Simulation of Correlated Horizontal Seismic Ground Motion Time Histories for a Given Scenario

I. Zentner
LAMSID UMR EDF-CNRS-CEA – EDF R&D, France

F. Poirion
ONERA The French Aerospace Lab, France

SUMMARY:
Reliable seismic vulnerability analysis is generally based on time-domain simulations of structural responses. The seismic load is then modelled by a stochastic process, representing seismic ground motion. For this purpose, the analyst can use recorded accelerograms or work with synthetically generated ones. When performing advanced probabilistic analyses, synthetic accelerograms are often used since available strong motion data is not sufficient. It is then necessary to have at our disposal methods that allow for generating synthetic accelerograms that realistically characterize earthquake ground motion. In particular, they have to accurately reproduce the natural variability of ground motion parameters related to the chosen site specific scenario. We present a method for generating two-dimensional synthetic ground motion reproducing variability in ground motion parameters and response spectra as observed for natural accelerograms. The proposed procedure uses Karhunen-Loève decomposition and a general (non-Gaussian) stochastic model in order to simulate ground motion time histories accounting for the space-time correlation structure of the two horizontal components This contribution presents an extension of an earlier work by the authors on seismic ground motion models.

Keywords: Simulation non Gaussian, non Stationary, Earthquake, variability

1. INTRODUCTION

This paper addresses the modelling and the simulation of correlated horizontal seismic ground motion time histories. In the framework of performance-based earthquake engineering, time-domain seismic and probabilistic analyzes have to be performed. The outputs of these analyses are quantities such as safety margins and fragility curves. It is acknowledged, that the accurate modelling of seismic load is a crucial point in order to determine “best-estimate” structural responses.
It has to be stressed that seismic ground motion exhibits non stationary behaviour as well in frequency content and as in amplitude (standard deviation is time-dependant). Thus, seismic ground motion has to be modelled by a non stationary stochastic process.
Most of the models for simulating artificial ground motion time histories are based on representations of a stochastic process by power spectral densities or equivalent time domain formulations (Pousse & al 2006, Rezaeian & Der Kiureghian 2010). This enables the analyst to generate as many time histories as necessary for the probabilistic analysis. On the other hand, one can use natural accelerograms in order to avoid approximations and discrepancies due to predefined models. The drawback of this rationale is the lack of sufficient natural accelerograms for performing probabilistic analysis. In a previous paper the authors (Zentner & Poirion, 2012) have proposed a new method based on Karhunen-Loève expansion, that allows for the enrichment of a natural ground motion database. The idea behind this approach is to use recorded ground motion data coming from the constantly growing international strong motion databases in order to feed an appropriate non Gaussian probabilistic model. The model thus allows for the enrichment of a data base in view of probabilistic analysis where repeated structural analyses have to be performed. This paper presents an extension of the earlier work by the authors to vector valued seismic ground motion models. We develop an empirical model, based on strong motion databases, for simulating of correlated horizontal ground motion components.
Penzien & Watabe (1975) argue that principal axes of earthquakes are usually directed towards the

general direction of the earthquake source and the corresponding perpendicular direction. In consequence, Rezaeian & Der Kiureghian (2010) propose to consider only uncorrelated components parallel and orthogonal to the source and to obtain other configurations by rotation.
However, when studying a particular building or industrial plant in moderate seismic zones, then its exact orientation with respect to the source is generally not well known. In consequence, it can be useful to have at our disposal statistical models accounting for the statistical distribution of correlation between the horizontal components.

2. SIMULATION OF NON GAUSSIAN STOCHASTIC PROCESSES

The simulation of non Gaussian non stationary stochastic processes is not an easy task. Some authors propose to use memoryless transformations of Gaussian processes (e.g Shields et al. 2011) but then the functional relationship linking the underlying Gaussian power spectral density function with the equivalent non Gaussian expression has to be determined. Another more general approach is the characterization of the non Gaussian process by its correlation function and using Karhunen-Loève expansion. However, in the general, non Gaussian, case, the distributions of the random variables of the K.-L. expansion are not known. In what follows, we present a ground motion simulation method based on Karhunen-Loève expansion where the empirical distributions are estimated from recorded ground motion. We furthermore discuss some theoretical properties of the resulting time histories.

2.1. Some elements on Karhunen-Loève expansion

Let D be a compact subset of $\mathcal{R}$ and $X(t) = (X_1(t),\ldots,X_d(t)), t \in D$, a second-order, zero mean stochastic process defined on a probability space $(B,\mathcal{B},P)$ with values in $\mathcal{R}^n$. We will assume that $X(t)$ is almost surely (a.s.) continuous. Since D is compact, the auto-correlation function

$$R_X(t,t') = \mathbf{E}(X(t)X(t')^T), \tag{2.1}$$

defines a continuous self-adjoint Hilbert-Schmidt linear operator Q of $\mathcal{H} = L^2(D,\mathcal{R}^n)$ (Guikman and Skorokhod,1979):

$$(Q\varphi)(t) = \int_D R_X(t,t')\varphi(t')dt' \quad ;\varphi \in \mathcal{H}, \tag{2.2}$$

which has a countable number of eigenvalues $\lambda_{1,}\lambda_2 \ldots \to 0$. The associated eigenfunctions are the solutions of the integral equation

$$\int_D R_X(t,t')\varphi(t')dt' = \lambda\varphi(t), \tag{2.3}$$

and constitute a Hilbertian basis $\{\varphi_\alpha\}_{\alpha\geq 1}$ of $\mathcal{H}$:

$$\prec \varphi_\alpha, \varphi_\beta \succ = \int_D < \varphi_\alpha(t), \varphi_\beta(t) > dx = \delta_\alpha^\beta, \tag{2.4}$$

in which $<,>$ denotes the inner product in $\mathcal{R}^n$. Then vector-valued random process $X(t)$ has the following expansion in $L^2(B,\mathcal{R}^n)$:

$$\forall t \in D, X(t) = \sum_{\alpha\geq 1} \sqrt{\lambda_\alpha}\,\xi_\alpha \varphi_\alpha(t), \tag{2.5}$$

in which $\xi_1, \xi_2, \ldots, \xi_\alpha, \ldots$ are uncorrelated real valued random variables given by

$$\xi_\alpha = \frac{1}{\sqrt{\lambda_\alpha}} \int_D < X(t), \varphi_\alpha(t) > dt. \tag{2.6}$$

2.2. Identification of the K.-L. expansion terms

We are dealing in this study with 2 dimensional ground motion fields: $X(t) = (x(t), y(t))^T$. Starting with a database containing N accelerograms according to given common feature (a given site specific scenario) yields: $\{X^{(l)}(t_i); i = 1,\dots,n\}$, $X^{(l)}(t_i) = (x^{(l)}(t_i), y^{(l)}(t_i))^T \in \mathcal{R}^n$, $l = 1, \dots, N$.

The first step to the construction of the K-L expansion model is to estimate the empirical autocorrelation matrix. The autocorrelation function of a given time history X reads:

$$R(t_i, t_j) = \frac{1}{\mathrm{N}} \sum_{l=1}^{\mathrm{N}} X^{(l)}(t_i) X^{(l)}(t_j)^T \tag{2.7}$$

Using the vectors $\mathrm{X}^{(l)} = (x^{(l)}(t_1), \dots, x^{(l)}(t_n))$, $\mathrm{Y}^{(l)} = (y^{(l)}(t_1), \dots, y^{(l)}(t_n))$ and $\mathrm{Z}^{(l)} = (\mathrm{X}^{(l)}, \mathrm{Y}^{(l)})^T$, we can construct directly an estimator of the autocorrelation matrix of the given accelerograms:

$$R_X = \frac{1}{\mathrm{N}} \sum_{l=1}^{\mathrm{N}} Z^{(l)} \mathrm{Z}^{(l)T} = \begin{pmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{pmatrix} \in \mathrm{Mat}_{\mathbf{R}}(2n, 2n) \quad , \tag{2.8}$$

We can check that we have taken into account the correlation between the 2 components of the accelerogram. Solving the discretized eigenvalue problem (2.3) yields $2n$ eigenvalues λ_α and $\Phi_\alpha \epsilon \mathcal{R}^{2\mathrm{n}}$, for $\alpha, i = 1, 2n$.

The second step is to construct samples $\xi_\alpha^{(l)}$ of the random variables appearing in the Karhunen-Loève expression. Expression (6) gives an explicit relation:

$$\xi_\alpha^{(l)} = \frac{1}{\sqrt{\lambda_\alpha}} \sum_{i=1}^{2n} < X^{(l)}(t_i), \Phi_\alpha(i) > \Delta t; \; l = 1, \mathrm{N} \;\; ; \alpha = 1, 2n \tag{2.9}$$

where Δt is the sampling time step.

The last step of the procedure is to construct the empirical estimate of the characteristic distribution of each random variable ξ_α:

$$F_\alpha(t) = \frac{1}{\mathrm{N}} \sum_{l=1}^{\mathrm{N}} I\{\xi_\alpha^{(l)} \le t\} \tag{2.10}$$

Let Ψ_α the 2 dimensional discrete function defined by $\Psi_\alpha(i) = (\Phi_\alpha(i), \Phi_\alpha(n+i))^T; i = 1, \dots, n$. At this point, if we consider for simplification that the random variables are independent, although the construction of $\{\xi_\alpha\}_\alpha$ has implicitly taken into account this dependency, the construction is finished. One is able to generate samples of each scalar random variable ξ_α appearing in the Karhunen-Loève expansion of X. Then, one is able to construct sample paths of the non stationary non Gaussian process using the truncated expression:

$$X(t_i) = (x(t_i), y(t_i))^T = \Sigma_{\alpha=1}^{M} \xi_\alpha \Psi_\alpha(i) ; i = 1, ..., n \tag{2.11}$$

2.3. Properties

It can be shown that the simulated time histories feature zero residual acceleration, velocity and displacement if the recorded accelerograms, contained in database used for the construction, do (Zentner & Poirion 2011). Baseline correction and other post-processing techniques should be applied to the original data if it does not feature the required or desired properties. Further studies of the statistical properties of natural and simulated time histories are investigated in section 3.

3. APPLICATION TO THE SIMULATION OF CORRELATED HORIZONTAL GROUND MOTION FIELDS

The construction of a pertinent database representing the seismic scenario and containing a sufficient number of accelerograms is a crucial point for the procedure. The evaluation of the cross-correlation matrix requires an important amount of data, in particular more than the evaluation of the autocorrelation alone. The use of too few accelerograms can lead to inaccuracies. For the purpose of testing the method, we consider a subset of events from NGA database corresponding to magnitudes *5<M<6*, focal distance *0<D<20* and all soil types with *Vs>200m/s*. This leads to a reduced database containing 220 pairs of accelerograms. Further work, relative to the choice of a pertinent database featuring the required properties, is in progress.

The median absolute correlation coefficient during strong motion phase was calculated to be 0.15. Some examples of pairs of accelerograms and their correlation coefficient are shown on figure 3.1. This figure also displays the empirical distribution (histogram) of the absolute value of the correlation coefficients determined for the strong motion phase. For the K.-L. expansion model, we retain 70 eigenmodes which corresponds to 99% of the total energy. The time dependent variance $R_{XX}(t,t)$ and the covariances $R_{XY}(t,t)$ are shown on figures 3.2 and 3.3.We show the curves obtained with the initial data, the resulting approximation with the K.-L. expansion as well as the empirical values obtained for 1000 simulated pairs (x,y) of ground motion time histories. Figure 3.4 displays examples of pairs of simulated accelerograms and their correlation coefficient. The histogram of the absolute value of the correlation coefficient is also shown.

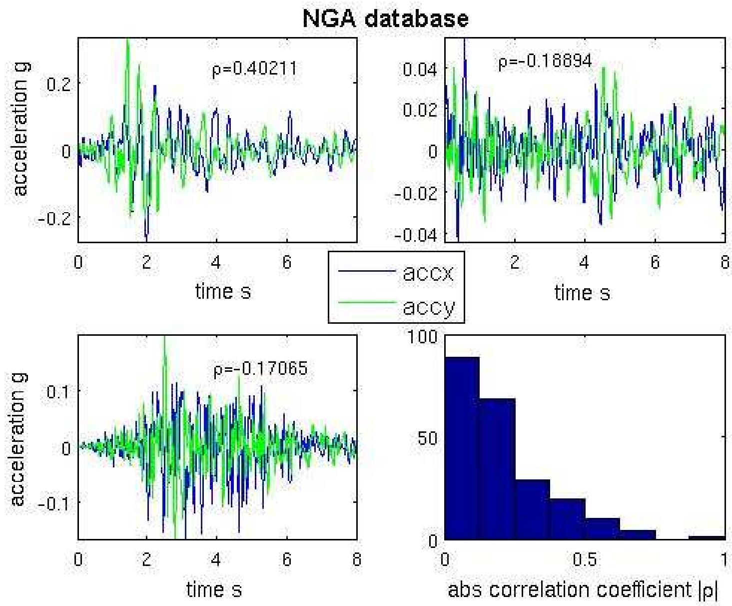

Figure 3.1 Examples of pairs of horizontal (x,y) accelerograms from NGA and distribution of absolute value of correlation coefficient

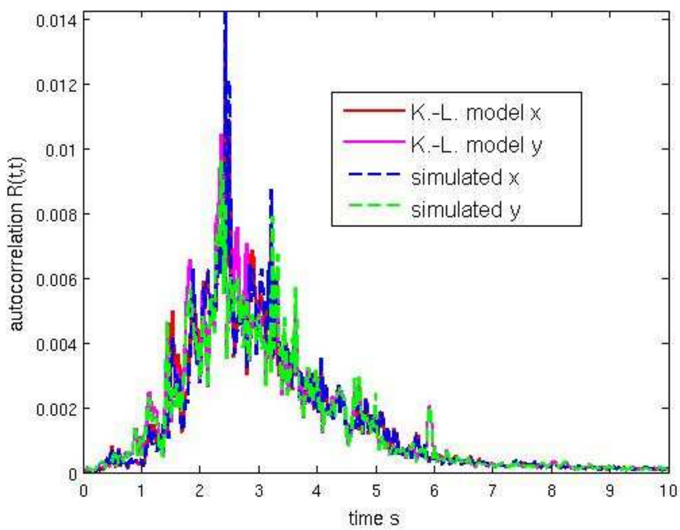

Figure 3.2 Time dependent variance: model (K.-L.) and from simulated time histories

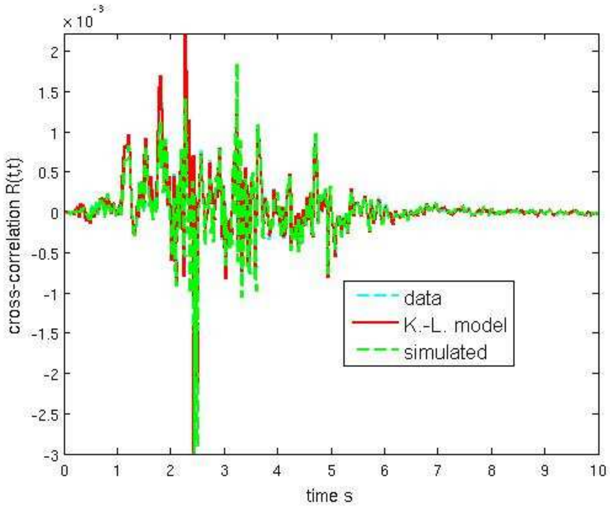

Figure 3.3 Time dependent covariance: data, model (K.-L.) and from simulated time histories

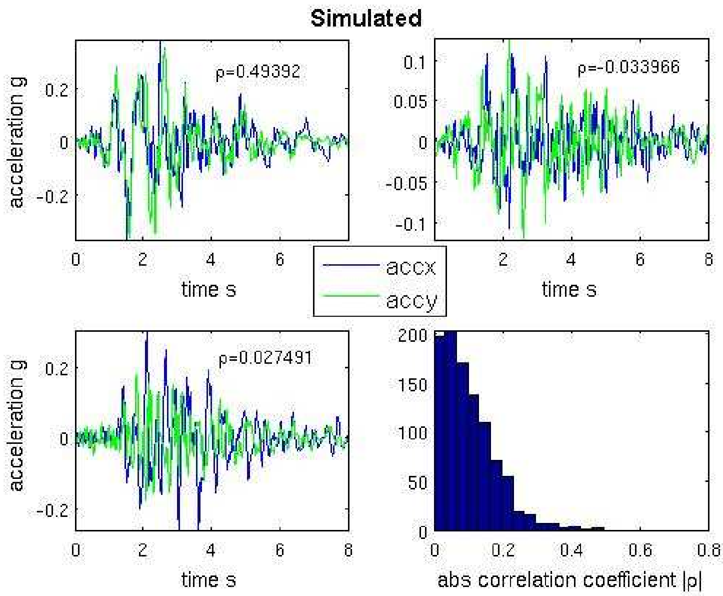

Figure 3.4 Examples of pairs of horizontal (x,y) simulated accelerograms and distribution of absolute value of correlation coefficient.

4. CONCLUSIONS

The use of recorded ground motion for performance-based seismic analysis is becoming more and more popular since it avoids approximations induced by using predefined ground motion models. Concurrently, data available through the international strong motion databases is constantly growing. The number of accelerograms available for a given scenario may, however, not be sufficient for extensive probabilistic analysis. This is why the authors have proposed a method, based on Karhunen-Loève expansion, which can be used for the enrichment of a ground motion database. This method is extended in this paper for the generation of horizontal 2D ground motion vector processes. The evaluation of the cross-correlation matrix requires more data than the autocorrelation. The construction of a pertinent database representing the seismic scenario and containing a sufficient number of accelerograms is thus a crucial point for the successful use of the procedure. Further work, relative to the choice of such a database, is in progress.

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US NGA strong-motion database (http://peer.berkeley.edu/nga/)