Simplified Dynamic Model for 3-dimensional Multi-story Steel Moment Frames with Eccentricity

D. Ehara, M. Yamanari & K. Ogawa *Graduate School Science and Technology, Kumamoto University, Japan*

Y. Sakai

Nippon Steel Engineering Co., Ltd., Japan

SUMMARY:

This paper presents a simplified dynamic model useful for the simulation of earthquake response of 3-dimensional multi-story steel moment frames with eccentricity. The simplified dynamic model can be completed with the methodology that all beams at each floor level are condensed into two couples of beams in orthogonal direction. All columns in each story are also condensed into one representative column. This procedure leads to reduction of the number of the freedom in each story. The accuracy of simplified dynamic model is calibrated for a couple of benchmark frames with two kinds of ground motions. Errors associated with the model are found to be negligible. Therefore, the model proposed here can provide the effective way to identify general earthquake response of multi-story steel moment frames with eccentricity.

Keywords: Eccentricity, Torsional vibration, 3-dimensional multi-story frames, Fishbone-shaped frame

1. INTRODUCTION

It is important to estimate the behavior of multi-story moment frames with eccentricity of the story under bidirectional ground motion for seismic design. Concerning the effect of eccentricity, many researchers have been investigating with one-mass system or one-story frames before. Conversely, some researchers who tried to study on multi-story frames only obtained qualitative results through case studies. In other words, a lot of numerical effort is needed to obtain a quantitative estimation of multi-story frames by means of complicated models covering wide range of physical parameters. Therefore, more simplified analytical model is needed to solve the problems without complicated and elaborate calculation. The purpose of the development of the model is to identify essential structural parameters that have effect upon earthquake response. The simplified dynamic model has only 6 degrees of freedom in each story. If the model can approximate earthquake response of multi-story moment frames with eccentricity, its dynamic characteristics can be represented by few structural parameters. Only stiffness eccentricity or only strength eccentricity can be considered in the model. Therefore, the analytical method with the model is valid way to clarify seismic response characteristics of multi-story moment frames with eccentricity.

2. PROPOSAL OF SIMPLIFIED DYNAMIC MODEL

In this chapter, outline of simplified dynamic model is showed.

2.1. Outline of Simplified Dynamic Model

The authors propose a simplified dynamic model, which is useful for the simulation of earthquake response of 3-dimensional multi-story steel moment frames with eccentricity. The model is shown Figure 2.1.1. The constitution of the model can be completed with the methodology that all beams at each floor level are condensed into two couples of beams in orthogonal direction. All columns in each story are also



condensed into one representative column. The effect of eccentricity by taking account of radius distance that expresses resistance against torsion in each story and eccentric distance are considered in the model. Here, the radius distance and the eccentric distance are concerned with inertial characteristics and with rigidity and strength of structural elements.



Figure 2.1.1. Simplified dynamic model

The simplified dynamic model has 6 degrees of freedom in each story. Degrees of freedom are given as follows.

1) Nodal rotation in the X-Y structural plane; θ_Y

2) Nodal rotation in the X-Z structural plane; θ_Z

3) Nodal rotation due to rotation of floor; ξ_X

4) Torsional rotation due to rotation of floor; θ_X

5) Horizontal displacement in Y-direction; v

6) Horizontal displacement in Z-direction; w

The nodal rotations θ_{Y} and θ_{Z} are shown in Figure 2.1.2 and Figure 2.1.3 respectively. It is assumed that the values of θ_Y of all nodal points lying on the floor should be the same. The values of θ_Z should be also obtained by the same way. The nodal rotations ξ_X are shown in Figure 2.1.4. This is represents nodal rotation due to rotation of floor about the origin of the model. The nodal rotation due to rotation of floor is the product of ξ_{χ} and the distance between the origin of the model and the structural plane of the original frame. Therefore, the nodal rotation considered torsion in the X-Y structural plane is the sum of the values, which is the product of ξ_X and the distance between the origin of the model and the structural plane of the original frame and the values of θ_{Y} . The nodal rotation considered torsion in the X-Z structural plane is also obtained by the same way. Torsional rotation due to rotation of floor θ_X is shown in Figure 2.1.5. θ_X is defined by the torsional rotation due to rotation of floor about the origin of the model. Horizontal displacement due to rotation of floor is the product of θ_{X} and the distance from the origin of the model to a structural plane of the original frame. Both displacements v and w are shown in Figure 2.1.6. The values of v and w are measured as the horizontal displacements due to translation of the floor. The values of all nodal points lying in one floor with respect to v and w respectively are the same. Therefore, the horizontal displacement considered torsional direction in Y-direction is the sum of the values, which is the product of θ_X and the distance from the origin of the model to a structural plane of the original frame and the values of v. The horizontal displacement considered torsional direction in Z-direction is also obtained by the same way.



Figure 2.1.2. Nodal rotation in the X-Y structural plane; θ_Y



Figure 2.1.3. Nodal rotation in the X-Z structural plane; θ_Z



Figure 2.1.4. Nodal rotation due to rotation of floor; ξ_X



Figure 2.1.5. Torsional rotation due to rotation of floor; θ_X



Figure 2.1.6. Horizontal displacement in direction Y and Z; v, w

2.2. Stiffness Matrix of A Beam Element in the Simplified Dynamic Model

The end moment versus rotation relationship of a half-beam of the original frame in X-Y structural plane can be denoted by the following equation.

$$M'_{Y,k,j} = B'_{Y,k,j} \cdot \theta'_{Y,k,j} , \quad B'_{Y,k,j} = \frac{6EI'_{b,Y,k,j}}{L'_{k,j}}$$
(2.2.1)

Here, $M'_{Y,k,i}$ is the both end moments of the *k*th beam in X-Y structural plane in the *i*th floor; $\theta'_{Y,k,i}$ is the nodal rotation of the *k*th beam at the X-Y structural plane in the *i*th floor; *E* is Young's modulus; and

 $I'_{b,Y,k,i}$ and $L'_{k,i}$ are the moment of inertia and length of the *k*th beam in X-Y structural plane in the *i*th floor. $\theta'_{Y,k,i}$ versus displacement vector in X-Y structural plane in the *i*th floor of the simplified dynamic model ($\{u_i\}$) and $M'_{Y,k,i}$ versus stress vector of the end of beam in X-Y structural plane in the *i*th floor of the simplified dynamic model ($\{P_{k,i}\}$) can be denoted by the following equation.

$$\theta'_{Y,kj} = \{T\}^{T} \{u_{i}\}, \{P_{kj}\} = \{T\}M'_{Y,kj}$$

$$\{P_{kj}\} = \{M_{Y,j} \quad M_{Z,j} \quad S_{i}\}^{T}, \{u_{i}\} = \{\theta_{Y,j} \quad \theta_{Z,j} \quad \xi_{X,j}\}^{T}, \{T\} = \{1 \quad 0 \quad z'_{k}\}^{T}$$

$$(2.2.2)$$

Here, z'_{k} is the distance between the origin of the model and the *k*th beam of the original frame in X-Y structural plane in the *i*th floor. Therefore, stiffness matrix of the *k*th beam element of the original frame at the X-Y structural plane in the *i*th floor for the simplified dynamic model ($[K_{Y,k,i}]$) can be denoted by the following equation.

$$\begin{bmatrix} K_{Y,k,j} \end{bmatrix} = B'_{Y,k,j} \{T\}^T \{T\} = B'_{Y,k,j} \begin{bmatrix} 1 & 0 & z'_k \\ 0 & 0 & 0 \\ z'_k & 0 & z'_k^2 \end{bmatrix}$$
(2.2.3)

Stiffness matrix of the beam element in X-Y structural plane in the *i*th floor for the simplified dynamic model ($[K_{Y_j}]$) is expressed as sum of $[K_{Y_{k,j}}]$

$$\left[K_{Y,j}\right] = \Sigma\left[K_{Y,k,j}\right] \tag{2.2.4}$$

Stiffness matrix of the beam element in X-Z structural plane for the simplified dynamic model is derived similarly.

2.3. Yield Surface of A Beam Element in The Simplified Dynamic Model

Assuming that the yield surface relating to the center of strength of a beam element in X-Y structural plane on the simplified dynamic model is an elliptical shape, its yield surface can be expressed by the following equation using stresses relating to the center of strength in X-Y structural plane and each full plastic strength.

$$F = \left(\frac{\overline{S_i}}{\overline{S_{p,b,Y_i}}}\right)^2 + \left(\frac{\overline{M_{Y_i}}}{\overline{M_{p,b,Y_i}}}\right)^2 - 1 = 0$$
(2.3.1)

$$\overline{M_{p,b,Y,i}} = \Sigma M'_{p,b,Y,k,i} \quad , \quad \overline{S_{p,b,Y,i}} = \Sigma \left| \overline{z'_k} \right| \cdot M'_{p,b,Y,k,i} = {_R}r_{b,Y,i} \cdot \overline{M_{p,b,Y,i}}$$

Here, $M'_{p,b,Y,k,i}$ is full plastic strength of the *k*th beam in X-Y structural plane in the *i*th floor; $\overline{z'_k}$ is the distance between the center of strength and the *k*th beam in X-Y structural plane in the *i*th floor.

The stresses relating to the origin of the model (M_{Y_i}) and (S_i) can be expressed by the following equations using the stresses relating to the center of strength of a beam element $(\overline{M_{Y_i}})$ and $(\overline{S_i})$.

$$M_{Y_j} = \overline{M_{Y_j}} , \quad S_i = \overline{S_i} + \overline{M_{Y_j}} \cdot_R z_{bj}$$
(2.3.2)

Here, $_{R}z_{bi}$ is the distance from the origin of the model and the center of strength of a beam element in X-Y structural plane. Thus, using Eqns.2.3.1 and 2.3.2, the yield surface relating to origin of the model can be expressed by Eqn.2.3.3.

$$F = \left(\frac{S_i - M_{Y_i} \cdot_R z_{b_i}}{\overline{S}_{p,b,Y_i}}\right)^2 + \left(\frac{M_{Y_i}}{\overline{M}_{p,b,Y_i}}\right)^2 - 1$$
(2.3.3)

The elasto-plastic stiffness matrix of a beam element in the simplified dynamic model is derived using plastic flow theory and Ziegler's kinematic hardening rule.

2.4. Stiffness Matrix of A Column Element in The Simplified Dynamic Model

Neglecting the axial deformation of columns, the elastic end force vector $(\{P'_{k,i}\})$ versus the end deformation vector $(\{u'_{k,i}\})$ relationship of a column in the original frame can be denoted by the following equation using elastic stiffness matrix $([K'_{k,i}])$.

$$\{P'_{kj}\} = [K'_{kj}]\{u'_{kj}\}$$

$$\{P'_{kj}\} = \{M'_{Ykj} \quad M'_{Zkj} \quad M'_{Xkj} \quad Q'_{Ykj} \quad Q'_{Zkj} \quad M'_{Yk,j} \quad M'_{Zk,j} \quad M'_{Xk,j} \quad Q'_{Yk,j} \quad Q'_{Zk,j}\}^{T}$$

$$\{u'_{kj}\} = \{\theta'_{Ykj} \quad \theta'_{Zkj} \quad \theta'_{Xkj} \quad v'_{kj} \quad w'_{kj} \quad \theta'_{Yk,j} \quad \theta'_{Zk,j} \quad \theta'_{Xk,j} \quad v'_{k,j} \quad w'_{k,j}\}^{T}$$

$$\{u'_{kj}\} = \{\theta'_{Ykj} \quad \theta'_{Zkj} \quad \theta'_{Xkj} \quad v'_{kj} \quad w'_{kj} \quad \theta'_{Yk,j} \quad \theta'_{Zk,j} \quad \theta'_{Xk,j} \quad v'_{k,j} \quad w'_{k,j}\}^{T}$$

Here, the components of the elastic end force vector of the original frame $(\{P'_{k,i}\})$ are forces to respond to the components of end deformation vector of the original frame $(\{u'_{k,i}\})$.

The elastic end force vector of the original frame $(\{P'_{k,i}\})$ versus the elastic the end force vector of the simplified dynamic model $(\{P_{k,i}\})$ and the end deformation vector of the original frame $(\{u'_{k,i}\})$ versus the end deformation vector of the simplified dynamic model $(\{u_i\})$ can be expressed by Eqn.2.4.2.

$$\{P_{kj}\} = [T]^{T} \{P'_{kj}\} , \ \{u'_{kj}\} = [T] \{u_{i}\}$$

$$\{P_{kj}\} = \{M_{Yj} \ M_{Zj} \ S_{i} \ T_{i} \ Q_{Yj} \ Q_{Zj} \ M_{Yj} \ M_{Zj} \ S_{j} \ T_{j} \ Q_{Yj} \ Q_{Zj}\}^{T}$$

$$\{u_{i}\} = \{\theta_{Yj} \ \theta_{Zj} \ \xi_{Xj} \ \theta_{Xj} \ v_{i} \ w_{i} \ \theta_{Yj} \ \theta_{Zj} \ \xi_{X,j} \ \theta_{X,j} \ v_{j} \ w_{j}\}^{T}$$

$$(2.4.2)$$

| | 1 | 0 | Z'_k | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
|-----------|---|---|------------------|------------------|---|---|---|---|------------------|------------------|---|---|--|
| | 0 | 1 | $\mathcal{Y'}_k$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | 0 | 0 | 0 | $-z'_k$ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| $[T]_{-}$ | 0 | 0 | 0 | $\mathcal{Y'}_k$ | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | z'_k | 0 | 0 | 0 | |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | $\mathcal{Y'}_k$ | 0 | 0 | 0 | |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-z'_k$ | 1 | 0 | |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\mathcal{Y'}_k$ | 0 | 1 | |

Consequently, using Eqns.2.4.1 and 2.4.2, elastic stiffness matrix in the *i*th floor of the simplified dynamic model can be expressed by Eqn.2.4.3.

$$\begin{bmatrix} K_i \end{bmatrix} = \sum \begin{bmatrix} T \end{bmatrix}^T \begin{bmatrix} K'_{ki} \end{bmatrix} \begin{bmatrix} T \end{bmatrix}$$
(2.4.3)

2.5. Yield Surface of A Column Element in The Simplified Dynamic Model

If it is assumed that the yield surface relating to the center of strength of a column element on the simplified dynamic model has a spheroid shape, its yield surface can be expressed by Eqn.2.5.1 deriving as well as beam element.

$$F = \left(\frac{S_i - M_{Y_j} \cdot_R z_{cj} - M_{Z_j} \cdot_R y_{cj}}{Rr_{cj} M_{pcj}}\right)^2 + \left(\frac{M_{Y_j}}{M_{pcj}}\right)^2 + \left(\frac{M_{Y_j}}{M_{pcj}}\right)^2 - 1$$

$$\overline{M_{pc,Y_j}} = \overline{M_{pc,Z_j}} = \overline{M_{pc,j}} = \sum M'_{pc,k,j}$$

$$\overline{S_{pc,j}} = \sum \sqrt{\overline{y'_k}^2 + \overline{z'_k}^2} M'_{pc,k,j} = Rr_{cj} \overline{M_{pc,k,j}}$$
(2.5.1)

The elasto-plastic stiffness matrix of a column element in the simplified dynamic model is also derived using plastic flow theory and Ziegler's kinematic hardening rule.

2.6. Mass Matrix in The Simplified Dynamic Model

The horizontal displacement vector at a certain point (y'_k, z'_k) in the *i*th floor level $(\{u'_{k,i}\})$ can be expressed by Eqn.2.6.1.

$$\{u'_{kj}\} = \begin{cases} v'_{kj} \\ w'_{kj} \end{cases} = \begin{bmatrix} 0 & 0 & 0 & -z'_k & 1 & 0 \\ 0 & 0 & 0 & y'_k & 0 & 1 \end{bmatrix} \{u_i\}$$

$$\{u_i\} = \{\theta_{Yj} \quad \theta_{Zj} \quad \xi_{Xj} \quad \theta_{Xj} \quad v_i \quad w_i\}^T$$

$$(2.6.1)$$

The kinematic energy can be expressed by Eqn.2.6.2 using mass at a certain point (y'_k, z'_k) in the *i*th floor

 $(m'_{k,i}).$

$$\sum \frac{1}{2} m'_{k,i} \left\{ u'_{k,i} \right\}^T \left\{ u'_{k,i} \right\} = \frac{1}{2} \left\{ u'_{k,i} \right\}^T \left[M_i \right] \left\{ u'_{k,i} \right\}$$
(2.6.2)

In consequence, using Eqns.2.6.1 and 2.6.2, the mass matrix in the *i*th floor of the simplified dynamic model $([M_i])$ can be expressed by Eqn.2.6.3.

3. RESULTS OF EARTHQUAKE RESPONSE ANALYSIS

In this chapter, the accuracy of the modeling is calibrated for two benchmark frames with two kinds of earthquake ground motions.

3.1. Benchmark Steel Moment Frames

To calibrate the accuracy of the simplified dynamic model, steel moment frames, which were provided to the full-scale shaking table test as shown Figure 3.1.1, were calculated by means of the proposed models. The original steel moment frames is non-eccentric frames. Making the frames with eccentricity is completed with the fact that some members are replaced by weak members. The weight of each floor is the same as well as the original frame. The member that make smaller is marked with a broken line in Figure 3.1.1. The size of the member cross section was changed as follows.

(a) Two columns in the second story were made smaller than the original frame.

RHS-300x300x9 => RHS-250x250x8 (Column eccentricity frame)

(b) One beam in the third floor level was made smaller than the original frame.

H-400x200x8x13 => H-300x150x6.5x9 (Beam eccentricity frame)



Figure 3.1.1. Benchmark steel moment frame

The results are compared with those obtained by finite element method analysis by means of ABAQUS. In the analysis, the size of cross section and yield strength of the members were determined according to the research in references, the strain hardening coefficient is 0.01, and P-delta effect is considered. As the steel moment frames have concrete slab, it was assumed that the floor is rigid in the plane.

3.2. Eigen Value Analysis

Table 3.2.1 summarizes the first natural periods of the frames obtained from the simplified dynamic model and ABAQUS. The values of the first natural period of a couple of analysis methods are very close, with the difference not greater than 2 per cent for the Y-direction and 3 per cent for the Z-direction. The reason is the assumption that all rotations at the joints lying at each floor level are identical. Figure 3.2.1 shows naturalmode obtained from the simplified dynamic model and ABAQUS. The vertical axis in this figure is story number, and the horizontal axis concerned with translational displacement as shown in Figure 3.2.1 (d) and Figure 3.2.1 (f), is the ratio of the roof displacement of center of gravity to the displacement of each story. The horizontal axis concerned with torsional rotation as shown in Figure 3.2.1 (b) and Figure 3.2.1 (d), is the torsional rotation when the roof displacement for Y-direction of the center of gravity is 10mm. The values of naturalmode obtained by the two analysis methods are very close.

| Frame | Direction | ABAQUS | Simplified dynamic model | Ratio |
|---------------------|-------------|--------|-----------------------------|-------|
| Column eccentricity | Y-direction | 1.109 | 1.094 | 0.986 |
| frame | Z-direction | 1.011 | 0.988 | 0.977 |
| Beam eccentricity | Y-direction | 1.142 | 1.127 | 0.987 |
| frame | Z-direction | 1.001 | 0.975 | 0.974 |

Table 3.2.1. Comparison of first natural periods



Figure 3.2.1. Comparison of naturalmodes between the proposed method and FEM (ABAQUS)

3.3. Earthquake Response Analysis

Two kinds of bi-directional ground motions shown in Table 3.3.1 are used for the analyses. For El Centro (1940), the maximum acceleration of ground motion was adjusted in order that its value in the north-south maximum velocity of ground motion takes 0.5 m/s. For JMA Kobe (1995), measured ground motion is used directly. The acceleration of ground motion in north-south direction (larger input energy) is made for the Y-direction of the analytical frame, which has eccentricity. The acceleration of ground motion in east-west direction (smaller input energy) is made for the Z-direction of the analytical frame. Rayleigh damping constant of 1 per cent for the first modes of Y-direction and Z-direction was adopted in the analysis. The step time of numerical integration of seismic response analyses is 0.002 s. The duration of the analyses is 15.0 s.

| Table 9.9.11 input ground motions | | | | | | |
|-----------------------------------|-------------|-----------------|--------------------------------|-------------------|--|--|
| Ground motion name | Direction | Input direction | Maximum acceleration (m/s^2) | Duration time (s) | | |
| El Contro 1040 | North-south | Y-direction | 5.11 | 15.0 | | |
| El Cellulo, 1940 | East-west | Z-direction | 3.14 | 15.0 | | |
| IMA Kaba 1005 | North-south | Y-direction | 8.21 | 15.0 | | |
| JMA KUUC, 1993 | East-west | Z-direction | 6.19 | 15.0 | | |

Table 3.3.1. Input ground motions

Figure 3.3.1 show the distribution of the maximum story drift angles (R_{max}) of column eccentricity frame and beam eccentricity frame. The distributions of the maximum inter-story drift angles of the two analysis methods are almost the same as shown in Figure 3.3.1. However, there is slight difference between two analysis methods. That reason is assumption that all rotations at joints lying at each floor level are identical.



Figure 3.3.1. Distribution of maximum story drift angles (R_{max})

Figure 3.3.2 show the distribution of maximum story torsional angles ($\Delta \theta_{max}$) of column eccentricity frame and beam eccentricity frame. $\Delta \theta_{max}$ is defined as the subtraction the twist rotation angle in the *i*th story from the torsional rotation angle in the story under the *i*th. The distribution of maximum story torsional angles of the two analysis methods is approximately equal in Figure 3.3.2.



Figure 3.3.2. Distribution of maximum story torsional angles ($\Delta \theta_{max}$)

4. CONCLUSIONS

In this paper, a simplified dynamic model useful for the simulation of earthquake response of 3-dimensional multi-story steel moment frames with eccentricity was proposed. The model makes the number of the degrees of freedom in an objective frame rather less than that of a conventional model in use of multi-purpose structural analysis program such as finite element analysis programs. The accuracy of the simplified dynamic model was calibrated by comparing the results obtained from this model with those obtained from finite element method analysis using ABAQUS. The errors associated with the model are negligible in light of the variability of responses by means of two kinds of earthquake ground motions and a couple of benchmark frames.

With these capacities, the simplified dynamic model is effective in conducting extensive numerical analyses needed for the identification and characterization of primary structural parameters that influence the earthquake responses of steel moment frames.

REFERENCES

- Nakashima, M., Ogawa, K. and Inoue, K. (2002). Generic model for simulation of earthquake responses of steel moment frames. *Journal of Earthquake Engineering and Structural Dynamics*. **31:3**,671-692.
- Suita, K., Matsuoka, Y., Yamada, S., Shimada, Y., Tada, M. and Kasai, K. (2009). Full scale shaking table collapse experiment on 4-story steel moment frame (Part 1 outline of the experiment). *Proceedings of the Sixth International Conference on Behavior of Steel Structures in Seismic Areas*. 125-130.