A New Method For Reducing Dimensionality of Finite Fault Inverse Problem In Time Domain

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SUMMARY:
Earthquake source inversion studies have played an important role in improving our understanding of the nature of earthquake sources. However, solving linear and non-linear inverse problems requires great computational effort. In this paper, a new method is proposed to reduce the dimensionality of linear and non-linear inverse problems. In this regard, the kinematic finite-fault inversion procedure in time domain is rewritten in a normed space with Green functions of the medium as the bases of the normed space. The method requires a fault plane with known geometry and prescribed dimensions to identify the coseismic distribution of slip. By taking the new formulation, the information included in a lengthy seismic waveform is compressed by projecting it into the bases of the normed space. It is shown that without changing the accuracy of the results, a great reduction in computational time and memory space is achieved through implementation of the proposed method.

Keywords: Inverse problem, Finite fault, Green

1. INTRODUCTION

Earthquake source inversion studies have played an important role in improving our understanding of the nature of earthquake sources. Source inversion studies for determining the spatial and temporal distribution of coseismic slip on relevant earthquake faults have increased dramatically in recent years because of their ability to give detailed source information. The slip histories of past earthquakes have key roles in simulating potential future earthquakes. However, solving linear and non-linear inverse problems requires great computational effort.

In this paper, a new method is proposed to reduce the dimensionality of linear and non-linear inverse problems. In this regard, the kinematic finite-fault inversion procedure in time domain is rewritten in a normed space with Green functions of the medium as the bases of the normed space. The method requires a fault plane with known geometry and prescribed dimensions to identify the coseismic distribution of slip. In conventional formulation, each time point on each record is explicitly included in the inversion. Thus, the dimension of the matrix of synthetics is the sum of the number of time points included in all waveforms, resulting in a very large matrix which requires large memory space and computational effort to manipulate. However, by taking the new formulation, the information included in a lengthy seismic waveform is compressed by projecting it into the bases of the normed space.

Another advantage of the proposed method is reducing ill-conditioning results from redundancies in the matrix which occur because many columns in the matrix are simply time shifts of each other. In the proposed method such redundancies are avoided since the whole waveform is projected into the bases of the normed space. The value of the proposed method is more highlighted in the case of non-linear inverse problems, where it is always necessary to solve many forward problems using different search algorithms. The results of the proposed method are verified with the results of conventional formulation from the viewpoints of accuracy and stability of the results. In this regard the accuracy and stability of the results were examined through a synthetic test using synthetic Green's functions (SGFs) calculated for homogenous half space. It is shown that without changing the accuracy of the results, a great reduction in computational time and memory space is achieved through implementation of the proposed method.

In the following, first, the conventional formulation of finite fault inverse problem is reviewed in brief and then the new formulation is presented. To examine the capabilities of new formulation, a comparison is made between the results of conventional method of solving inverse problem and the new formulation through a synthetic test. A brief discussion is also made at the end of the paper.

2. FINTE FAULT INVERSE PROBLEM

Faulting is characterized by the slipping of one side of a fault surface with respect to the other. If the earth is modeled as an elastic solid, then the displacement field due to a point dislocation can be taken as a Green's function for the earthquake faulting problem. The displacement field at all points in the earth due to an arbitrary distribution of slip on a fault is expressed as an integral over the fault surface of the slip distribution convolved with the Green's function. The slip distribution enters linearly into the integrand so that it may be obtained as the solution to a linear inverse problem in which recorded ground motion at the earth's surface is taken as data. The representation theorem provides an expression for the radiation in an elastic media resulting from the creation of a discontinuity in the displacement and stress fields across a fault surface. The general form of representation theory is (Aki, and Richards 2002, Kostrov and Das 1988):

$$u_n(x,t) = \int d\tau \int_S \Delta u_i(\zeta,\tau).c_{ijkl}.\nu_j.G_{nk,l}(x,t;\zeta,\tau).ds \tag{2.1}$$

where u_n is displacement field in the domain, Δu is dislocation field on fault surface *S*, c_{ijkl} is linear coefficients of the medium and *G* represents Green function of the space.

The fault surface is usually divided into a set of cells, each cell being a rectilinear planar zone. Locations within each cell are assumed to undergo the same slip within a specified time shift. Within each cell, the slip is described by a two-component vector in the plane of the fault having unknown time dependence. In Fig. 2.1, the general geometry of finite fault model is illustrated.

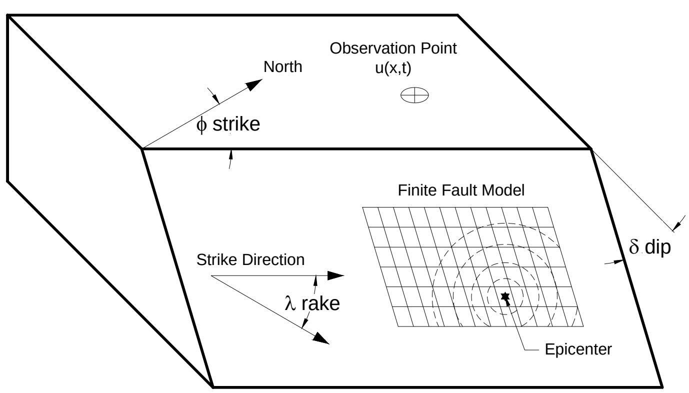

Figure 2.1. General geometry of finite fault model

It is possible to write discrete form of representation theorem as follow (Olson and Apsel 1982):

$$\Delta u(x,t) = \sum_{j=1}^{Nc} X_j(x).\sum_{k=-K}^{K} u_{jk}.P_k(x,t) \tag{2.2}$$

$$X_j(x) = \begin{cases} 1 & if\, x\ is\ \ in\ \ jth\ cell \\ 0 & otherwise \end{cases} \tag{2.3}$$

$$P_k(x,t) = F(t - T(x) + k\delta t) \tag{2.4}$$

The first sum in Eqn. 2.2 is over the Nc cells representing the fault surface. The second sum defines the slip within the j^{th} cell. The vector u_{jk} is the slip direction of the j^{th} cell at the k^{th} time point and has two components in the plane of the fault. The function $P_k(x,t)$ contains the time dependence of the k^{th} slip. Each cell is allowed to slip $2K + 1$ times at successive increments in time. Each slip varies according to the specified time function $F(t)$. The absolute time at which slip takes place within the cell is centered about $T(x)$. Substituting Eqn. 2.2 in Eqn. 2.1 we have

$$u_n(x,t) = \sum_{j=1}^{Nc} \sum_{k=-K}^{K} u_{jk} \, . g_j^n(x, t + k.\delta t) \tag{2.5}$$

$$g_j^n(x, t + k.\delta t) = \left(\int_{S_j} G_{nk,l}(\zeta, \tau; x, t).ds \right) \otimes F(t - T(x) + k\delta t) \tag{2.6}$$

where $\otimes$ represents convolution operator.

It is possible to write Eqn. 2.5 in the matrix form. The dimension of corresponding coefficient matrix will be $(2K+1)\times Nc \times 2$ columns and $3 \times N\,stations \times N_{rec}$ rows, where $N\,stations$ represents number of seismic or strong motion stations considered in inversion and N_{rec} represents number of sampling points in each record. As it is clear from Eqn. 2.5, for each sampling point of each record, it is necessary to add one row in the matrix form. Although the difference between successive rows is very small, however, it is necessary to keep all rows in this formulation. As a result the size of memory storage that is needed to keep such large matrixes is relatively large in real applications. Since in most applications, it is not possible to keep such large function in the memory of the computers, it is necessary to use hard drives in computation of matrix inversion, which result in a large increase in the time of processing, even with the use of recent, high technology computers.

The small difference between successive rows makes determinant of coefficient matrix to be very small and the inverse problem be ill-conditioned as well. In the following section, a new formulation is proposed which condenses the information represented by similar rows.

3. THE NEW FORMULATION OF INVERSE PROBLEM IN TIME DOMAIN

To reduce the redundancy in coefficient matrix of inverse problem, we applied the notion of inner product of normed spaces represented in mathematics of functional analysis (Griffel 2002). A (real or complex) inner product space is a (real or complex) vector space V with an inner product specified. An inner product is a rule which, given any $x, y \in V$, specifies a (real or complex) number (x,y), called the inner product of x and y. This definition is an abstract version of the scalar product of elementary vector algebra. Using the definition of inner product in a normed space, called usually Hilbert space, it is possible to define the notion of weak convergence as well (Griffel 2002). A sequence (x_n) in a Hilbert space H converges weakly if for every $y \in H$ the sequence of numbers (x_n,y) converges. The notion of weak convergence is actually the basis of almost all numerical method like finite element method (FEM) and boundary element method (BEM). We apply the notion of weak convergence in the formulation of representation theorem. It is possible to rewrite Eqn. 2.1 in the following form:

$$u_n(x,t) = \int d\tau \int_S \Delta u_i(\zeta,\tau).c_{ijkl}.\nu_j.G_{nk,l}(\zeta,\tau;x,t).ds$$

$$= \int d\tau \int_S \left\{ \begin{array}{l} \mu.\left(\Delta u_1(\zeta,\tau).\nu_2 + \Delta u_2(\zeta,\tau).\nu_1\right).\underbrace{\left(G_{n1,2}(\zeta,\tau;x,t) + G_{n2,1}(\zeta,\tau;x,t)\right)}_{G_{12n}} + \\ \mu.\left(\Delta u_1(\zeta,\tau).\nu_3 + \Delta u_3(\zeta,\tau).\nu_1\right).\underbrace{\left(G_{n1,3}(\zeta,\tau;x,t) + G_{n3,1}(\zeta,\tau;x,t)\right)}_{G_{13n}} + \\ \mu.\left(\Delta u_2(\zeta,\tau).\nu_3 + \Delta u_3(\zeta,\tau).\nu_2\right).\underbrace{\left(G_{n2,3}(\zeta,\tau;x,t) + G_{n3,2}(\zeta,\tau;x,t)\right)}_{G_{23n}} + \\ 2\mu.\left(\begin{array}{l} \Delta u_1(\zeta,\tau).\nu_1.G_{n1,1}(\zeta,\tau;x,t) + \Delta u_2(\zeta,\tau).\nu_2.G_{n2,2}(\zeta,\tau;x,t) \\ + \Delta u_3(\zeta,\tau).\nu_3.G_{n3,3}(\zeta,\tau;x,t) \end{array} \right) \end{array} \right\}.ds \qquad (3.1)$$

In the case where the surface of the fault is divided into N_{cells} cells, we have

$$u_{jn} = \sum_{i=1}^{N_{cells}} \left\{ \begin{array}{l} \underbrace{\left(\upsilon_2.G_{12n}^{ij} + \upsilon_3.G_{13n}^{ij} + \upsilon_1.G_{11n}^{ij}\right)}_{\varphi_{1n}^{ij}} u_1^i + \underbrace{\left(\upsilon_1.G_{12n}^{ij} + \upsilon_3.G_{23n}^{ij} + \upsilon_2.G_{22n}^{ij}\right)}_{\varphi_{2n}^{ij}} u_2^i + \\ \underbrace{\left(\upsilon_1.G_{13n}^{ij} + \upsilon_2.G_{23n}^{ij} + \upsilon_3.G_{33n}^{ij}\right)}_{\varphi_{3n}^{ij}} u_3^i \end{array} \right\} \quad \left\{ \begin{array}{l} j = \overline{1, N_{rec}} \\ n = 1,2,3 \end{array} \right. \qquad (3.2)$$

Or equivalently,

$$u_{jn} = \sum_{i=1}^{N_{cells}} \varphi_{1n}^{ij}.u_1^i + \varphi_{2n}^{ij}.u_2^i + \varphi_{3n}^{ij}.u_3^i \qquad (3.3)$$

If we write the dislocation vectors of fault surface in the direction of strike and dip angles, we have

$$u_{jn} = \sum_{i=1}^{N_{cells}} \varphi_{1n}^{ij}.(\theta_{1s}.u_s^i + \theta_{1d}.u_d^i) + \varphi_{2n}^{ij}.(\theta_{21s}.u_s^i + \theta_{2d}.u_d^i) + \varphi_{3n}^{ij}.(\theta_{3s}.u_s^i + \theta_{3d}.u_d^i) \qquad (3.4)$$

$$u_{jn} = \sum_{i=1}^{N_{cells}} (\underbrace{\varphi_{1n}^{ij}.\theta_{1s} + \varphi_{2n}^{ij}.\theta_{2s} + \varphi_{3n}^{ij}.\theta_{3s}}_{\eta_{sn}^{ij}}).u_s^i + (\underbrace{\varphi_{1n}^{ij}.\theta_{1d} + \varphi_{2n}^{ij}.\theta_{2d} + \varphi_{3n}^{ij}.\theta_{3d}}_{\eta_{dn}^{ij}}).u_d^i \qquad (3.5)$$

$$u_{jn}(t) = \sum_{i=1}^{N_{cells}} \eta_{sn}^{ij}(t).u_s^i + \eta_{dn}^{ij}(t).u_d^i \qquad (3.6)$$

The formulation represented by Eqn. 3.6 is actually functional expansion of known displacement field $u_{jn}(t)$ across the known functions η_{sn}, η_{dn} and unknown fault surface dislocation values u_s, u_d. Thus, with analogy to weighted residual method in FEM, it is possible to use the notion of inner product and write Eqn. 3.6 in weak convergence form as

$$< u_{jn}(t), \eta_{sn}^{i'j}(t) > = \sum_{i=1}^{N_{cells}} < \eta_{sn}^{ij}(t), \eta_{sn}^{i'j}(t) > .u_s^i + < \eta_{dn}^{ij}(t), \eta_{sn}^{i'j}(t) > .u_d^i$$

$$< u_{jn}(t), \eta_{dn}^{i'j}(t) >= \sum_{i=1}^{N_{cells}} < \eta_{sn}^{i\,j}(t), \eta_{dn}^{i'j}(t) > u_s^i < \eta_{dn}^{i\,j}(t), \eta_{dn}^{i'j}(t) > .u_d^i$$
$$i = \overline{1, N_{cells}} \qquad j = \overline{1, N_{rec}} \qquad n = 1,2,3 \tag{3.7}$$

In the matrix form, it is possible to write Eqn. 3.7 as

$$\begin{bmatrix} <\eta_{sn}^{11}(t),\eta_{sn}^{11}(t)> & <\eta_{sn}^{21}(t),\eta_{sn}^{11}(t)> & \cdots & <\eta_{sn}^{N_{cells}1}(t),\eta_{sn}^{11}(t)> \\ <\eta_{dn}^{11}(t),\eta_{dn}^{11}(t)> & <\eta_{dn}^{21}(t),\eta_{dn}^{11}(t)> & \cdots & <\eta_{dn}^{N_{cells}1}(t),\eta_{dn}^{11}(t)> \\ <\eta_{sn}^{11}(t),\eta_{sn}^{21}(t)> & <\eta_{sn}^{21}(t),\eta_{sn}^{21}(t)> & \cdots & <\eta_{sn}^{N_{cells}1}(t),\eta_{sn}^{21}(t)> \\ <\eta_{dn}^{11}(t),\eta_{dn}^{21}(t)> & <\eta_{dn}^{21}(t),\eta_{dn}^{21}(t)> & \cdots & <\eta_{dn}^{N_{cells}1}(t),\eta_{dn}^{21}(t)> \\ \vdots & \vdots & \vdots\cdots & \vdots \\ <\eta_{sn}^{1N_{rec}}(t),\eta_{sn}^{N_{cells}N_{rec}}(t)> & <\eta_{sn}^{2N_{rec}}(t),\eta_{sn}^{N_{cells}N_{rec}}(t)> & \cdots & <\eta_{sn}^{N_{cells}N_{rec}}(t),\eta_{sn}^{N_{cells}N_{rec}}(t)> \\ <\eta_{dn}^{1N_{rec}}(t),\eta_{dn}^{N_{cells}N_{rec}}(t)> & <\eta_{dn}^{2N_{rec}}(t),\eta_{dn}^{N_{cells}N_{rec}}(t)> & \cdots & <\eta_{dn}^{N_{cells}N_{rec}}(t),\eta_{dn}^{N_{cells}N_{rec}}(t)> \end{bmatrix}_{(2\times N_{celss}\times 3\times N_{rec})\times(2\times N_{cells})} \times$$

$$\begin{Bmatrix} u_s^1 \\ u_d^1 \\ \vdots \\ u_s^{N_{cells}} \\ u_d^{N_{cells}} \end{Bmatrix}_{2\times N_{cells}} = \begin{Bmatrix} <S_{1n}(t),\eta_{sn}^{11}(t)> \\ <S_{1n}(t),\eta_{dn}^{11}(t)> \\ <S_{1n}(t),\eta_{sn}^{21}(t)> \\ <S_{1n}(t),\eta_{dn}^{21}(t)> \\ \vdots \\ <S_{N_{rec}n}(t),\eta_{sn}^{N_{cells}N_{rec}}(t)> \\ <S_{N_{rec}n}(t),\eta_{dn}^{N_{cells}N_{rec}}(t)> \end{Bmatrix}_{2\times N_{celss}\times 3\times N_{rec}} \tag{3.8}$$

where $S_{jn}(t) = u_{jn}(t)$.

The main and primary advantage of the formulation represented in Eqn. 3.8 is major reduction of computation effort and memory storage. To better highlight this point, as it was mentioned previously, the size of coefficient matrix in conventional formulation of Eqn. 2.6 is $((N_{cells} \times 2) \times (3 \times N_{stations} \times N_{rec})$, taking *K=1*. On the other hand, the size of coefficient matrix of Eqn. 3.8 is $(N_{cells} \times 2) \times (2 \times N_{cells} \times 3 \times N_{stations})$. Thus, the ratio of matrix size of conventional method and the proposed formulation is $N_{rec} / (2 \times N_{cells})$. Taking $N_{rec} = 3000$ samples and $N_{cells} = 100$, this ratio is equal to 15. In the other words, the coefficient matrix of the conventional method is 15 times greater than the matrix of the proposed method.

Inner product of Green function vector and observes motion has a physical meaning as well. The number of inner product indicates the degree that the observed motion is correspond to Green function of the space. The bigger the value of inner product of observed motion and Green function vector is, the more is the influence of the Green function of that cell in total construction of synthetic record.

4. SYNTHETIC TEST OF PROPOSED FORMULATION

To better examine the advantages and disadvantages of the proposed method, a synthetic test is performed. In Table 4.1, the characteristics and parameters of synthetic forward problem are presented. In addition, in Fig. 4.1, the slip distribution of fault surface and the corresponding rake angle of slip in each cell are presented.

Table 4.1, Parameters used in synthetic test of proposed method

Strike angle	323 degree
Dip angle	90 degree
Fault length	42 Km
Fault width	10.5 Km
Number of cell in strike direction	14
Number of cells in dip direction	4
Cell length in strike direction	3 Km
Cell length in dip direction	2.6 Km
Distance from upper point of the fault to free surface	1 Km
Rupture velocity	2500 m/s
Velocity of P waves	5900 m/s
Velocity of S waves	3400 m/s

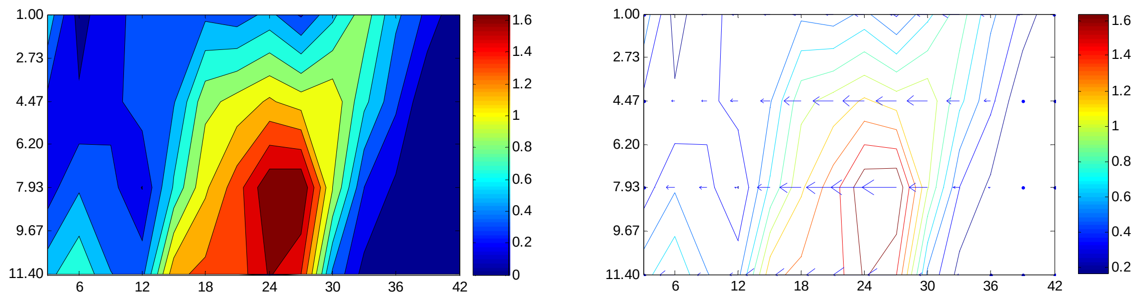

Figure 4.1. Fault slip distribution and corresponding rake angle

To solve the forward problem, 11 stations are considered on the ground surface. The relative location of these stations and fault plane is presented in Fig. 4.2. Also shown in Fig. 4.3 is the synthetic displacement obtained by solving forward problem. The Green function used in this test is isotropic and homogenous Green function obtained in half space (Johnson 1974).

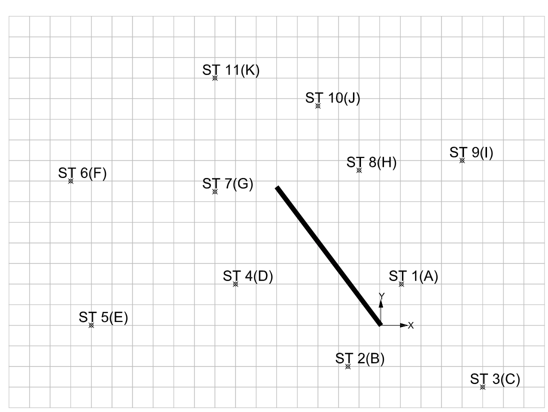

Figure 4.2. Relative location of stations and fault plane in synthetic test

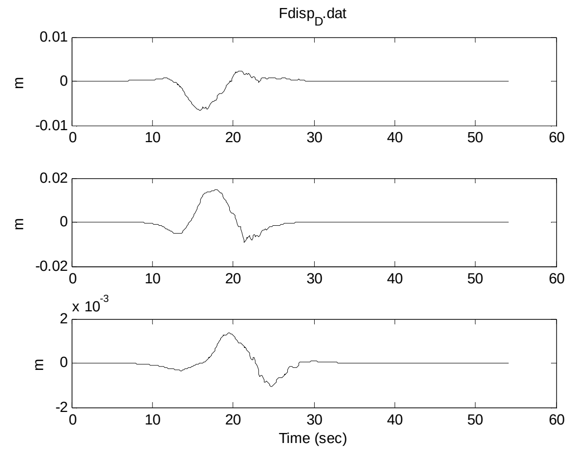

Figure 4.2. Synthetic displacement at station D

In Fig 4.3, displacement time histories obtained by solving inverse problem using conventional method and proposed formulation are compared with the true hypothetical displacement of point D. As it is clear from this figure, both conventional and proposed method give identical results, however, as previously mentioned, the execution time of the proposed method is much less than the conventional method. The time required to solve inverse problem with conventional formulation was 6.5156 seconds with a Intel Corei5 processor and 8 Gb of RAM memory. The corresponding execution time for proposed formulation was 3.3752 second which is approximately half the time required for solving using conventional method.

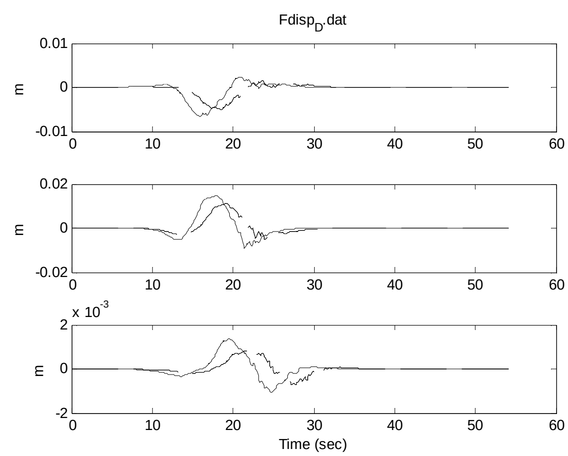

Figure 4.2. Synthetic displacement (solid line) and two displacement time histories obtained by conventional and proposed formulation of inverse problem at station D

It is worth mentioning that the proposed method has also value in solving non-linear inverse problems. In solving non-linear inverse problem by grid search method, it is usually common to solve many linear inverse problems by different input parameters and then compute the error of each solution. By finding the minimum of the error function, it is possible to find the best solution of non-linear inverse problem. As a result, the proposed method could also be used in solving non-linear inverse problem as well.

6. CONCLUSION

In this paper, a new method is proposed to reduce the dimensionality of linear and non-linear inverse problems. In this regard, the kinematic finite-fault inversion procedure in time domain is rewritten in a normed space with Green functions of the medium as the bases of the normed space. The method requires a fault plane with known geometry and prescribed dimensions to identify the coseismic distribution of slip. In conventional formulation, each time point on each record is explicitly included in the inversion. Thus, the dimension of the matrix of synthetics is the sum of the number of time points included in all waveforms, resulting in a very large matrix which requires large memory space and computational effort to manipulate. However, by taking the new formulation, the information included in a lengthy seismic waveform is compressed by projecting it into the bases of the normed space. It is shown that by using the proposed method, it is possible to solve linear and non-linear inverse problem with much less computational efforts.

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