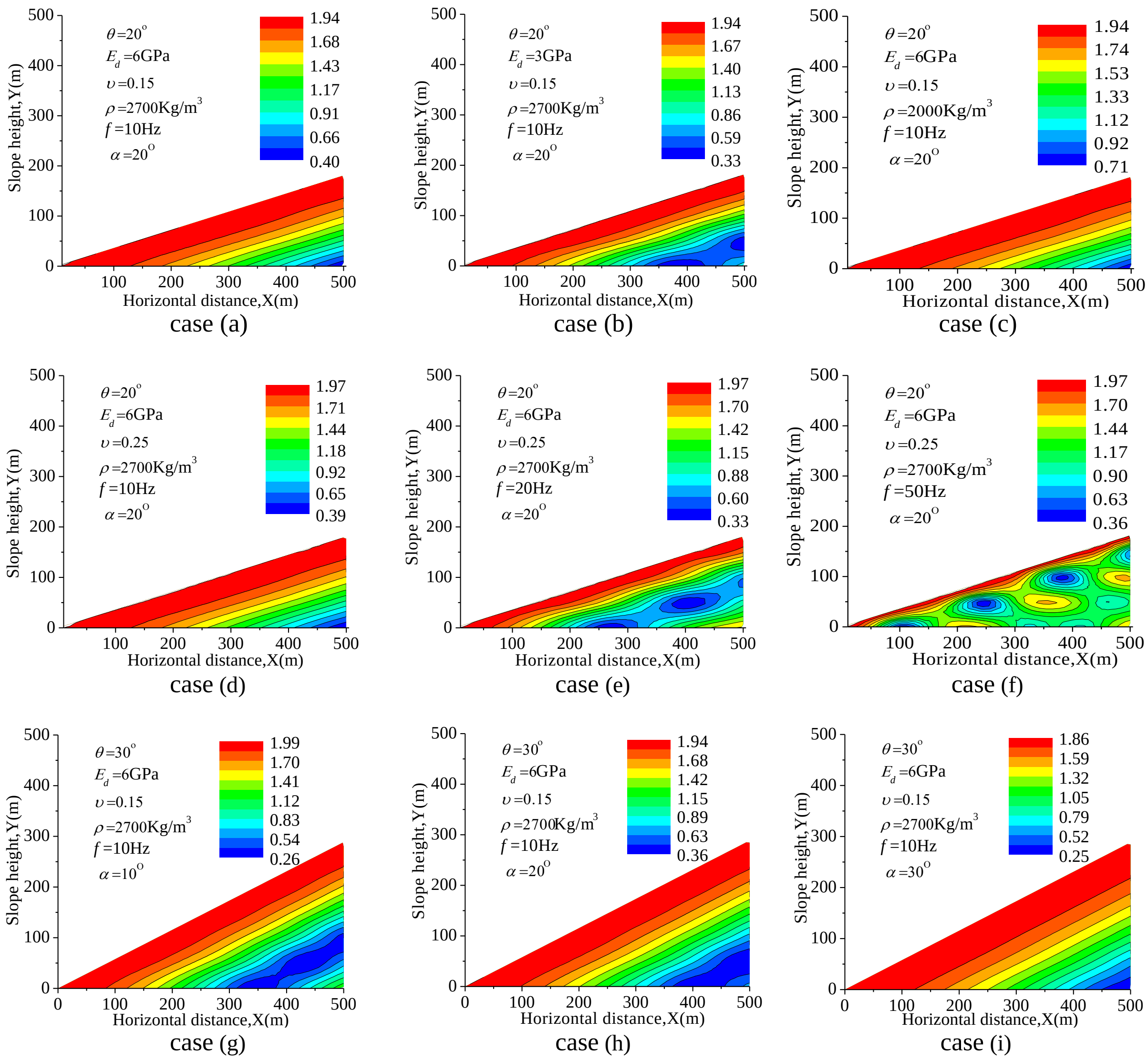

Figure 4.1. Distribution regularity of displacement amplification factor related to different parameters(red color denotes the salient region affected by earthquake)

Analytical results show us that elevation amplification effect has been proved by this theoretical solution, displacement amplification factor increases with elevation in vertical section, meanwhile, amplification factor increases from inner slope to the outer in the horizontal section. As shown in the Max(η_d) column of **Table 4.1.**, the maximum of displacement amplification factor only relies on Poisson's ratio and input angle. According to Eqn.3.5, the reason is inferred that displacement amplification factor depends on the reflection coefficients of wave mode conversion, herein, reflection coefficients are decided by Poisson's ratio and input angle, as shown in Eqn.(2.3) and (2.4).

Compared **Figure 4.1.(a)** and **Figure 4.1.(b)**, it suggests the distribution regularity of displacement amplification factors and maximum value are almost the same even if dynamic Young's modulus reducing an half, but with dynamic Young's modulus increase, the thickness of the saliently affected region(represented by red color in **Figure 4.1.**) becomes thicker. The effect of density can be revealed by comparison between **Figure 4.1.(a)** and **Figure 4.1.(c)**. When density reduces 26%, the distribution regularity of displacement amplification factors is similar to each other, maximum value is the same and minimum slightly increases. In a word, density has smaller effect on slope dynamic responses than dynamic Young's modulus, which can be explained by following Eqn.4.2:

$$v_p = \sqrt{\frac{E_d(1-\upsilon)}{\rho(1-2\upsilon)(1+\upsilon)}} \text{ and } v_s = \sqrt{\frac{E_d}{2\rho(1+\upsilon)}} \tag{4.2}$$

Eqn.4.2 suggests the velocity of P-wave and S-wave increases with dynamic Young's modulus and decreases with the increment of density, if the parameters of seismic wave, slope angle and Poisson's ratio keep constant, the seismic wave superposition point with the same displacement amplification factor will be deeper as a result of the velocity increase. Therefore, the saliently affected region in **Fig.4.1.(a)** and **(c)** is thicker than that in **Fig.4.1(b)**. In generally, the increment absolute value of dynamic Young's modulus is larger than that of density when slope material changing, therefore, dynamic Young's modulus is more influential than density to slope dynamic responses.

The comparison of **Figure 4.1.(a)** and **Figure 4.1. (d)** suggests with the increase of Poisson's ratio, the maximum of displacement amplification factor slightly increases and its distribution regularity is almost the same. With increment of seismic wave frequency, isolines of displacement amplification factors change from almost parallel to the slope inclined surface to rhythm distribution with multi-extreme values, the thickness of saliently affected region becomes shallower with frequency increase, as shown in **Figure 4.1.(d), Figure 4.1.(e),** and **Figure 4.1.(f)**, which is consistent with that lower frequency causes more damage during earthquake. **Figure 4.1.(g), Figure 4.1.(h),** and **Figure 4.1.(i)** reveal that the maximum of displacement amplification factor decreases with increment of input angle, and the thickness of saliently affected region becomes thicker. Compared **Figure 4.1.(a)** and **Figure 4.1.(h)**, the result suggests when slope angle increases, the distribution regularity and value of displacement amplification factors are almost the same, and the saliently affected region gradually becomes shallower.

5. DISCUSSION AND CONCLUSIONS

Due to wave mode conversion on the slope free air surface, complicated seismic wave field exists in the slope. Although, slope shape was simplified and the damping was not taken into consideration, the theoretical solution of this paper may be referred. Conclusions are follows: (1) The parameters of slope material and seismic wave are more influential to slope dynamic responses than those of slope geometry shape. The distribution regularity of displacement amplification factors is determined by seismic wave frequency; the maximum of displacements relies on Poisson's ratio and wave input angle; slope angle, dynamic Young's modulus and density have effect on the thickness of the saliently affected region. (2) Seismic wave mode conversion and wave superposition cause slope dynamic responses. Lower frequency of seismic wave induces that the isolines of slope displacement (velocity, acceleration) amplification factors are parallel to slope inclined surface. Higher frequency brings out rhythm distribution with multi-extreme values; Lower frequency induces more remarkable dynamic responses. (3) Theoretical solution has proved the elevation amplification effect. With elevation increase, displacement (velocity, acceleration) amplification factor becomes larger, meanwhile, the theoretical solution reveals that amplification factor turns larger from inner slope to the outer.

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