# Seismic Progressive Collapse of Reinforced Concrete Framed Structures

### S.M. Al Hafian and I.M. May

School of The Build Environment, Heriot-Watt University, Edinburgh, UK



#### SUMMARY:

In order to avoid structural collapse during earthquakes, the potential collapse mechanisms and the key factors that affect the collapse process should be understood. The Applied Element Method, AEM, is an innovative method, for direct progressive collapse simulation, in which material and strong geometric nonlinearity, element separation and collision can automatically be considered. This study focuses on the verification of the AEM for seismic progressive collapse and on the investigation of the effect of randomness in material parameters on the time at incipient collapse and on the possible failure modes. The AEM model of a scaled frame is verified by comparing the analytical and experimental results. The results indicate that the AEM can reliably predict the nonlinear response. The sensitivity study shows that the most important parameter is the mass, followed by the parameters that control the column-beam strength ratio. The collapse modes strongly depend on the location and type of the first member lost due to failure.

Keywords: Applied Element Method, Seismic progressive collapse, Sensitivity analysis, Failure modes.

### 1. INTRODUCTION

Seismic collapse is defined as the inability of a structural system or a part of it to sustain gravity loads under earthquake loadings. Earthquake loadings may trigger vertical or lateral dynamic instability (sidesway) collapse. The former may occur because of the loss of axial capacity of one or several vertical structural members following their shear failure (Elwood and Moehle, 2003), whereas the latter is generally triggered by large displacement of a single or multiple storeys due to a combination of P-delta effects and excessive component deterioration.

The emphasis of most studies of seismic collapse is on sidesway collapse using the incremental dynamic analysis approach, IDA (Vamvatsikos and Cornell, 2002). Uncertainties in ground motions as well as in simulating the seismic collapse behaviour associated with modeling the parameters that define lumped plasticity models of the structural component are the focus of these studies (Zareian and Krawinkler, 2009, Haselton et al., 2011, Liel et al., 2011). Other collapse modes such as column shear failure can be indirectly incorporated in estimating the collapse fragility curve by post-processing the analytical results using component limit state checks (Liel, 2008). Sophisticated analytical tools for simulating direct collapse using element removal criteria, which are associated with different failure modes, and incorporating large deformation as well as effects of contact and impact are the subject of a few studies (Tagel-Din and Meguro, 2000a, Talaat and Mosalam, 2009).

This paper demonstrates the validation of the Applied Element Method developed by (Tagel-Din and Meguro, 1999), AEM, in modeling the non-linear response of structures under seismic loadings and in

simulating seismic progressive structural collapse. Applied element analyses are performed on a sixstorey, three-bay reinforced concrete plane frame previously shake-table tested in order to validate the AEM model using the experimental results obtained by (Lu, 1996, Lu et al., 1999, Lu, 2002). Sensitivity analyses of the collapse behaviour of this seismically designed frame are also conducted. The Extreme Loading for Structure software, ELS, which is based on the AEM is used in these analyses (Applied Science International, 2010).

### 2. BRIEF REVIEW OF THE APPLIED ELEMENT METHOD (AEM)

The Applied Element Method adopts the discrete cracking approach. This method combines the advantages of both the Finite Element Method (FEM) and the Discrete Element Method (DEM). The AEM is a reliable analytical tool for simulating different collapse modes. By using the AEM, the structural behaviour can be automatically traced throughout all response stages including elastic, crack initiation, reinforcement yielding, element separation and collision as well as the effect of debris loading on the structural system (Meguro and Tagel-Din, 2000, Meguro and Tagel-Din, 2001, Meguro and Tagel-Din, 2002, Tagel-Din and Meguro, 1999, Tagel-Din and Meguro, 2000a, Tagel-Din and Meguro, 2000b).

In the AEM, the structure is discretized into series of relatively small rigid elements connected together along their faces through a set of three non-linear contact springs (normal and two shear springs), which are based on the material characteristics, as shown in Fig. 2.1. The springs are located at contact points. The springs represent stresses, strains and deformations of a certain portion. The AEM is a stiffness-based method. The equilibrium equations are implicitly solved utilizing a step-by-step integration (Newmark-beta) approach. In reinforced concrete structures, the concrete and reinforcements are represented by matrix springs and reinforcement bar springs, respectively. In the AEM, fully nonlinear path-dependent constitutive models are adopted for concrete and reinforcement bars. The Maekawa compression model, an elasto-plastic and fracture model, is utilized for concrete in compression (Okamura and Maekawa, 1991). While for concrete in tension, a linear stress–strain relationship is used until reaching the cracking point, when the stresses drop to zero. For reinforcement bars, the model presented by Ristic et al is adopted (Ristic et al., 1986).



Figure 2.1. Modeling of structures using AEM

### 3. SELECTION OF THE REFERENCE STRUCTURE

A six-storey RC frame structure, which was seismically designed according to EC8(Eurocode 8 (EC8), 1988 (draft); 1994), is selected as a case study to investigate the collapse behaviour using the AEM. The nonlinear response of a structure strongly depends on the structural system, loading pattern, material properties and longitudinal and transverse reinforcement details. The collapse behaviour is expected to be structure specific. Therefore, the decision was made to choose a typical structure, which has been designed and analyzed by other researchers. The use of this structure enables the conclusions to be more general and can therefore be applied to similar structures. In addition, the available shaking table results of 1/5.5 scale specimen allow the verification of the AEM model.

The prototype frame, BF1 frame, was a structure of height 20 m. The height of all storeys except the first storey is 3 m, while the first storey height is 5m. The planar frame consists of columns spaced at 5m with three bays. The cross-section dimensions and the corresponding reinforcement details for the BF1 frame are given in Table 3.1. The slab thickness is 140 mm with two layers of Ø15/250 mm. The slab contribution is taken into account in the analysis by considering the effective slab width as well as the slab reinforcements within this width. Concrete class C20/25 was adopted for the design of the frame and the steel Grades used were S400 and S220 for flexural and transverse reinforcement, respectively. The vertical loads considered in addition to the structure self weight were 6 and 8 KN/m for dead and live load calculated from the three dimensional model of the structure assuming a span length of 4 m in the perpendicular direction. In the test, the vertical loads consisted of the dead plus 30% of the live loads and these loads were combined with seismic loads. The mass of each storey was calculated based on the aforementioned consider loads.

Model	Storey level	Column			Beam			
		Cross- section dimensions	Longitudinal steel	Lateral steel	Cross- section dimensions	Longitudinal steel	Lateral steel	
BF1	5,6	350X350	4Ø32	2Ø15/110	350X400	2Ø32	2Ø15/100	
	3,4	450X450	4Ø32	2Ø15/110	400X450	2Ø32	2Ø15/110	
	2	500X500	4Ø32	2Ø15/100	400X500	2Ø32	2Ø15/125	
	1 (top)	600X600	4Ø32	2Ø15/100	400X500	2Ø32	2Ø15/125	
	1 (bottom)		8Ø32	3Ø15/125				

**Table 3.1.** Reinforcement details for the BF1 frame (Zhang, 1996)

### 4. VERIFICATION OF THE AEM MODEL

The seismic tests were constructed as 1/5.5 scale replicas of the aforementioned designed frame using materials with similar properties to those expected in the typical frame, but the gravity loads were augmented by additional masses in order to have similar gravity stresses as in the typical frame (Lu et al., 1999). For the seismic loads, the N-S component of El Centro 1940 was used as a base acceleration for the shaking table, with gradually increasing intensity from 0.1 g to 0.9 g and a time scale factor of the square root of the test scale factor (5.5) (Lu, 2002). More details about the seismic test are available in (Lu, 1996, Lu, 2002, Lu et al., 1999). The scaled model is used instead of the typical frame in order to directly compare the analytical and measured results and avoid the differences, which may result from similitude design. Material nonlinearity, large deformations, element separation and contact are considered in the AEM model. To achieve the best agreement between the experimental and analytical results, appropriate values for the material parameters are used, as shown in Table 4.1.

Material properties	Concrete	Steel (longitudinal-transverse)
Young's modulus (MPa)	29700	200000
Tensile strength (MPa)	3	448-195
Compressive strength (MPa)	30	

**Table 4.1.** Material parameters used in the AEM model

The structure has been modeled by 1716 three-dimensional cubical sub-elements. 10 sets of contact springs have been assigned to each two adjacent element interfaces resulting in 126602 springs in the entire model. All reinforcement details, longitudinal and transverse reinforcement of the beams and columns, have been explicitly modeled as well as the slab reinforcement as shown in Fig. 4.1. Steel springs representing the reinforcement bars have been automatically assigned to interfaces of the cuboids at the exact location of the steel bars. A mesh sensitivity check of the AEM model has carried out. It has been found that the adopted mesh size yields converged results. Distributed masses and weights have been used. The bases of the first storey columns are fully restrained. An elastic damping ratio of 0.05, which is a mass proportional, has been considered for the first mode of vibration.

Due to the limited amount of available data, the input ground motion used in the analyses is based on scaling the shaking table acceleration of 0.3g to obtain the other required levels of intensity. Therefore, the response of the analytical models is expected to be less than those obtained from the experiment as the peak acceleration of the adopted input motion is often smaller than the shaking table acceleration (Zhang, 1996). Also, some of the details and accuracy of the experimental displacement records have been lost in digitizing process since the experimental results were available only in paper format. The loading stages involve both static and dynamic loading. Firstly, the gravity loads are applied to allow the structure to deform under static loads and then the scaled ground motion records are applied in sequence through stage 2 to 7, as shown in Fig 4.1. The time step used in the dynamic analysis is 0.00833 sec. Because of this loading scenario, the damage sustained by the elements during the previous loading stage will be used as the initial condition for the following loading stage. The verification of AEM model is performed in terms of the global response, the displacement time histories. Comparisons between the storey displacements of the AEM and test models for different peak acceleration levels are shown in Fig. 4.2.



Figure 4.1. AEM models of BF1 frame and corresponding seismic loading scenarios

In some cases, the AEM model slightly underestimate the measured storey displacements particularly at low PGA levels as expected because of the slightly small seismic load. Despite these small differences, good agreement between the analytical and experimental results is generally obtained especially for the lower storeys. It is worth noting based on the comparison above that the AEM is determined to be an accurate tool for predicting the highly nonlinear response of RC structures under dynamic loading.



Figure 4.2. Comparison between analytical and experimental storey-displacement histories

### 5. UNCERTAINTIES IN THE COLLAPSE PROCESS

In general, the assessment of the collapse of RC structures using nonlinear time history analysis is highly uncertain due to various sources of uncertainty, for example, material properties, design variables, structural modeling, simulation and analysis method and defined limit states (Kwon and Elnashai, 2006). In addition, the ground motion input in terms of frequency content, characteristics (called record to record variability), and intensity (given by the hazard curve for a specific site) can cause a significant variability in the structure response. Deterministic methods for collapse assessment are not sufficient for evaluating structural safety under seismic loading due to these uncertainties. Using a complete probabilistic approach to account for all random variables is computationally very expensive and time consuming.

This study does not address all the random variables. The emphasis here is on the effects of uncertainties in material properties and the mass and gravity loads on the collapse process. The reason for this selection is related to the abilities of the utilized analytical tools. AEM modeling does not require definition of element stiffness, strength or deformation capacity in contrast to the generally used analytical tools, the Drain-2DX software and the OpenSees platform, in which lumped plasticity models are often adopted. AEM modeling depends on the constitutive relationship used for representing each of the concrete and steel materials. Thus, the aforementioned effects of element stiffness, strength and deformation are inherent in the model due to the corresponding material properties.

The focus of attention of this sensitivity study is on the time at the onset of collapse and the corresponding failure pattern rather than the collapse capacity, which requires around hundred analyses to be determined. It is concluded by (Talaat and Mosalam, 2009) that using the time of incipient collapse is more appropriate than using the maximum inter-story drifts prior to the onset of failure as a measure for the sensitivity of progressive collapse response. In order to identify the most influential parameters and to rank them with regard to their relative importance on the structural failure behaviour, a deterministic sensitivity analysis called tornado diagram analysis is conducted due to its simplicity and efficiency. Several researchers have studied the effects of modeling uncertainties on the seismic structural performance using tornado diagram analysis, for example (Binici and Mosalam, 2007, Haselton et al., 2008, Lee and Mosalam, 2005).

#### 6. SEISMIC INPUT

The collapse process is very sensitive to the input ground motion in terms of the intensity and ground motion profile. Several factors have been taken into account for selecting the seismic input for this sensitivity study focusing on the collapse limit state only. An artificial accelerogram generated using SIMQKE-1 (Gasparini and Vanmarcke, 1976) is adopted as it can be more appropriate than a real ground motion record for two reasons. Firstly, real records are generally scaled based on the spectral acceleration at the first mode period of vibration, which is significantly changed due to stiffness and strength degradation during the damage propagation. An accelerogram, which matches the EC8 elastic response spectrum, can capture the changes in the dynamic characteristics. Secondly, the distribution of the intensity of the selected time history record should be nearly uniform so the structure will be subjected to strong ground shaking during a long duration and will not be governed by the arrival of strong shaking amplitude of real records.

The seismic intensity level should be large enough to cause element separation resulting in progressive collapse. In addition, the seismic demand on the structure does need to be limited. The record intensity should not be very high, otherwise it will control the collapse process and cause instantaneous sidesway collapse regardless of the structure properties. The tornado diagram analysis will be performed at two intensity levels that satisfy all these requirements. The two desired intensity levels, which are sufficient to cause the structure collapse, are obtained by scaling the accelerogram by a factor of 2.85 and 2.35. Fig. 6.1 shows the unscaled accelerogram and the corresponding elastic response spectrum.



Figure 6.1. Reference accelerograms and corresponding elastic response spectrum

### 7. TORNADO DIAGRAM ANALYSIS

The full-scale six-storey RC frame, BF1, is selected for the Tornado diagram analysis. This analysis gives considerable insights into the relative importance of the random variables. Firstly, a deterministic analysis is performed using the mean values of all random variables to compute the response of interest (e.g. the time at onset of collapse), which establishes the baseline output. Then the probability distribution of each uncertain variable is selected and each input parameter is varied between two extreme values defining the values of the upper and lower bounds of the input probability distribution (e.g. the 90<sup>th</sup> percentile and the 10<sup>th</sup> percentile). These random variables are assumed uncorrelated. Next, the analysis is repeated for each variable for the two extreme values, while the remaining input parameters are set to their mean values. The output of interest is measured and plotted as a horizontal bar associated with each random variable, which is called the swing. The length of the swing represents the sensitivity of the output to the variation in the corresponding random variable. Thus, the random input variables are sorted regarding to their swings (Binici and Mosalam, 2007).

The dimensions and reinforcement details of the AEM model are similar to the designed structure, but the material properties are slightly different. The mean values of the material properties obtained based on previous studies (Kappos et al., 1999), which are less than the design values, are utilized and are listed in Table 7.1.

The mass of each storey, which is calculated based on the 3 D structure gravity loads (the dead and live loads), is treated as a random variable in similar way to the study conducted by (Talaat and Mosalam, 2008). The dead load is assumed constant while three cases of the live load are considered namely; full occupancy with a live load factor of one, LL=1, no occupancy with a live load factor of zero, LL=0 and the live load corresponding to the expected occupancy adopted in the EC 8 design code, LL=30%. The reference mass is calculated using the full dead load plus 30% of the live load. Two cases of correlation between the parameters for the different types of structural members are investigated. In the first case, similar material parameters are used in all of the structure members, both beams and columns. While in the second case, the beams and columns are treated separately.

Parameter			COV	Distribution	Minimum	Maximum
Concrete	Compressive strength, $f_c$ (MPa)	28	0.18	Normal	21.55	34.45
	Tensile strength, f <sub>t</sub> (MPa)	2.2	0.22	Normal	1.6	2.80
	Initial modulus of elasticity, E <sub>c</sub> (MPa)	30305	0.08	Normal	27200	33410
Longitudinal	Yield strength, $f_y$ (MPa), with $\frac{f_u}{f_y} = 1.15$	440	0.06	Normal	406	474
reinforcing	Ultimate strength, $f_u$ (MPa)	506	0.06	Normal	467	545
steel	Ultimate strain, u	9%	0.09	Normal	8%	10%
Transverse	Yield strength, $f_{ys}$ (MPa), with $\frac{f_{us}}{f_{ys}} = 1.15$	195	0.06	Normal	180	210
reinforcing	Ultimate strength, $f_{us}$ (MPa)	225	0.06	Normal	208	242
51001	Ultimate strain, u <sub>a</sub>	9%	0.09	Normal	8%	10%

Table 7.1. Statistics of random input variables

## 7.1. Selected Response Parameters

Defining the collapse limit state quantitatively is very important. The structure generally does not collapse due to the loss of only one or two members. Collapse occurs when the structure or a large portion of it becomes unstable. The time at incipient collapse is defined as the onset of the unrestrained increase of either vertical or horizontal displacements at one or more storey levels. The average vertical and horizontal storey displacements, calculated at the top ends of all columns at the considered storey, are used to define the collapse criteria of the vertical and sidesway collapse, respectively.

### 7.2. Tornado Diagram Results

The variability of the time at incipient collapse due to uncertainty in modeling input parameters is represented in the tornado diagram analysis as shown in Figs. 7.1 and 7.2. The markers illustrate if the input parameter is associated with positive or negative effects. The reference values for this tornado diagram analysis are 15 and 4.2 sec for PGA level of 0.7 g and 0.86 g, respectively. A relatively small increase in the seismic intensity results in significantly earlier collapse and different failure modes. In addition, it alters the order of the important parameters.

One of the most important variables is the structure gravity load and hence the corresponding mass. In the case of full occupancy, the seismic forces as well as the gravity loads are higher leading to decrease in the column capacities and resulting in earlier soft storey mechanisms. On the contrary, the demands on the structure are less in the case of no occupancy resulting in avoiding or delaying the occurrence of collapse. The least important parameters are the parameters related to the transverse reinforcing steel. It is noted that changing most of the parameters related to the beam's strength as well as the structure mass have an inverse effects on the time at incipient collapse. Thus, decreasing their values often result in delaying the time of the collapse onset and vice versa. On the other hand, increasing the values associated with the column strength and stiffness (e.g.  $E_{c,c}, f_{c,c}, f_{u,c}$  and  $f_{y,c}$ ) generally lead to postponing the time at incipient collapse or avoiding the structure collapse. In some cases, decreasing the column strength or increasing the beam strength may delay the collapse. This

could be attributed to the variation in the collapse mode. For example, reducing the value of  $f_{u,b}$  modifies the collapse mode from a multi-storey mechanism to a fifth soft-storey mechanism, which takes place 11 sec earlier at 0.7 g. It is worth noting that the effect of the parameters is not symmetric. Also, it is found that the effects of the selected parameters are governed by the collapse mode, the localization of damage and the secondary effects during the collapse process such as impact between different elements and force redistribution following the element separation. The effects of changing the input modeling parameters can either lead to more concentration of damage in one or two storeys resulting in earlier collapse, or to distributing the structural damage throughout the whole structure and thus the structure collapse will be avoided or the time at the collapse onset will be delayed. Further investigation of some of these counterintuitive effects is required.



Figure 7.1. Tornado diagram analysis for sensitivity of time at incipient collapse for case 1



Figure 7.2. Tornado diagram analysis for sensitivity of time at incipient collapse for case 2

### 7.3. Collapse Mechanisms

The potential collapse mechanisms predicted by the nonlinear dynamic analyses are summarized in Table 7.2. Nonlinear time history analyses revealed that modeling uncertainties could lead to at least five different collapse mechanisms at each PGA level. The predominant collapse mode at a PGA level of 0.86 g is a global collapse mode due to a single first storey mechanism, while a multi-storey mechanism at upper storeys takes place at a PGA level of 0.7 g. It is noted that the soft storey mechanism often migrates from the lower to the upper storyes as a result of decreasing the seismic intensity for a specific accelerograms (Zareian et al., 2010). At a lower intensity, most members in the upper part of the structure exhibit severe damage before the onset of collapse, (generally vertical or multi-storey collapse modes). Sidesway collapse is more likely to occur at high intensity levels.

Even though the collapse behaviour of this structure is govened by two predominant failure modes associated with the PGA level, different collapse mechanisms such as sidesway and vertical collapse mechanism may occur due to uncertainties in input parameters. The propagation of failure is strongly affected by the type of the structural element which initiates the collapse process (a beam or a column), the location of this element (a beam at an exterior or an interior bay, an exterior or an interior column, at lower or upper stories), the number of the damaged elements and the capacity of the neighbouring members. Moreover, the collapse senarioes may change dramatically due to secondary effects such as impact between elements and force redistribution. Ignoring these effects may lead to inaccurate prediction of the collapse modes or unconservative estimation of the collapse probabilites due to the exclusion of some possible collapse mechanisms. For example, impact of falling beams on the beams or columns of the lower storeys is the main cause of collapse in several cases. Failure of most of beams at one storey level can result in a formation of tall columns and consequently a collapse mechanism. In all collapse cases, complete collapse occurs due to the impact forces resulting from collision between the falling elements and the severely damaged members at lower storeys.

			X	Ħ		THE REAL PROPERTY AND A DECEMBER OF A DECEMB	
0.86 g	45	$+f_{u,c}$ , $+f_{c}$ , $-f_{u}$	$-f_{c,b}, -u_s, +f_{y,c},$	$-f_{c'}+f_{u}$	-	-f <sub>us,b</sub>	-
0.7g	$-f_{y,c'}+f_{us,c}$ + $f_t$	$\begin{array}{c} -f_{u,b}, -f_{u,c} \\ -f_{y'}, -f_{u} \end{array}$	-f <sub>us</sub>	+f <sub>us,b</sub> , +f <sub>y</sub>	$+f_{u}$ , $+f_{us}$ , $+u_b$	-	42

Table 7.2. Potential collapse mechanisms

Although the design of the structure satisfies the strong-column weak-beam principle and a complete ductile collapse mode is expected under the earthquake forces, plastic hinges form in columns in most collapse cases. It was concluded by (Haselton et al., 2008) that seismic design provisions can delay the formation of column plastic hinges, but do not prevent the occurrence of soft storey mechanisms. It is also worth noting that increasing the beam strength alone can cause earlier collapse and an undesirable failure mode. However, if the failure of beams occurs first then the collapse process can be delayed. The column to beam strength ratio plays an important role in determining the seismic collapse behaviour (Zareian and Krawinkler, 2009). (Dooley and Bracci, 2001) suggest utilizing a ratio of 2.0 or more to avoid a storey mechanism. A proper collapse mode involving most of the structure storey could be the result by using this value in low-rise buildings (Haselton et al., 2011). The desirable failure mode can be obtained provided the severe damage is uniformly distributed throughout the structure beams and column hinging is limited.

### 8. CONCLUSION

This study provides an insight into the importance of modeling parameters on the collapse modes. Ignoring modeling uncertainties can be unconservative. The effects of modeling uncertainties are asymmetric and depend on the structural system and the potential failure modes. The most important parameter is the mass of the structure, followed by the parameters that control the column-beam strength ratio. Decreasing the beam strength is more effective in delaying or avoiding the collapse than increasing the column strength. In addition, it can be concluded that the collapse mode strongly depends on the location and the type of the first member lost and the impact forces resulting from falling debris. It can also be concluded that the AEM is a reliable simulation tool, which can model the different collapse modes.

#### ACKNOWLEDGEMENT

Acknowledge the use of the academic license of the Extreme loading for structures software provided by Applied Science International LLc.

#### REFERENCES

Applied Science International, ASI. (2010). Extreme Loading for Structures, 3.1.

- Binici, B. and Mosalam, K. M. (2007). Analysis of Reinforced Concrete Columns Retrofitted with Fiber Reinforced Polymer Lamina, *Composites Part B: Engineering*, **38:2**, 265-76.
- Dooley, K. L. and Bracci, J. M. (2001). Seismic Evaluation of Column-to-Beam Strength Ratios in Reinforced Concrete Frames, *ACI STRUCTURAL JOURNAL*, **98:6**, 843-51.
- Elwood, K. J. and Moehle, J. P. (2003). Shake Table Tests and Analytical Studies on the Gravity Load Collapse of Reinforced Concrete Frames, *Report No. PEER 2003/01*.

Eurocode 8 (EC8) (1988 (draft); 1994). Design provisions for earthquake resistance of structures. CEN (European Commission for Standardization)/TC250/SC8, Brussels, Belgium.

- Gasparini, D. and Vanmarcke, E. H., (1976). SIMQKE -1: A Program for Artificial Motion Generation, Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, MA.
- Haselton, C. B., Goulet, C. A., Mitrani-Reiser, J., Beck, J. L., Deierlein, G. G., Porter, K. A., Stewart, J. P. and Taciroglu, E. (2008). An Assessment to Benchmark the Seismic Performance of a Code-Conforming Reinforced Concrete Moment-Frame Building, *Report No. PEER 2007/12*.
- Haselton, C. B., Liel, A. B., Deierlein, G. G., Dean, B. S. and Chou, J. H. (2011). Seismic Collapse Safety of Reinforced Concrete Buildings. I: Assessment of Ductile Moment Frames, *Journal of Structural Engineering*, 137:4, 481-91.
- Kappos, A. J., Chryssanthopoulos, M. K. and Dymiotis, C. (1999). Uncertainty Analysis of Strength and Ductility of Confined Reinforced Concrete Members, *Engineering Structures*, **21:3**, 195-208.
- Kwon, O.-S. and Elnashai, A. (2006). The Effect of Material and Ground Motion Uncertainty on the Seismic Vulnerability Curves of RC Structure, *Engineering Structures*, **28:2**, 289-303.
- Lee, T.-H. and Mosalam, K. M. (2005). Seismic Demand Sensitivity of Reinforced Concrete Shear-Wall Building Using FOSM Method, *Earthquake Engineering and Structural Dynamics*, **34:14**, 1719-36.
- Liel, A. B. (2008). Assessing the Collapse Risk of California's Existing Reinforced Concrete Frame Structures: Metrics for Seismic Safety Decisions, *Department of Civil and Environmental engineering, Stanford University*, Ph.D. Thesis.
- Liel, A. B., Haselton, C. B. and Deierlein, G. G. (2011). Seismic Collapse Safety of Reinforced Concrete Buildings. II: Comparative Assessment of Non-Ductile and Ductile Moment Frames, *Journal of Structural Engineering*, **137:4**, 492-502.
- Lu, Y. (1996). Study of Seismic Behavior of Reinforced Concrete Frames Having Vertical Irregularities, *National Technical Univ. of Athens*, **Ph.D Dissertation**.
- Lu, Y. (2002). Comparative Study of Seismic Behavior of Multistory Reinforced Concrete Framed Structures, *Journal of Structural Engineering*, **128:2**, 169-78.
- Lu, Y., Tassios, T. P., Zhang, G.-F. and Vintzileou, E. (1999). Seismic Response of Reinforced Concrete Frames with Strength and Stiffness Irregularities, *ACI, Structural Journal*, **96:2**, 221-29.
- Meguro, K. and Tagel-Din, H. (2000). Applied Element Method for Structural Analysis: Theory and Application for Linear Materials, *Structural Eng./Earthquake, JSCE*, **17:1**, 21s-35s.
- Meguro, K. and Tagel-Din, H. (2001). Applied Element Simulation of RC Structures under Cyclic Loading, *Journal of Structural Engineering*, **127:11**, 1295-305.
- Meguro, K. and Tagel-Din, H. S. (2002). Applied Element Method Used for Large Displacement Structural Analysis, *Journal of Natural Disaster Science*, **24:1**, 25-34.
- Okamura, H. and Maekawa, K. (1991). Nonlinear Analysis and Constitutive Models of Reinforced Concrete, Gihodo Co. Ltd.
- Ristic, D., Yamada, Y. and Iemura, H. (1986). Stress-strain based modeling of hysteretic structures under earthquake induced bending and varying axial loads, *Research report no. 86-ST-01. Kyoto (Japan): School of Civil Engineering. Kyoto University.*
- Tagel-Din, H. and Meguro, K. (1999). Applied Element Simulation for Collapse Analysis of Structures, *Bulletin* of Earthquake Resistant Structure Research Center, **32**, 113-23.
- Tagel-Din, H. and Meguro, K. (2000a). Applied Element Method for Dynamic Large Deformation Analysis of Structures, *Structural Eng./Earthquake*, *JSCE*, **17:2**, 215s-24s.
- Tagel-Din, H. and Meguro, K. (2000b). Nonlinear Simulation of RC Structures Using Applied Element Method, *Structural Eng./Earthquake, JSCE*, **17:2**, 137s-48s.
- Talaat, M. and Mosalam, K. M. (2009). Modeling progressive collapse in reinforced concrete buildings using direct element removal, *Earthquake Engineering & Structural Dynamics*, **38:5**, 609-34.
- Talaat, M. M. and Mosalam, K. M. (2008). Computational Modeling of Progressive Collapse in Reinforced Concrete Frame Structures, *Report No. PEER 2007/10*.
- Vamvatsikos, D. and Cornell, C. A. (2002). Incremental dynamic analysis, *Earthquake Engineering & Structural Dynamics*, **31:3**, 491-514.
- Zareian, F. and Krawinkler, H. (2009). Simplified Performance-Based Earthquake Engineering, *Report NO. TB* 169, John A. Blume EarthquakeEngineering Research Center, Department of Civil and Environmental Engineering, Stanford University, **Ph.D Dissertation**.
- Zareian, F., Krawinkler, H., Ibarra, L. and Lignos, D. (2010). Basic Concepts and Performance Measures in Prediction of Collapse of Buildings under Earthquake Ground Motions, *The Structural Design of Tall* and Special Buildings, **19**, 167-81.
- Zhang, G. (1996). Seismic Behaviour Factor of Vertically Irregular Reinforced Concrete Frames with or without Infill Walls, *National Technical University of Athens*, **Ph.D Dissertation**.