Extended CCA Method for Estimating Phase Velocity Using Arbitrarily Shaped Arrays

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SUMMARY:
We propose a new method for estimating phase velocity using arbitrarily shaped array configurations based on the CCA method. In general, it is necessary to set the radius of array as a constant value for determining the phase velocity by the CCA method. On the other hand, the proposed method fixes the frequency as a constant value instead of radius of array. By calculating the spectral ratios in several radii (i.e. any two sensors), the phase velocity can be obtained to remedy the constraint of the array design. In order to confirm the applicability of the proposed method, we carried out numerical experimentation and measurements at a site where PS-logging is available. The phase velocity dispersions obtained using both conventional and the proposed methods were compared with those calculated from the PS-logging data. From this study, we conclude that the proposed method gives a promising result in wide frequency range.

Keywords: Microtremor, Phase Velocity, CCA Method, SPAC Method

1. INTRODUCTION

From the viewpoint of earthquake disaster mitigation, it is important to evaluate the dynamic characteristics of the ground. There are a number of methods available that are used for estimating S-wave velocity profile. Among others, microtremor measurement is one of the simplest and the most inexpensive ways and has been conducted extensively. The joint inversion of array data (i. e. dispersion curve (e.g., Aki, 1957; Capon, 1969; Cho et al., 2006) and horizontal to vertical amplitude ratio (H/V spectrum (e.g., Nakamura, 1989; Tokimatsu et al., 1998; Arai et al., 2004; Sánchez-Sesma et al., 2010)) in order to obtain the S-wave velocity profile has been successfully conducted by several authors (e.g., Arai et al, 2005; Nakagawa et al., 2011). Cho et al.(2006) have introduced the CCA (centerless circular array) method that enables to estimate phase velocities of Rayleigh waves by analyzing vertical component records of microtremors that are obtained with an array of three or five sensors placed around a circumference. In this method, it possesses higher resolution in long-wavelength ranges than the other methods such as the SPAC (spatial autocorrelation) method (Aki, 1957) and the F-k (frequency-wavenumber) method (Capon, 1969).

However, there are problems for practical use to estimate phase velocity by the CCA method. One is that it is not easy to apply the CCA method for data from arbitrarily shaped arrays such as strong ground motion records. It is well known that the F-k method and the ESPAC method can deal with the arbitrarily shape configurations. In this article, we proposed the new method based on the CCA method using arbitrarily shaped configurations. In order to examine the applicability of the proposed method, the observation data were analyzed by the use of both conventional and the proposed method at a site where the PS-logging is available.

2. METHODOLOGY

There are several methods to estimate the phase velocity from data set of a microtremor array measurement. In particular, SPAC and F-k method are well known and can be found in many articles so far. In this article, we propose the new method to estimate the phase velocity based on the CCA method, which was developed from the SPAC method by Aki (1957). Therefore, the methods for estimating phase velocity from microtremor measurement are briefly reviewed below.

2.1. Spatial Autocorrelation (SPAC) Method

Aki (1957) has introduced the correlation method to estimate the phase velocity by having an array of receivers equally spaced on a circle of radius r and having an extra receiver at the center as shown in Fig.1. Using a Fourier transform, observation records at receiver A and B can be expressed as follows:

$$X_A(f) = U_A(f)\exp(-i\phi_A(f)) \tag{2.1}$$

$$X_B(f) = U_B(f)\exp(-i\phi_B(f)) \tag{2.2}$$

where, f indicates frequency, i is imaginary number, U, ϕ denote amplitude and phase, respectively, suffixes denote observation points. The coherence between receiver A and B can be expressed as following:

$$coh(f,r) = \frac{X_A(f)\cdot X_B^*(f)}{U_A(f)\cdot U_B(f)} = \exp\{i(\phi_B - \phi_A)\} = \exp\left\{i\frac{2\pi fr}{c(f,\psi)}\right\} \tag{2.3}$$

where, the superscript "*" denotes complex conjugate, ψ indicates azimuth angle between incoming wave and the line that links the point A and B, c denotes apparent phase velocity. However, in practice, we do not know arrival direction of incoming wave before a data processing. In order to solve directional dependency, the coherence between each of the receivers on the circle and the one in the center are averaged to obtain the SPAC coefficient, ρ, which is related to the function:

$$\rho(f,r) = \frac{1}{2\pi}\int_0^{2\pi}\exp\left\{i\frac{2\pi fr}{c(f,\psi)}\right\}d\psi = J_0\left(\frac{2\pi fr}{c(f)}\right) \tag{2.4}$$

where, $J_0(\cdot)$ stands for the zeroth-order Bessel Functions of the first kind. Once the SPAC coefficient is obtained from observation data, the phase velocity, c, can be determined by fitting the function appearing on the right-hand side of eq. (2.4) for each frequency, f.

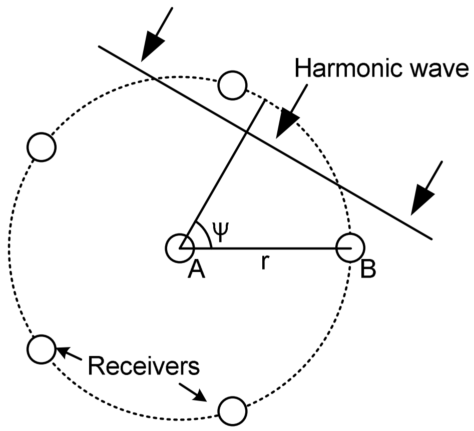

Figure 1. Configuration of array and incoming plane harmonic wave

2.2. Centerless Circular Array (CCA) Method

Cho et al. (2006) have introduced the new technique in order to obtain phase velocity from microtremor measurement by the use of circular arrays. The essential part of this approach is to extract the higher Fourier coefficients from circular array. In the SPAC method, it only utilizes the zeroth-order Bessel Functions of the first kind. In contrast, the CCA method can be exploited both the zeroth- and first-order Bessel Functions of the first kind. Hence, it possesses higher resolution in long-wavelength ranges than the SPAC method (see Fig.2).

Suppose we deploy a circular array of radius r as shown in Fig. 1, and let the vertical component of microtremor be denoted by x_j, where suffix j indicates the sensor number located on the array circle. The average value along the circumference is given by:

$$\alpha_0(t) = \frac{1}{N}\sum_{j=1}^{N} x_j(t) \tag{2.5}$$

and, a weighted average of x_j over the azimuth angle θ is expressed as:

$$\alpha_1(t) = \frac{1}{N}\sum_{j=1}^{N} x_j(t)\exp(i\theta_j) \tag{2.6}$$

By computing the power spectral ratio, we can obtain the CCA coefficient, ρ_{cca}. If the single Rayleigh wave mode dominates the vertical component of the microtremor wave field, ρ_{cca} is related to the function:

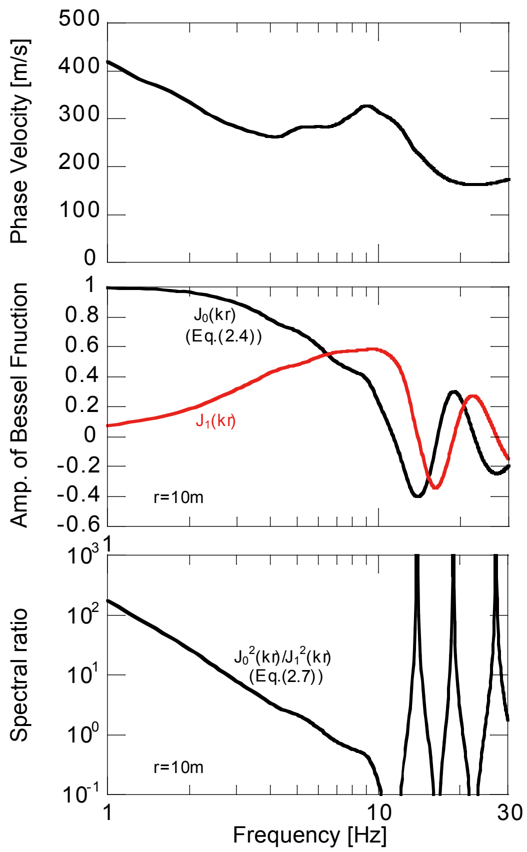

Figure 2. Phase velocity of Rayleigh waves (top), SPAC and CCA coefficient appearing on the right-hand side of eqs. (2.4) and (2.7)

$$\rho_{cca}(f) = \frac{J_0^2(rk(f))}{J_1^2(rk(f))} \tag{2.7}$$

where, k indicates wave number, $J_0(\cdot)$ and $J_1(\cdot)$ are the zeroth- and first-order Bessel Functions of the first kind, respectively. Thus, it is possible to determine rk by inverting Eq. (2.7) for each frequency f, if the CCA coefficient, ρ_{cca}, is obtained from measurement records. Since r is known, the phase velocity, c, is then determined by $c = 2\pi f / k$. In other ways, if the radius r is very small compared to a target wavelength, ρ_{cca} is related to the phase velocity, c, by the following expression (Tada et al., 2007):

$$c(f) = \pi f r \sqrt{2 + \rho_{cca}(f)} \tag{2.8}$$

2.3. Extended CCA Method

As mentioned above, generally, in determining the phase velocity by the CCA method, it is necessary to set the radius of array as a constant value. On the other hand, the proposed method fixes the frequency as a constant value instead of radius of array. By calculating the spectral ratios in several radii (i.e. any two sensors), the phase velocity can be obtained to remedy the constraint of the array design. In the SPAC method, similar topic was discussed by the previous researchers (e. g., Ling et al., 1993; Ohori et al., 2002).

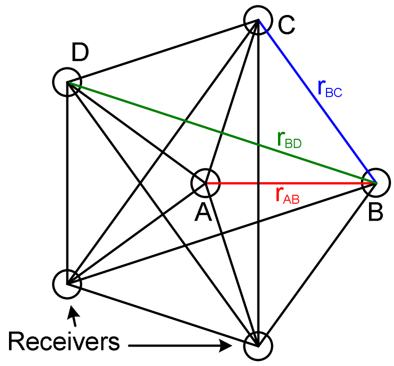

Figure 3. Combinations for calculating CCA coefficient based on the extended CCA method. CCA coefficients can be obtained from several radii for each frequency.

3. NUMERICAL EXPERIMENTAION

Firstly, we applied the proposed method to synthetic wave data. Table 1 shows the velocity profile for calculating synthetic waveform. Synthetic waves were calculated by the 2.5-dimensional thin layered element method (Nakagawa et al., 2010) due to incident fundamental mode of Rayleigh wave. The incident angles considered are 0°, 30°, 45° and 60°. Figure 4 shows estimation result of phase velocities by both ESPAC and the proposed method. Black lines indicate the theoretical phase velocity of fundamental Rayleigh wave from the model in Table 1. It is confirmed that the proposed method gives a promising result in wide frequency range when compared to the conventional method. It is also confirmed that the estimation accuracy becomes low when the array configuration is flat.

Table 1. Velocity profile for calculating synthetic waves to estimate phase velocity

Layer No.	V_p [m/s]	V_s [m/s]	Density [kg/m^3]	Thickness [m]
1	1500	200	1500	12
2	1500	700	1500	∞

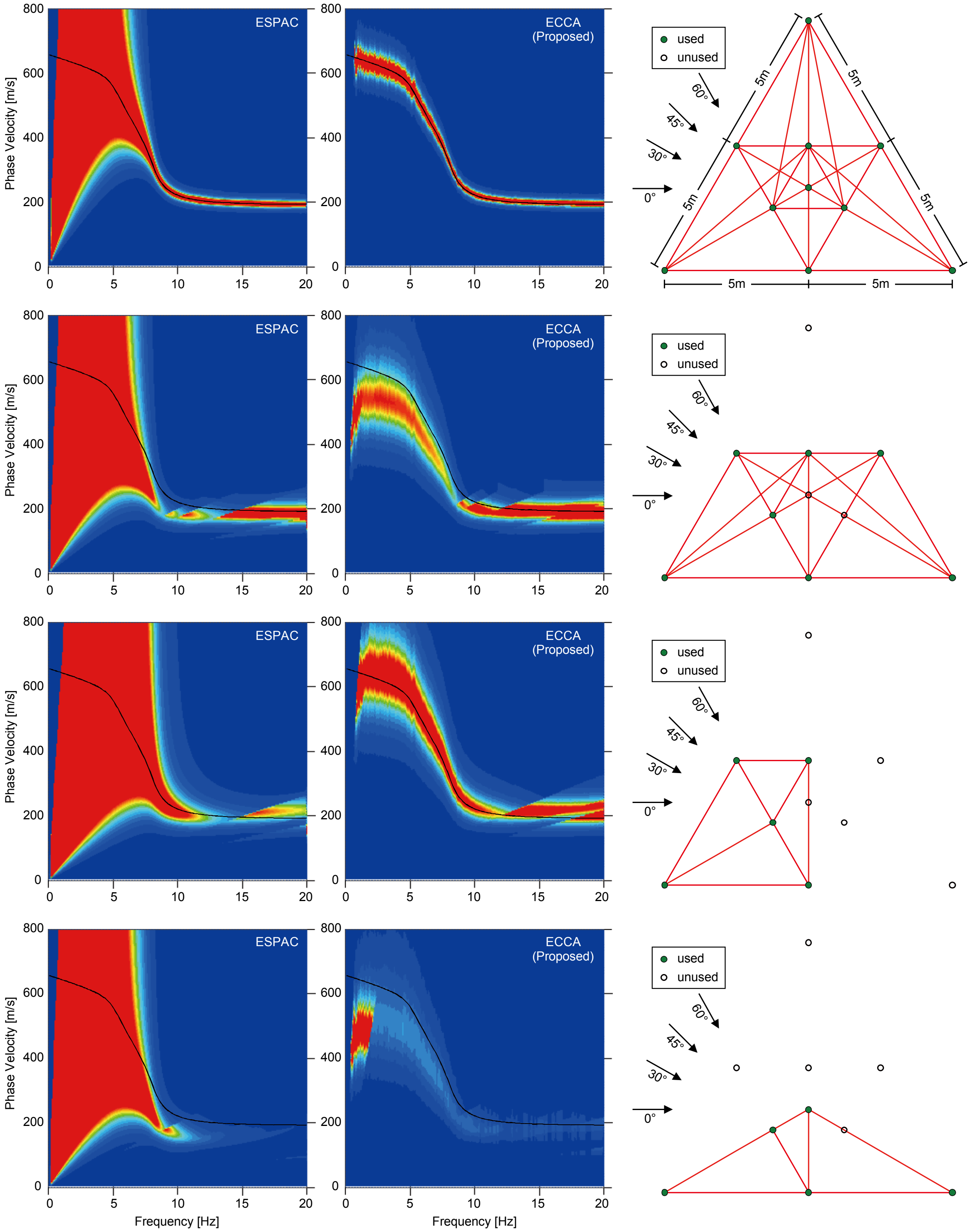

Figure 4. Estimated phase velocities from the synthetic waveform data by (left) extended SPAC method and (center) the proposed (extended CCA) method. Contour maps express the residuals. Black lines indicate the theoretical phase velocity of fundamental mode of Rayleigh wave from the model in Table 1. (right) Array configurations for estimating phase velocity. Green closed circles denote receivers. Open circles are unused receivers. Red lines indicate the combinations for calculating SPAC and CCA coefficient. Arrows represent the incident angles of fundamental mode of Rayleigh wave.

4. FIELD IMPLEMENTATION

4.1. Microtremor Array Data

Next, we applied the proposed method to real microtremor data from the field located in Tsukuba, Japan. From the seismic reflection/refraction exploration results, shallow underground structure of the field is mainly horizontally layered medium. Table 2 shows the velocity profile from the PS-logging data at the site. Microtremor measurement was carried out during the day on 29 August 2011. Array configuration is shown as the top right of Figure 5. The data sampling rate and the duration time, respectively, were set to 500Hz and 10 min. In calculating the coherence and the spectral ratio, 10 sets of data segments with 16,384 points each were selected and transformed into the frequency domain with the fast Fourier transform. Then the cross spectrum and the power spectrum for each segments were computed and smoothed by Parzen window with a width of 0.3 Hz.

Figure 5 shows the estimation result of phase velocities by both ESPAC and the proposed method. Black lines indicate the phase velocity of superposed Rayleigh waves from the PS-logging model in Table 2. In the superposition of multiple Rayleigh waves, we used the medium responses (Harkrider, 1964) as weighting factor. As can be seen in the figure, the proposed method gives a promising estimation. It is also confirmed a tendency of accuracy degradation in frequency below about 2Hz. This tendency is mainly attributed to sensor characteristics.

Table 2. Velocity structure of the test field obtained from PS-logging

Layer No.	V_p [m/s]	V_s [m/s]	Density [kg/m^3]	Thickness [m]
1	430	140	1500	4
2	1680	270	1800	44
3	1680	370	1800	54
4	1740	430	1900	82
5	1740	510	1900	∞

4.2. Coda Part of Strong Ground Motion Records

Finally, we applied the proposed method to coda part of the strong ground motion records from the field as shown in Fig.6. The earthquakes considered in the analysis are listed in Table 3. In calculating the coherence and the spectral ratio, 30 sets of data segments with 16,384 points each were selected. Then the cross spectrum and the power spectrum for each segments were computed and smoothed by Parzen window with a width of about 0.01 Hz.

Figure 7 shows the estimation result of phase velocities by both ESPAC and the proposed method. Black lines represent the estimated phase velocity using microtremor records by SPAC method (Suzuki et al., 1999). It is found from the result that estimated phase velocity by the proposed method is in good agreement with the previous survey. According to Suzuki et al. (1999), they have carried out microtremor exploration using the several equilateral triangle configurations. The base length of the largest array is 1,000 m. This implies that the proposed method possesses higher resolution in long- wavelength ranges than the conventional method.

Table 3. Earthquake information used in the analysis

Date (mm/dd/yy)	Origin Time	Latitude(N)	Longitude (E)	Depth [km]	M_{jma}*
03/16/11	13:14:29.6	37.533	141.580	25	5.6
03/22/11	18:19:05.2	37.315	141.910	43	6.4
04/07/11	23:32:43.4	38.203	141.920	66	7.2
07/10/11	09:57:07.3	38.032	143.507	34	7.3
07/25/11	03:51:25.3	37.708	141.627	46	6.3
08/19/11	14:36:31.6	37.648	141.797	51	6.5
01/01/12	14:27:52.0	31.427	138.565	397	7.0

*M_{jma} is a magnitude determined by the Japan Meteorological Agency.

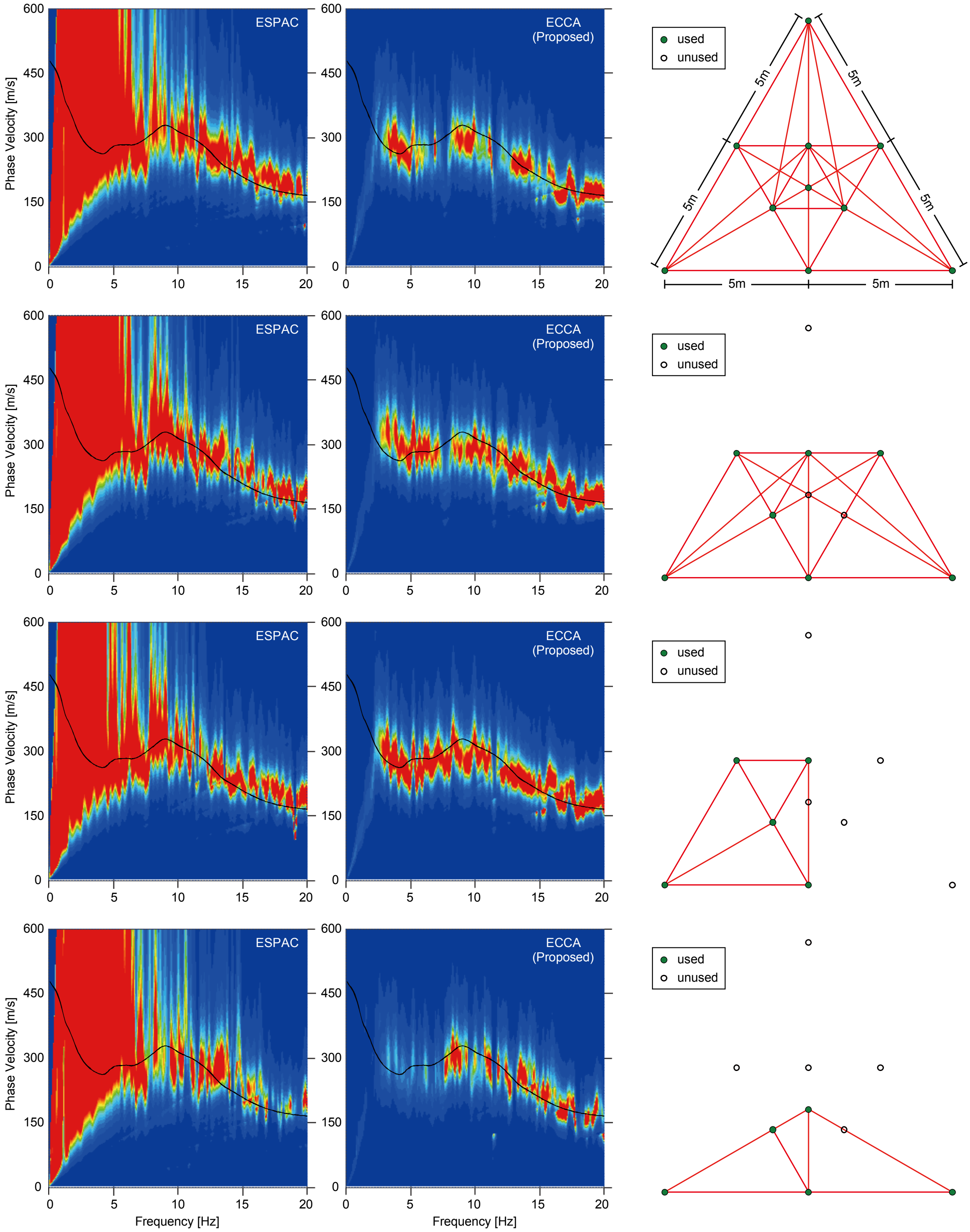

Figure 5. Estimated phase velocities from the observed microtremor data by (left) extended SPAC method and (center) the proposed (extended CCA) method. Contour maps show the residuals. Black lines indicate the phase velocity of superposed mode of Rayleigh waves from the PS-logging model in Table 2. (right) Array configurations for estimating phase velocity. Green Circles denote receivers. Open Circles are unused receivers. Red lines indicate the combinations for calculating SPAC and CCA coefficient.

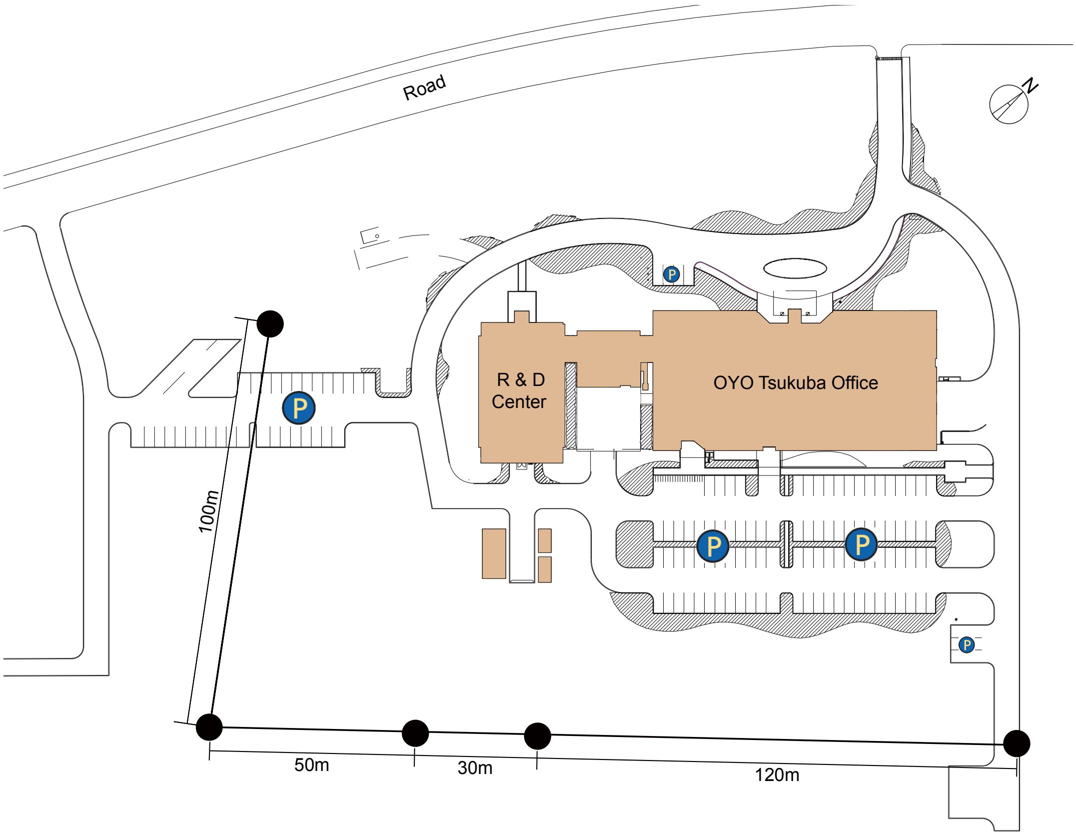

Figure 6. Seismometer array configurations at OYO Tsukuba office located in Tsukuba, Japan. Closed circle indicates the seismometer.

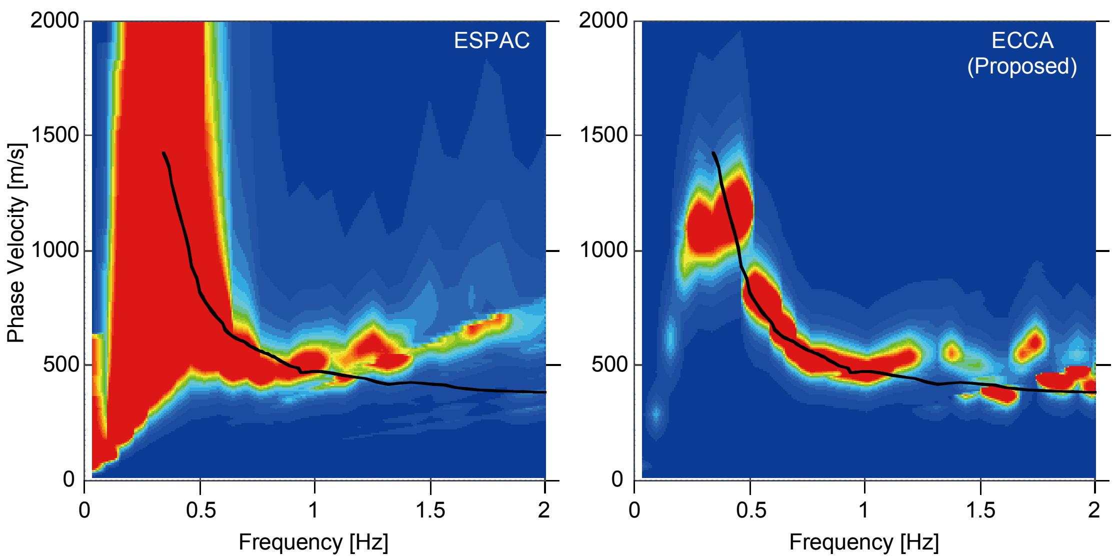

Figure 7. Estimated phase velocities from coda part of the strong ground motion data by (left) extended SPAC method and (right) the proposed (extended CCA) method. Contour maps show the residuals. Black lines indicate the estimated phase velocity using microtremor records (after Suzuki et al., 1999).

5. CONCLUSION

In this paper, we proposed the new method for estimating phase velocity using arbitrarily shaped array configurations based on the CCA method. In order to confirm the applicability of the proposed method, we carried out numerical experimentation and microtremor measurements at a site where PS-logging is available. In addition, we have applied the proposed method to coda part of strong ground motion data. The phase velocity dispersions obtained using both conventional and the proposed methods were compared with those calculated from the PS-logging data and with a literature. From this study, we conclude that the proposed method gives a promising result in wide frequency range.

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