Ultra-Low Forced-Vibration Testing of a Large Building

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SUMMARY
Over the past five years, the authors have performed forced vibration testing on a variety of modest, low-rise building structures. The experimental setup consists of a small linear shaking device that gently (below human perception) vibrates the building structure and highly sensitive (but inexpensive) hardware and software to collect and process the results. In this paper, the procedure to experimentally determine the modal parameters (natural frequency, mode shape, and damping) of a much larger building structure is described. The chosen building not only is much larger in scale, it also contains many irregularities that make the placement of the shaker to maximize the mode of interest particularly difficult. This paper describes the process, which is on-going, that the authors used to accommodate these challenges and produce the experimentally derived mode shapes and periods to be used for comparison to a computational model of the structure.

Keywords: Ultra, Low, Forced, Vibration, Testing

1. INTRODUCTION

Over the past several years, the authors have performed Ultra-Low Forced Vibration Tests (UL-FVT) on a variety of low-rise building structures using a small linear shaker that puts out a modest 30 lb sinusoidal force (McDaniel and Archer 2009, Roskelley 2010). The UL-FVT research is funded by NSF-NEESR. UL-FVT is a low-cost, non-intrusive, easily-portable, and practical means to experimentally determine a building's natural mode shapes, periods, and damping. The building structures tested to date (McDaniel and Archer 2010) include steel moment resisting frames, steel concentric braced frames, reinforced concrete moment-resisting frames, and reinforced concrete shear walls. The majority of the structures were rectangular or L-shaped 2 and 3 story on-campus buildings with around 20,000 ft^2 of floor space. In the current work (Rendon 2011), the authors have sought to discover the limit of the experimental setup by testing a much larger structure; a 5-story 180,000 ft^2 library building (Fig. 1.1).

Figure 1.1. Robert E. Kennedy Library

The building under investigation is a five story reinforced concrete shear wall structure built in 1977. Aside from the sheer size of the structure, this building contains many irregular shear walls and a large atrium located near the center that runs the entire height of the building and takes up roughly 10-16% of the entire floor area (Fig. 1.2).

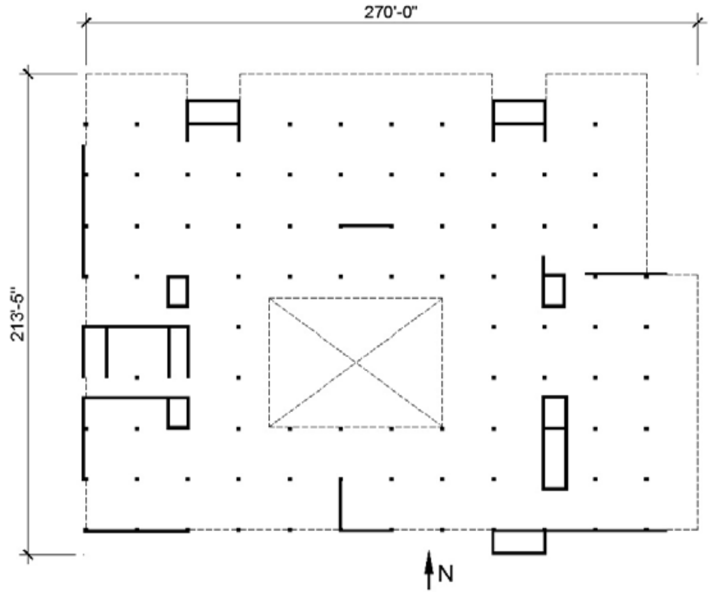


Figure 1.2. 2^{nd} Floor Plan of Building

These building features create many challenges. In order for the UL-FVT procedure to capture the dynamic properties of a building structure, the induced floor accelerations must be clearly identified above the ambient vibrations. To achieve this, placement of the shaker to maximize the mode of interest and minimize the effect of the other modes is critical. Often, the optimal placement for some modes is in the vicinity of the centers of mass and stiffness. In this library structure, the non-uniform distribution of mass, the irregular shear wall placement, and the large central atrium have resulted in both the center of mass and center of stiffness located in the atrium void. Furthermore, two modal periods were found to be relatively close to one another. This presents a very difficult problem with regard to isolating one mode from the other.

In this paper, the initial pass in the incrementally improved procedure to experimentally determine the modal parameters (natural frequency, mode shape, and damping) by means of a variety of UL-FVT's is described. The first three mode shapes and periods of the building are determined experimentally and the shaker locations for the next iteration are presented.

2. EXPERIMENTAL SETUP

The UL-FVT procedure consists of four phases: 1) an initial broad-range ambient vibration test (AVT) to determine the likely range of structural frequencies; 2) an FVT over a more narrow range of frequencies to accurately determine the natural modal frequencies; 3) a very narrow range FVT to determine modal damping ratios; and 4) a fixed frequency FVT to determine the mode shape.

The heart of the UL-FVT test equipment is a small portable linear shaker (Fig. 2.1) with a total weight of about 80 lbs. The shaker is capable of putting out a relatively constant sinusoidal force of only 30 lbs over a frequency range of 2-20 Hz. The sensors used are piezoelectric flexural accelerometers with frequency ranges of less than 1 Hz to greater than 200 Hz and advertised broadband resolutions of 1-3μg rms. Three sensors are used, two perpendicular to each other and one parallel but separated from the first to capture floor rotations about a vertical axis. The total cost for UL-FVT equipment is modest -- less than $15,000.

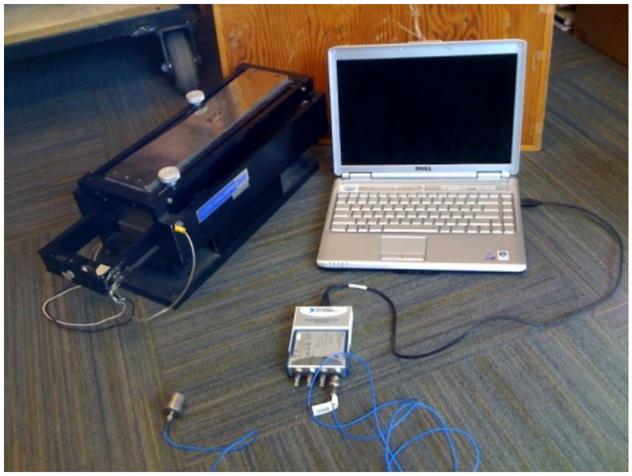


Figure 2.1 Test Equipment

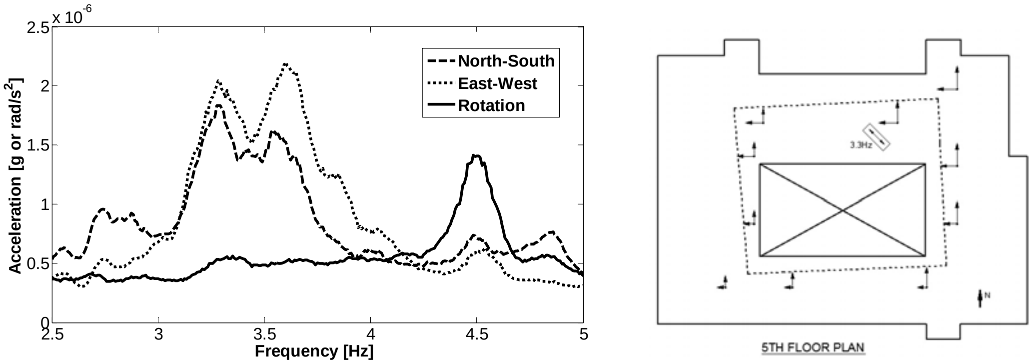
The constant battle with the UL-FVT is with the always present ambient noise. The shaker's 30 lb harmonic force produces structural accelerations that are only significantly above the ambient noise at resonance. Thus FVT methods involving white noise are not applicable. The white noise put out by such a small shaker is not sufficient to drown out the variable ambient noise. Thus once the structure's natural frequencies are identified by the AVT and FVT sweep test, the shaker is set at the natural frequency and placed to maximize its effect on a single mode and minimize its effect on adjacent modes. The placement is an educated trial and error process. The resulting floor accelerations are then measured at selected locations throughout the building to estimate the mode shape. Even with careful shaker placement, it is not possible to excite a single mode. The shaker will undoubtedly not be orthogonal to some modes. Given modal coupling through damping, these mode shapes will be present in the measured values. Thus they are numerically swept out using a Modified Gramm-Schmidt algorithm (Golub 1989).

3. EXPERIMENTAL TESTING

Ambient vibration tests (AVT) often are useful (Foschaar 2008) in guiding the selection of shaker locations for forced vibration tests (FVT). However for this building the AVT was inconclusive. The AVT test identified three possible natural frequencies between 3.2 and 3.6 Hz. However at each of these frequencies, the NS and EW motions were of a similar magnitude and the rotational accelerations were minimal. Therefore for the first FVT, the shaker location was chosen randomly on the 5th floor (the roof was inaccessible) as shown in Fig. 3.1b.

The shaker was then set to sweep from 2.5 to 5 Hz and the Fast Fourier Transform (FFT) of the resulting 5th floor accelerations was recorded as shown in Fig. 3.1a. Accelerations induced by the shaker were recorded as high as 60μg, about 12 times the average ambient acceleration readings. Induced accelerations were detected across the entire diaphragm at the 5th, 4th, and 3rd floors, with the shaker operating at the 5th floor. Readings decreased in acceleration by approximately 20% per floor. To further validate the acceleration readings, a modal analysis was performed to produce theoretical data to compare to the measured accelerations at the same location (and due to the same shaker frequency, orientation and location). Measured acceleration magnitudes matched the theoretical acceleration magnitudes to within about 10% and the relative directions were the same.

The first three natural frequencies of this building are at 3.3, 3.65, and 4.56 Hz. The first two natural frequencies are close together and appear to be associated with motion in both the EW and NS directions. The third natural frequency is primarily associated with rotation. Also, it is apparent that this shaker location is not ideal for isolating modes 1 and 2. Had this been a good location to isolate mode 1, the FFT would have shown a large peak at 3.3 Hz and vastly diminished peaks at 3.65 Hz and 4.56 Hz. Therefore, the next step is to find a better shaker location to isolate mode 1.

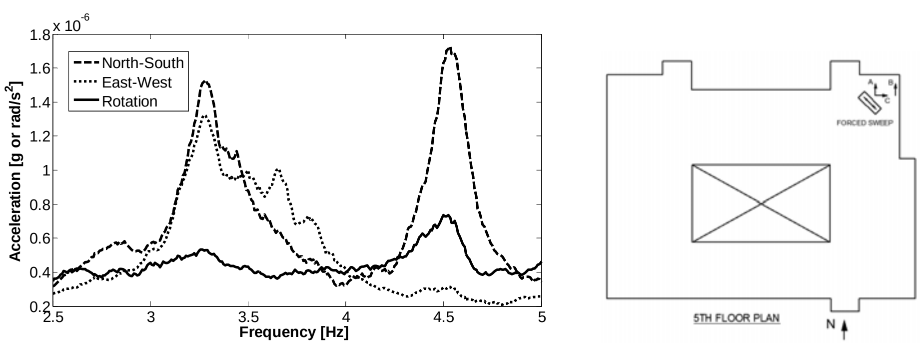


(a) (b)

Figure 3.1. (a) Initial FFT; (b) shaker location/orientation

4. MODE 1

To determine mode shape 1, acceleration mapping was performed with the shaker oscillating at 3.3 Hz at the current location/orientation. The resulting shape is a mixture of any number of natural mode shapes of the structure, although it will be dominated by mode 1. The largest accelerations are at the north-eastern corner (in the NW-SE direction) of the diaphragm and that the accelerations approach zero at the south-western corner. These accelerations indicate that a good shaker location to isolate this shape is at the north-eastern corner of the building as shown in Fig. 4.1b. It is important to note that taking more than one reading is necessary only when the diaphragm is not expected to exhibit rigid behavior as is the case for Mode 1.



(a) (b)

Figure 4.1. (a) Mode 1 FFT; (b) Mode 1 shaker location/orientation

The FFT in Fig. 4.1a shows that this location is a much better place to isolate the first natural frequency from the second. Had the second natural frequency shown up as a larger peak, forced vibration sweeps would have been performed with different shaker orientations until an orientation was discovered that reduced this peak. Looking again at Fig. 4.1a, the third natural frequency has a larger influence than at the previous location (see Fig. 3.1a). However, complete isolation of a given mode is unlikely if a building is not symmetric. Acceleration mapping of the diaphragm was then performed with the shaker oscillating at 3.3 Hz at the location/orientation shown in Fig. 4.1b to uncover what will be called apparent mode shape 1 (AMS 1). The term *apparent* is used here to highlight the fact that the acceleration readings are not only due to one mode, but all modes that have directional components common to the directional components excited by the shaker. The results of the acceleration mapping are shown in Fig. 4.2. The solid line represents the undeformed position and the dashed line represents AMS 1. Also, while taking acceleration readings, it was discovered that the diaphragm is essentially rigid in AMS 1. This rigidity was confirmed by constant readings for rotation at multiple locations on the diaphragm. Readings for rotation that are different from one location to the next indicate flexibility in the diaphragm.

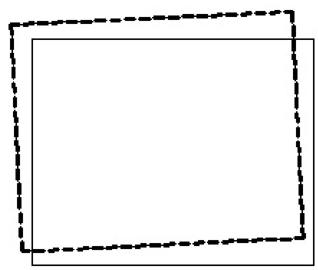
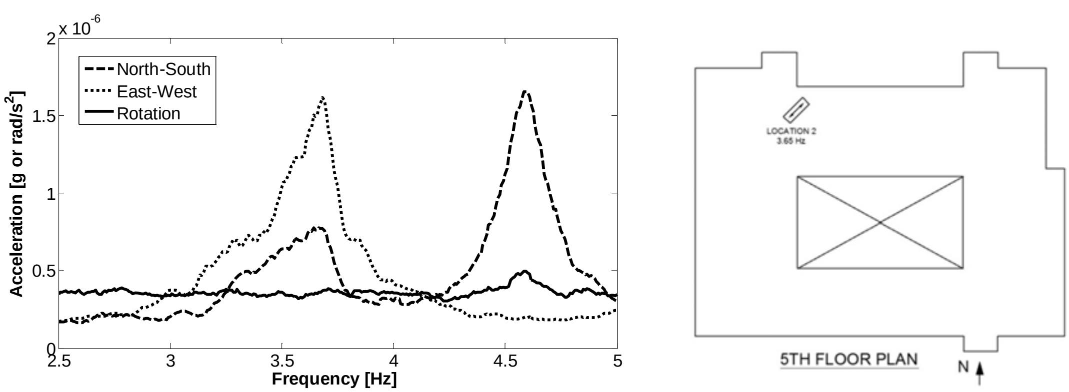


Figure 4.2. Apparent Mode Shape 1 (AMS 1)

5. MODE 2

The shaker location for mode 2 was chosen to be orthogonal to apparent mode shape 1. Since mode 1 is primarily a motion in the NW-SE direction, an orientation in the NE-SW direction was chosen as shown in Fig. 5.1b. As can be seen from the FFT, this was successful at diminishing the contribution at 3.3 Hz (mode 1). However, as it was in the FFT for mode 1, this location also results in a significant excitation of mode 3.



(a) (b)

Figure 5.1 (a) Mode 2 FFT; (b) Mode 2 shaker location/orientation

Leaving the shaker at location 2 and setting it to oscillate at 3.65 Hz, apparent mode shape 2 (AMS 2) was mapped. AMS 2 is shown in Fig. 5.2. It is primarily a diagonal motion from the SW corner to the NE corner of the building with very little rotation. Like AMS 1, AMS 2 is a rigid diaphragm motion.

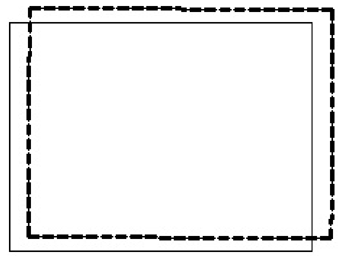
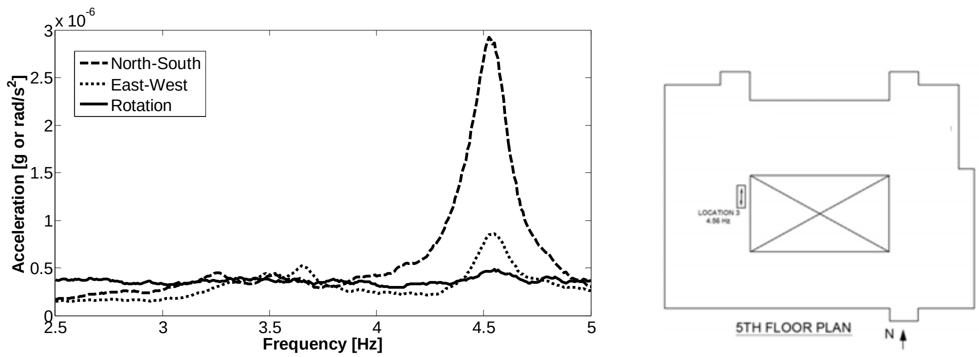


Figure 5.2 Apparent Mode Shape 2 (AMS 2)

6. MODE 3

Apparent mode shape 3 (AMS 3) was the most interesting of the three modes because it exhibited flexible diaphragm behavior. Thus the shaker location that would excite a shape orthogonal to apparent mode shapes 1 and 2 is not easy to visualize; therefore, shaker location 3 was chosen based on the FFTs resulting from forced vibration sweeps performed at several locations. Shaker location 3, shown in the Fig. 6.1b produced the FFT shown in Fig. 6.1a with the best isolation of the third natural frequency from the first two.



(a) (b)

Figure 6.1 (a) Mode 3 FFT; (b) Mode 3 shaker location/orientation

The results of Fig. 6.1a indicate that this shaker location/orientation simultaneously produces a large effect for mode 3 and little detectable excitation of modes 1 and 2. This is an apparent contradiction to the previous results for locations 1 and 2 that produced a significant response in mode 3. Thus it would be reasonable to believe that location 3 should excite modes 1 and 2 with equal strength. Fig. 6.1a disputes this assumption. However, if one ignores the flexible nature of the mode 3 response, it can be seen (Fig. 6.2) that the rigid-body portion of the shape is essentially a rotation about the center of the building. Thus positioning the shakers at the periphery of the structure perpendicular to the radius arm from the center, as done in the shaker locations for AMS 1 and AMS 2, is unfortunately an ideal way to excite mode 3. Thus mode 3 is strongly represented as rotational accelerations in AMS 1 and AMS 2 (see Fig. 4.1a and 5.1a). Conversely location 3, while not orthogonal to modes 1 and 2, is a poor way to excite modes 1 and 2. Similarly location 3 is a poor place to measure the NS and EW motions. Thus modes 1 and 2 are only mildly represented by in the FFT for location 3 shown in Fig. 6.1a.

The results of the acceleration mapping due to the shaker oscillating at 4.56 Hz at shaker location 3 are shown in Fig. 6.2. The solid line represents the undeformed position and the dashed line represents the deformed position (based on recorded accelerations). There is significant flexibility in this apparent mode shape; the east side and the west side of the library move opposite to each other in the NS direction. This flexibility is also the reason why finding shaker location 3 was done in a trial-and-error fashion. The flexibility made it so that the shaker location and orientation weren't representative of the shape that it excited (representative in the sense that the shape could be easily deduced by the shaker's positioning).

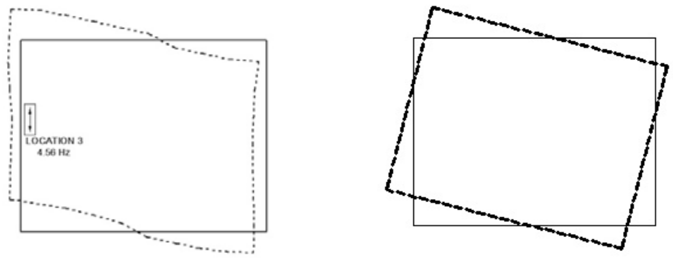


Figure 6.2 Apparent Mode Shape 3 (AMS 3) and the Rigid-Body Approximation

7. MODAL COUPLING

By definition, mode shapes are orthogonal to each other. As previously noted, the shapes that were derived experimentally were *apparent* mode shapes. Apparent mode shapes aren't the pure mode shapes; although through the efforts of shaker placement they are primarily comprised of the mode shape of interest. However, due to structural damping and the near-impossibility of optimally placing the shaker, the apparent mode shapes contain components of all modes. Thus AMS's are not orthogonal to each other.

A good test for coupling is the mass-weighted modal assurance criterion (MAC) (Allemang 2003, McDaniel and Archer 2010). The MAC compares two modal vectors (ϕ_i and ϕ_j) and results in a number between 0 and 1. Two orthogonal modes have a MAC number of 0 and two identical modes have a MAC number of 1. The mass-weighted MAC formula is provided in Eqn. 7.1 where the mass matrix of the structure is M.

$$MAC = \frac{\left(\phi_i^T M \phi_j\right)^2}{\left(\phi_i^T M \phi_i\right)\left(\phi_j^T M \phi_j\right)} \tag{7.1}$$

Table 7.1 gives the mass weighted MAC numbers that compare the three apparent mode shapes (AMS) to one another.

Table 7.1. Mass Weighted MAC Numbers Comparing the Apparent Mode Shapes

AMS	1 and 2	1 and 3RB	2 and 3RB
Mass Weighted MAC number	0.0076	0.9832	0.0463

Table 7.1 shows that AMS 1 and 2 and AMS 2 and 3RB are virtually uncoupled from each other. However AMS 1 and 3 are strongly coupled. Since AMS 2 is essentially uncoupled from both 1 and 3, AMS 2 can be considered the true mode 2 (within experimental error). However, the strong coupling between AMS 1 and 3RB indicates that the rigid body portion of AMS 3 is nearly identical to AMS 1. Since AMS 1 is a rigid diaphragm motion (and is orthogonal to mode 2) and AMS 3 is a flexible diaphragm motion it is clear that AMS 1 is the true mode 1 and that AMS 3 is a combination of mode 3 and mode 1. In other words, shaker location 3 excited both modes 3 and 1.

Fig. 7.1 shows the shaker locations that should be investigated next. Shaker location 2 (used to isolate mode 2) should be left in its original position because this was a good location to isolate mode 2 from mode 1 (see Fig. 4.1a and 5.1a). In order to decouple modes 1 and 3 further, shaker location 3 should be moved to where the accelerations for mode 1 are smallest. This location may have the effect of removing the previously noted 1st mode contribution from AMS 3. Shaker location 1 should be moved to the SE corner of the building because this location might excite an AMS 1 that is more orthogonal to AMS 2. Looking again at Fig. 6.2, diaphragm motion at the SE corner in mode 1 is primarily NS. Therefore, a NS orientation of the shaker at shaker location 1 in Fig. 7.1 should also be investigated because it may produce a purer mode 1. Shaking at these locations, taking readings every 16′ (to capture diaphragm flexibility), describing the apparent mode shapes with several (more than three) degrees of freedom per floor, and exploring higher modes, may uncover new behavior that could explain the discrepancy noted earlier in this section.

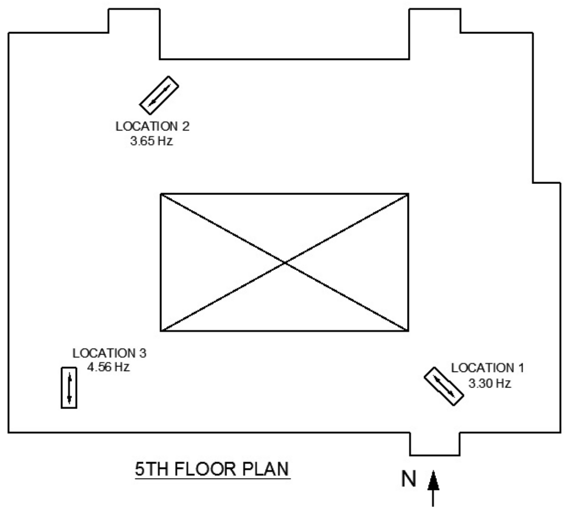


Figure 7.1 Shaker Locations for Further Investigation

8. SWEEPING

While great care is taken to position the shaker to eliminate as much of the effect of the other modes, the Apparent Mode Shapes (AMS) will contain some influence of the other modes. These hopefully small quantities of the other modes need to be swept out of the shape. Given two mode shapes ϕ_i and ϕ_j, the formula used for sweeping ϕ_i out of ϕ_j is the Modified Gram-Schmidt (MGS) provided in Eqn. 8.1.

$$\phi_j^{'} = \phi_j - \left(\frac{\phi_i^T M \phi_j}{\phi_i^T M \phi_i} \right) \phi_i \tag{8.1}$$

Where $\phi_j^{'}$ is the mode shape resulting from sweeping ϕ_i out of ϕ_j, and
M is the mass matrix of the structure.

The general procedure is to start with a pure mode and sweep it out of the remaining shapes. Then in turn, each purified shape is swept out of the remaining shapes. As was described earlier, AMS 2 is considered the true second mode shape. It is now swept out of AMS 1 and AMS 3 using equation 2. Due to the small MAC numbers listed in Table 1 for AMS 1 and 2 and AMS 2 and 3RB, this will provide only a minor modification to the shapes. The revised AMS 1 can now be considered the true first mode shape. Ideally at this point, the revised AMS 1 would be swept out of AMS 3 to provide the third mode shape. However, due to the extremely high correlation between AMS 1 and AMS 3 shown in Table 7.1, it is unclear if the portion of AMS 3 that remains after the sweeping process could be considered significant. Thus it was decided to retest using the new shaker locations outlined in Fig. 7.1. The work is ongoing.

9. CONCLUSIONS

The UL-FVT procedure, which was designed to be used on smaller low-rise building structures, is in general able to produce the lower mode shapes and periods of a much larger complicated structure. The larger structure presented many challenges with regard to the selection of locations for the placement of the shaker. Typically the center of mass is a good first guess. The large central atrium precluded this choice. The irregular layout of the building in plan and non-symmetric arrangement of the lateral resisting walls results in the first two modal periods to be close to each other. Furthermore, the first two mode shapes appear to be combinations of both translational motions and rotation. Thus it was extremely difficult to find shaker locations that simultaneously excited the mode of interest but had minimal effect on the adjacent modes. The first two mode shapes and periods and the third modal period were successfully determined. Based on the results, a suggested shaker arrangement is suggested for further work.

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