Rocking Response and Overturning Criteria for Free Standing Blocks to Single – Lobe Pulses

E. Voyagaki, I.N. Psycharis
Laboratory for Earthquake Engineering, National Technical University, Athens, Greece

G.E. Mylonakis
University of Patras, Greece

SUMMARY:
The classical problem of rocking behavior of a rigid, free-standing block to earthquake shaking containing significant pulses, as is the case in near-field ground motions, is revisited. A rectangular block resting on a perfectly rigid base is considered, subjected to a suite of idealized acceleration pulses expressed by a generalized function controlled by a single shape parameter. The problem is treated analytically in the realm of the linearized equations of motion, under the assumption of slender block geometry and rocking without slippage. Simple overturning criteria for different earthquake waveforms are presented in the form of dimensionless closed-form expressions and graphs that provide insight into the physics of the response. Two parameters are employed to express the results: a dimensionless pulse duration f (actual pulse duration times characteristic block frequency) and a dimensionless uplift strength η (ratio of minimum required acceleration for initiation of uplift over peak pulse acceleration).

Keywords: rocking, overturning, seismic response, closed-form solution, idealized pulse

1. INTRODUCTION

Starting with the pioneering studies by Milne (1893), Kirkpatrick (1927) and Housner (1963), the problem of determining the rocking response and overturning of a rigid, free-standing block to a ground pulse has been the subject of intense research for several decades. A basic aim of these investigations is the estimation of a minimum and a maximum level of ground motion intensity given the overturning or survival of slender blocks during an earthquake. Despite the apparently simple nature of the problem, solving analytically the equations of motion has proven to be a formidable task – even for very simple waveforms – due to the non-linear nature of the impact and the transcendental character of the response functions. As a result, the majority of available studies are either numerical (Yim et al 1980, Ishiyama 1982, Psycharis & Jennings 1983, Spanos & Koh 1984, Tso & Wong 1989) or experimental in nature (Priestley et al 1983, Peña et al 2007). Whereas certain mixed analytical-numerical solutions are available (Anooshehpoor et al 1996, Zhang & Makris 2001, Dimitrakopoulos & DeJong 2012), to be discussed in the ensuing, pure analytical investigations leading to exact closed-form solutions are limited to rectangular and sinusoidal pulses of half-cycle duration (Housner 1963, Shi et al 1996), which can induce overturning only in the forward sense (i.e., without impact).

Despite the aforementioned difficulties, analytical investigations are known to be exceptionally valuable as they can cast light into the complex physics of the response, including certain counterintuitive trends observed in numerical and experimental studies. For example: (a) overturning of a block subjected to a particular ground motion does not imply that the block will necessarily overturn with decreasing size or increasing slenderness ratio (Yim et al 1980); (b) overturning of a block does not imply overturning for an increase in ground motion amplitude (Psycharis and Jennings 1983) and (c) reduction in restitution coefficient, and thus increase in system damping, does not imply a decrease in response. The very limited number of exact analytical solutions to the problem provided the initial motivation for the herein-reported work.

With reference to a free-standing rigid block subjected to a generalized waveform that can simulate an infinite set of symmetric pulses of half-cycle duration ranging from a perfect rectangle (“box”) to a perfect impulse (“spike”), an exact analytical solution is presented in this paper for the rocking response of the block and the conditions of overturning. It will be shown that, despite the idealized nature of the excitation, complex response patterns-leading to useful results-can be identified as demonstrated in the ensuing.

2. PROBLEM AND PARAMETER DEFINITION

The problem considered in this work involves a rigid block resting on a perfectly rigid horizontal plane, subjected to an idealized ground acceleration pulse acting parallel to the plane. The block, shown in Fig 2.1, is of rectangular shape having mass m, dimensions (2xb) by (2xh) – leading to a radial distance from the center of rotation to the center of gravity $R = (b^2+h^2)^{1/2}$ and a dimensionless slenderness angle $\alpha = \tan^{-1}(b/h)$. The interface between the block and the rigid base is both tensionless and cohesionless, obeying a linear friction law characterized by a Coulomb coefficient μ, which is assumed to be sufficiently large to prevent sliding during rocking. As shown by Housner (1963), the dynamic characteristics of the block can be expressed by its characteristic frequency $p = (3g/4R)^{1/2}$ (measured in units of 1/Time), g being the gravitational acceleration. The dependence of the characteristic frequency solely on block size and gravitational action (not on mass) is analogous to the behavior of a pendulum – hence the term “inverted pendulum” coined by Housner for the problem at hand.

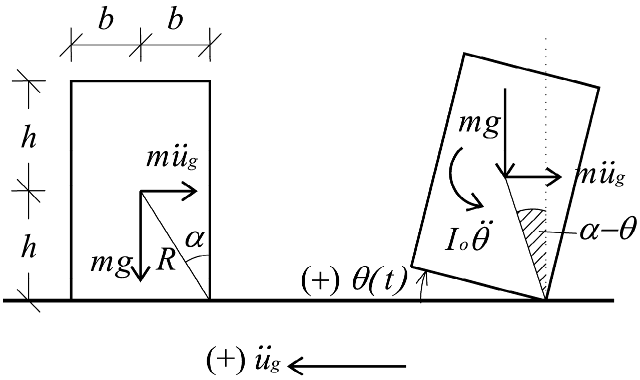

Figure 2.1. Problem considered: rocking motion of a free-standing block resting on a rigid base.

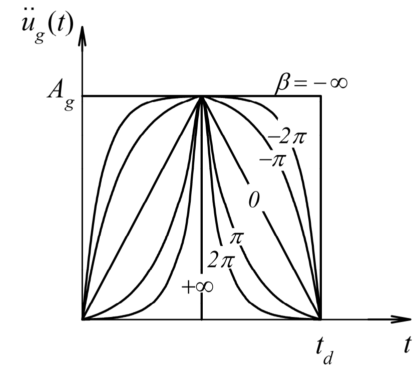

Figure 2.2. Ground acceleration time history for a generalized exponential pulse of half-cycle duration.

Ground excitation consists of an idealized horizontal ground acceleration pulse described by a shape parameter β (to be discussed below), amplitude A_g (normalized amplitude $a_g = A_g/g$) and half-cycle duration t_d. With reference to Fig 2.2, the pulse at hand is described as (Voyagaki et al 2012):

$$\ddot{u}_g(t) = \frac{A_g}{1-e^{\beta}}[1-e^{2\beta t/t_d} + (e^{2\beta t/t_d} - e^{2\beta(1-t/t_d)})H(t-t_d/2) - (1-e^{2\beta(1-t/t_d)})H(t-t_d)] \tag{2.1}$$

where $H(\,)$ denotes the Heaviside (step) function and β a shape parameter.

The excitation function in Eq. (2.1) was apparently first employed by Jacobsen & Ayre (1958) for the investigation of linear shock spectra. Its use is extended here for the case of rocking blocks, as it can simulate by means of a single parameter (β) an infinite set of symmetric pulses (Fig 2.2). This appears desirable as a single solution suffices to describe dynamic response to vastly different waveforms.

The following dimensionless quantities are employed to describe the results presented below: rocking angle θ and peak rocking angle θ_{max}, uplift strength $\eta = \alpha/\alpha_g$, dimensionless pulse duration $f = p\ t_d$, time of rocking initiation $\tau_l = t_l/t_d$, time of maximum response $\tau_m = t_m/t_d$, critical time $\tau_c = t_c/t_d$ when $\theta(\tau_c)=\alpha$.

From a dimensional analysis viewpoint, given that the problem is described by 5 dimensional variables (i.e. m, R, g, A_g, t_d) and the rank of the dimensional matrix is 3 (i.e. Mass, Time, Length), Buckingham's theorem suggests that 2 (= 5–3) dimensionless parameters (in addition to the waveform parameter β and the inherent dimensionless geometric parameter α) suffice for expressing peak rocking response. In the present study the parameters η and f are selected for this purpose. Accordingly one can write

$$\theta_{max} = F(f, \eta, \alpha, \beta) \tag{2.2}$$

which can be established *a priori* i.e., without knowing the details of the solution. It should be noticed, however, that due to the complexity of the problem, dimensional analysis alone cannot contribute much more than Eq. (2.2), so tackling the equations of motion is essential.

2.1. Rocking Initiation

Depending on the characteristics of the excitation and the properties of the frictional interface, the block can either: stay still, slide (without rocking), rock (without sliding), slide and rock at the same time, or upthrow (Ishiyama 1982, Shenton 1996, Pompey al 1998). Our analysis focuses solely on rocking behavior and, thus, assumes a sufficiently large frictional coefficient so that sliding is prevented. To trigger rocking motion, the overturning moment due to inertial action should exceed the restoring moment due to gravity, which can be written as $\ddot{u}_g/g \geq \tan\alpha$; for slenderness angles α of less than approximately 20 degrees (Yim et al 1980), the trigonometric argument can be dropped to give

$$\ddot{u}_g / g \geq \alpha \tag{2.3}$$

This criterion can be expressed in terms of the uplift strength η as follows: if $\eta > 1$ no rocking occurs, if $\eta < 1$ rocking initiates at $\tau = \tau_l$, when $\ddot{u}_g(\tau_l) = \alpha\, g$.

Substituting into Eq. (2.1), the time of uplift can be easily obtained as

$$\tau_l = \frac{1}{2\beta}\ln[1-\eta(1-e^{\beta})] \tag{2.4}$$

Evidently if uplift takes place (i.e., if $\eta < 1$), corresponding times for the pulses examined herein, occur during rising pulse (i.e. $0 \leq \tau_l \leq 1/2$). Naturally, for a rectangular pulse ($\beta \to -\infty$) rocking initiates at $\tau_l = 0$, and for a perfect spike pulse ($\beta \to +\infty$) at $\tau_l = 1/2$.

2.2. Equations of Motion

The non-linear equations governing rocking response of a rigid block are (Housner 1963)

$$I_o\ddot{\theta} + mgR\sin(\alpha-\theta) = +m\ddot{u}_g R\cos(\alpha-\theta)\,, \qquad \theta(t) > 0 \tag{2.5}$$

$$I_o\ddot{\theta} - mgR\sin(\alpha+\theta) = +m\ddot{u}_g R\cos(\alpha+\theta)\,, \qquad \theta(t) < 0 \tag{2.6}$$

where θ is the rocking angle and I_o is the moment of inertia of the block with respect to the corner of the base (pivot point). For rectangular geometry, $I_o = (4\,/3)mR^2$. The positive sign in the right-hand side of Eqs (2.5) and (2.6) is to ensure positive response for positive ground acceleration, as evident from the reference system of Fig 2.1.

For slender blocks angles θ and α are small; the equations of motion can be linearized using the first-order approximations $sin(\alpha \pm \theta) \cong \alpha \pm \theta$ and $cos(\alpha \pm \theta) \cong 1$. Accordingly, Eqs. (2.5) and (2.6) can be cast in the compact form:

$$\ddot{\theta} - p^2\theta = +p^2\ddot{u}_g / g - p^2\alpha\,\mathrm{sgn}(\theta) \tag{2.7}$$

where *sgn*( ) denotes the signum function. It should be noticed that the second term in the right-hand side of Eq. (2.7) refers to a constant, in each response branch, restoring force, whereas the second term in the left-hand side can be interpreted as a negative (geometric) stiffness. As mentioned in the Introduction, the above equation has been solved in an exact manner only for two cases: (1) a half-cycle rectangular pulse (Housner 1963); (2) a half-cycle sinusoidal pulse (Shi et al 1996). [It is worth mentioning that an elegant (yet still approximate) closed-form analysis procedure for a full-cycle trigonometric pulse has recently been published by Dimitrakopoulos and DeJong (2012).] An attempt to provide analytical solutions for the earthquake waveforms in Eq. (2.1) is presented in the ensuing.

3. SOLUTIONS
3.1. Rocking Response

For the generalized exponential pulse in Eq. (2.1), upon introducing dimensionless peak ground acceleration $\alpha_g = A_g/g$ and considering positive rocking angles (the only case of interest for half-cycle excitation), Eq. (2.7) can be cast in the form

$$\ddot{\theta} - p^2\theta = +p^2\alpha_g(1 - e^{2\beta[\tau+(1-2\tau)H(\tau-1/2)-(1-\tau)H(\tau-1)]})/(1-e^{\beta}) - p^2\alpha \tag{3.1}$$

which holds under uplift conditions i.e., when $\alpha_g > \alpha$ or, equivalently, $\alpha/\alpha_g = \eta < 1$. Enforcing the initial conditions of zero angular velocity, $\dot{\theta}(\tau_l) = 0$, and rotation, $\theta(\tau_l) = 0$ at time of uplift $\tau=\tau_l$ provided by Eq. (2.4), the time histories of angular displacement and velocity are obtained as:

$$\frac{\theta(\tau)}{\alpha} = 1 - \frac{x_1 + x_2 H(\tau-1/2) + x_3 H(\tau-1)}{\eta\left(1-e^{\beta}\right)\left(f^2 - 4\beta^2\right)} \tag{3.2a}$$

where

$$x_1 = \beta[(f+2\beta)[1-\eta(1-e^{\beta})]^{1-\frac{f}{2\beta}} e^{f\tau} - (f-2\beta)[1-\eta(1-e^{\beta})]^{1+\frac{f}{2\beta}} e^{-f\tau}] + f^2 - 4\beta^2 - f^2 e^{2\beta\tau} \tag{3.2b}$$

$$x_2 = f^2(e^{2\beta\tau} - e^{2\beta(1-\tau)}) - 2\beta f e^{\beta}(e^{f(\tau-1/2)} - e^{-f(\tau-1/2)}) \tag{3.2c}$$

$$x_3 = f^2 e^{2\beta(1-\tau)} - (f^2 - 4\beta^2) + \beta[(f-2\beta)e^{f(\tau-1)} - (f+2\beta)e^{-f(\tau-1)}] \tag{3.2d}$$

and

$$\frac{\dot{\theta}(\tau)}{\alpha f} = -\frac{\dot{x}_1 + \dot{x}_2 H(\tau - 1/2) + \dot{x}_3 H(\tau - 1)}{\eta f \left(1 - e^{\beta}\right)\left(f^2 - 4\beta^2\right)} \tag{3.3}$$

where dot denotes differentiation with respect to time (i.e., $\dot{x}_i = dx_i / d\tau$, i=1, 2, 3).

For the limit case of a rectangular pulse, $\beta \to -\infty$; Eqs (3.2) and (3.3) simplify to

$$\frac{\theta(\tau)}{\alpha} = (\frac{1}{\eta} - 1)(\cosh(f\tau) - 1) - \frac{1}{\eta}(\cosh[f(\tau - 1)] - 1)H(\tau - 1) \tag{3.4}$$

$$\frac{\dot{\theta}(\tau)}{\alpha f} = (\frac{1}{\eta} - 1)\sinh(f\tau) - \frac{1}{\eta}\sinh[f(\tau - 1)]H(\tau - 1) \tag{3.5}$$

On the other hand, for a triangular pulse, $\beta \to 0$, Eqs (3.2) and (3.3) yield

$$\begin{aligned}\frac{\theta(\tau)}{\alpha} = 1 - \frac{1}{\eta}(2\tau - \frac{2}{f}\sinh[f(\tau - \frac{\eta}{2})]) - \frac{1}{\eta}(2 - 4\tau + \frac{4}{f}\sinh[f(\tau - \frac{1}{2})])H(\tau - \frac{1}{2}) - \\ - \frac{1}{\eta}(2(\tau - 1) - \frac{2}{f}\sinh[f(\tau - 1)])H(\tau - 1)\end{aligned} \tag{3.6}$$

$$\begin{aligned}\frac{\dot{\theta}(\tau)}{\alpha f} = -(\frac{2}{\eta f}(1 - \cosh[f(\tau - \frac{\eta}{2})])) - (\frac{4}{\eta f}(-1 + \cosh[f(\tau - \frac{1}{2})]))H(\tau - \frac{1}{2}) - \\ - (\frac{2}{\eta f}(1 - \cosh[f(\tau - 1)]))H(\tau - 1)\end{aligned} \tag{3.7}$$

Note that in the limit $\beta \to +\infty$, corresponding to a perfect spike pulse, both θ and $\dot{\theta}$ are zero (no rocking) regardless of pulse amplitude and overall duration.

3.2. Overturning Criterion

For unilateral excitations like the half-cycle pulse at hand, if the response angle θ exceeds the block slenderness α, the restoring moment due to self weight will switch into a driving moment and the block will overturn. Accordingly, a limit acceleration amplitude to overturn the block can be defined – as a sufficient condition – by setting the angle $\theta(\tau_c)$ in Eq. (3.3) equal to α.

Enforcing this condition at the end of pulse (τ_c = *1*), corresponding to the end of the decreasing acceleration branch in Fig. 2.2, the following discriminant equation is obtained

$$\beta[(f + 2\beta)[1 - \eta(1 - e^{\beta})]^{1 - \frac{f}{2\beta}} e^{f} - (f - 2\beta)[1 - \eta(1 - e^{\beta})]^{1 + \frac{f}{2\beta}} e^{-f}] = 4\beta^2 - f^2 + 2\beta f e^{\beta}(e^{\frac{f}{2}} - e^{-\frac{f}{2}}) \tag{3.8a}$$

which can be solved numerically in the form $\eta = F(f)$ *or* $f = F(\eta)$. Repeating the analysis for the end of rising acceleration branch (τ_c =1/2), yields the analogous expression

$$\beta[(f + 2\beta)[1 - \eta(1 - e^{\beta})]^{1 - \frac{f}{2\beta}} e^{\frac{f}{2}} - (f - 2\beta)[1 - \eta(1 - e^{\beta})]^{1 + \frac{f}{2\beta}} e^{-\frac{f}{2}}] = 4\beta^2 - f^2 + f^2 e^{\beta} \tag{3.8b}$$

For $\beta \to -\infty$, referring to a rectangular pulse, Eqs. (3.8a, b) yield, respectively

$$\eta = 1 - 1/\cosh(f), \quad \eta = 1 - 1/\cosh(f/2) \tag{3.9a, b}$$

In the case of a triangular pulse ($\beta \to 0$) the corresponding expressions are

$$\eta = 2 + \frac{1}{f}\ln[1 + 2f(f - \sqrt{f^2 + 1})], \quad \eta = 1 + \frac{1}{f}\ln[1 + \frac{f}{2}(f - \sqrt{f^2 + 4})] \tag{3.10a, b}$$

As overturning over the duration of the pulse does not represent the most critical condition for the system at hand, the corresponding limit criterion (*safety wall*) against overturning can be defined by the condition $\theta(\tau_c) = \alpha$ under zero rocking velocity, after the end of the pulse. This condition corresponds to instantaneous immobility $(\dot{\theta} = 0)$ and equilibrium $(\ddot{\theta} = 0)$, which can be expressed as

$$\dot{\theta}(\tau_c) = 0 \tag{3.11a}$$

$$\ddot{\theta}(\tau_c) = 0 \text{ or } \theta(\tau_c) = \alpha \tag{3.11b}$$

that define a *saddle point* in the response – provided that $\dddot{\theta}(\tau_c) \neq 0$ – at critical time $\tau = \tau_c$. For the single-lobe pulses at hand, it is straightforward to show that the saddle point is always located at infinity (i.e., $\tau_c \to \infty$).

By virtue of Eqs. (3.2) and (3.3), the above conditions yield the closed-form solution

$$\eta = \frac{1}{1 - e^{\beta}}\left\{1 - \left[\frac{e^{f}(f + 2\beta)}{2f e^{\frac{f}{2} + \beta} - (f - 2\beta)}\right]^{\frac{2\beta}{f - 2\beta}}\right\} \tag{3.12}$$

which expresses the overturning criterion for the infinite suite of pulses at hand.

Considering the simple case of a rectangular pulse ($\beta \to -\infty$), Eq. (3.12) yields

$$\eta = 1 - e^{-f} \tag{3.13}$$

which coincides with the solution by Housner (1963). In the same spirit, for a triangular pulse ($\beta \to 0$) Eq. (3.12) yields

$$\eta = 2 - \frac{2}{f}\ln[2e^{\frac{f}{2}} - 1] \tag{3.14}$$

Figure 3.1 depicts the areas of safety (denoted by letter *S*) and overturning (denoted by letter *O*) as function of uplift strength η and pulse duration f for a rocking block under an exponential-like pulse obtained for a shape factor $\beta = \pi$. Evidently, for large values of uplift strength, overturning requires large pulse durations and vice versa. Regions O_1 and O_2 correspond to overturning during pulse (the former taking place before $t_d/2$ and the latter after $t_d/2$), whereas region O_3 corresponds to overturning after pulse. The line separating area O_2 and O_3 corresponds to the solution of Eq. (3.8a) whereas the line separating area O_3 from the area of safety S corresponds to Eq. (3.12). Finally the line separating areas O_1 and O_2 is defined by the solution of Eq. (3.8b). Corresponding results for the cases of a rectangular ($\beta \to -\infty$) and a triangular ($\beta = 0$) pulse are provided in Figs 3.2 and 3.3.

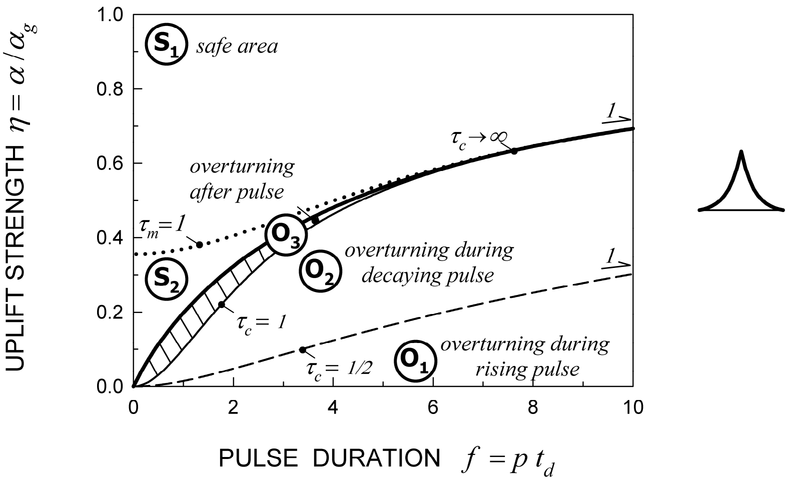

Figure 3.1. Areas of Safety (S) and Overturning (O_1 to O_3) for a rocking block under an exponential pulse excitation of half-cycle duration ($\beta = \pi$).

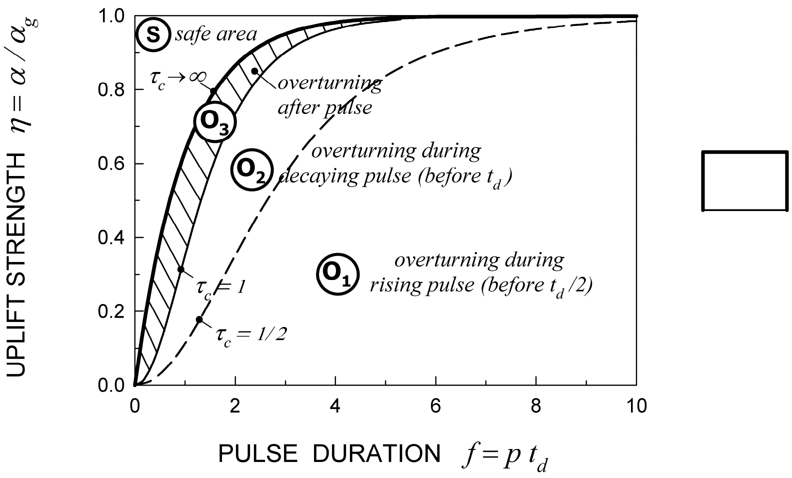

Figure 3.2. Areas of Safety (S) and Overturning (O_1 to O_3) for a rocking block under a rectangular pulse excitation of half-cycle duration ($\beta \to -\infty$).

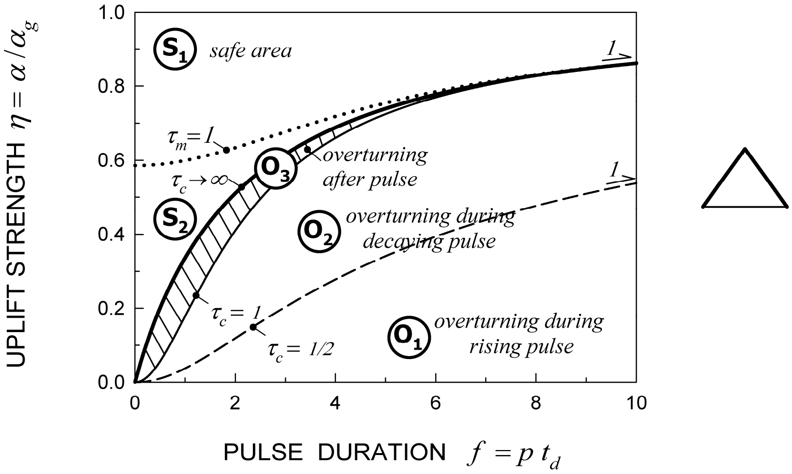

Figure 3.3. Areas of Safety (S) and Overturning (O_1 to O_3) for a rocking block under a triangular pulse excitation of half-cycle duration ($\beta \to 0$).

3.3. Peak Rocking Response

With reference to systems that do not overturn (i.e., blocks in region S, having strength higher than η in Eq. 3.12), time of peak response $\tau_m = t_m/t_d$, is determined from Eqs (3.3) by setting the angular velocity equal to zero. For systems belonging to region S_1, (i.e., $1/2 \le \tau_m \le 1$), τ_m is obtained by solving the equation

$$
\begin{aligned}
&(f+2\beta)[1-\eta(1-e^{\beta})]^{1-\frac{f}{2\beta}} e^{f\tau_m} + (f-2\beta)[1-\eta(1-e^{\beta})]^{1+\frac{f}{2\beta}} e^{-f\tau_m} = \\
&= -2fe^{2\beta(1-\tau_m)} + 2fe^{\beta}\left(e^{f(\tau_m-1/2)} + e^{-f(\tau_m-1/2)}\right)
\end{aligned}
\tag{3.15a}
$$

which can only be tackled numerically except for certain special cases (e.g., $\beta\to-\infty$, 0, $+\infty$) to be discussed below.

For systems in region S_2 (i.e., $\tau_m \ge 1$) the following closed-form solution can be derived

$$
\tau_m = \frac{1}{2f}\ln\left[\frac{\left(f-2\beta+2fe^{\frac{f}{2}+\beta}\right)\left[e^{f}(f+2\beta)-2fe^{\frac{f}{2}+\beta}+(f-2\beta)[1-\eta(1-e^{\beta})]^{1+\frac{f}{2\beta}}\right]}{4f^2e^{2\beta}-e^{-f}\left(f-2\beta\right)^2-(f+2\beta)(f-2\beta+2fe^{\frac{f}{2}+\beta})[1-\eta(1-e^{\beta})]^{1-\frac{f}{2\beta}}}\right]
\tag{3.15b}
$$

Time of peak rocking response for the limit cases of a rectangular ($\beta\to-\infty$) and a triangular pulse ($\beta\to 0$) are provided by Eqs. (3.16) and (3.17a,b) respectively. Note that for a rectangular pulse, peak response always occurs during free rocking ($\tau_m>1$), whereas for a triangular pulse Eq. (3.17a) refers to $1/2 \le \tau_m \le 1$ and Eq. (3.17b) to $\tau_m>1$. Results are provided in Fig. 3.4.

$$
\tau_m = \frac{1}{2f}\ln\left[\frac{1-\eta-e^{f}}{1-\eta-e^{-f}}\right]
\tag{3.16}
$$

$$
\tau_m = \frac{1}{f}\ln\left[\frac{1+2\sqrt{\cos[\frac{f}{2}(1-\eta)]-1}}{2e^{-\frac{f}{2}}-e^{-\frac{f\eta}{2}}}\right], \quad \tau_m = \frac{1}{2f}\ln\left[\frac{2e^{\frac{f}{2}}-e^{\frac{f\eta}{2}}-e^{f}}{2e^{-\frac{f}{2}}-e^{-\frac{f\eta}{2}}-e^{-f}}\right]
\tag{3.17a, b}
$$

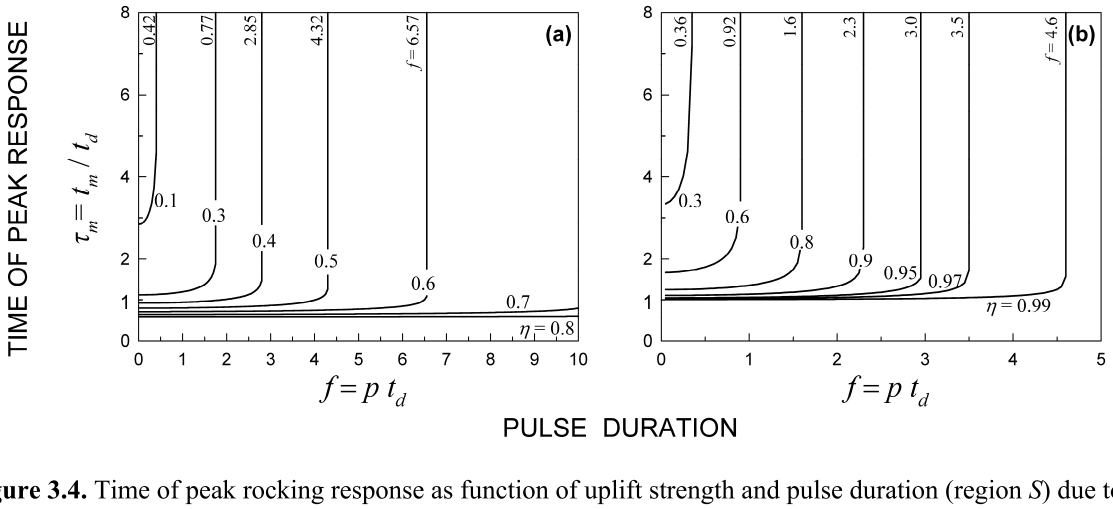

Figure 3.4. Time of peak rocking response as function of uplift strength and pulse duration (region S) due to a half-cycle generalized exponential pulse of : (a) exponential shape ($\beta=\pi$) and (b) rectangular shape ($\beta=-\infty$).

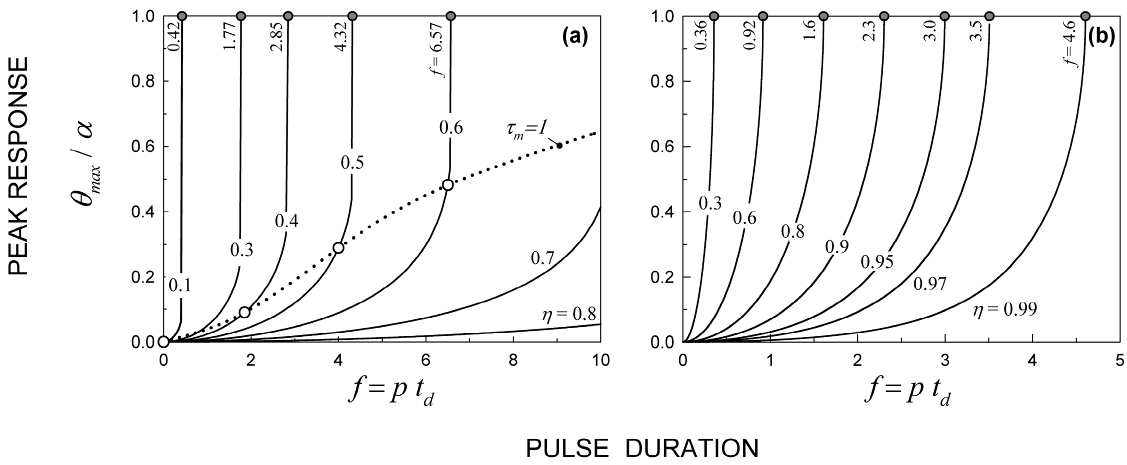

Figure 3.5. Peak rocking response ("rocking spectra") as function of uplift strength and pulse duration (region *S*) due to a single-lobe generalized pulse of : (a) exponential shape ($\beta = \pi$) and (b) rectangular shape ($\beta = -\infty$).

Maximum rocking response can be obtained from Eqs (3.2) and (3.15) as fraction of the slenderness angle α. For example, in the simple case of a rectangular pulse, maximum rocking angle is given by

$$\theta_{max} = \frac{\alpha}{\eta}\left[\eta + (1-\eta)\cosh(f\tau_m) - \cosh[f(\tau_m - 1)]\right] \tag{3.18}$$

Figure 3.5 illustrates maximum rocking response plotted in terms of pulse duration and uplift strength. Evidently, systems in region *S* rock to angles which are smaller than α. Corresponding dimensionless times naturally exceed 1 in all cases. Note that when θ_{max} reaches α, the system lies on the critical limit (safety wall), as defined by Eq. (3.12).

Figure 3.6 shows the areas of safety as function of uplift strength η and pulse duration f for a rocking block under a generalized exponential pulse of half-cycle duration for different values of the shape factor β, ranging from $-\infty$ (a perfect rectangle) to $+\infty$ (a perfect spike). For large values of uplift strength overturning requires large pulse durations and vice versa.

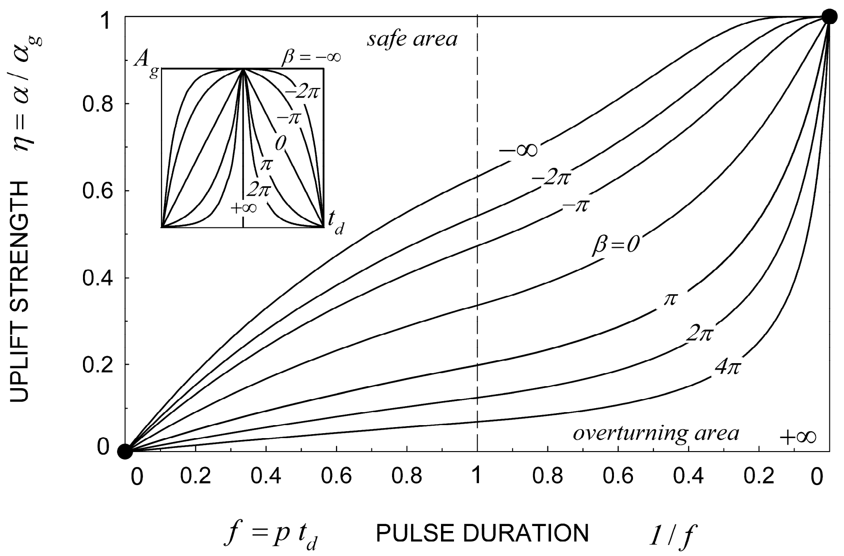

Figure 3.6. Areas of Safety (*S*) and Overturning (*O*) for a rocking block under an exponential pulse excitation of half-cycle duration for different values of shape factor β.

4. CONCLUSIONS

The main conclusions of the study are:

1. By virtue of the generalized single-lobe pulse in Eq. (2.1), novel analytical solutions for rocking response where derived covering an infinite number of pulses ranging from a perfect spike ($\beta \to +\infty$) to a perfect rectangle ($\beta \to -\infty$).
2. It was shown that peak response and block overturning are controlled by two dimensionless parameters: dimensionless pulse duration f (i.e., actual pulse duration times characteristic block frequency) and dimensionless uplift strength η (i.e., ratio of minimum required acceleration for initiation of uplift over peak pulse acceleration). On the other hand, the absolute slenderness parameter α is not an independent variable in the realm of the linearized equations of motion in Eq. (2.7).
3. Rocking response was found to exhibit complex patterns even for idealized pulses such as the ones at hand. Nevertheless, contrary to known counterintuitive trends observed with full-cycle pulses, overturning of a block subjected to a half-cycle pulse, does actually imply that the block will overturn with decreasing size, increasing slenderness ratio, or increasing ground motion amplitude.
4. Analytical formulas were provided for the determination of the Safe (S) and Overturning (O) regions. Regions O may be divided into three sub-groups (Figs 3.1-3.3): Region O_1 corresponding to instability in response during the rising pulse branch, O_2 corresponding to instability in response during the decaying branch, and O_3 where critical angle is attained after the end of the pulse. Regions S may be subdivided into subgroups S_1 for peak rocking response attained during pulse and S_2 for peak response attained during free rocking response.
5. Solutions for peak block rotation ("rocking spectrum") and associated time were presented.

REFERENCES

Anooshehpoor, A., Heaton, T.H., Shi, B., Brune, J.N. (1999). Estimates of the Ground Accelerations at Point Reyes Station during the 1906 San Francisco Earthquake. *BSSA*. **89:4**, 845-853.

Dimitrakopoulos, E., DeJong M. (2012). Revisiting the Rocking Block: Closed- Form Solutions and Similarity Laws. *Proceedings of the Royal Society*. published online (doi: 10.1098/rspa.2012.0026).

Housner, G.W. (1963). The behavior of inverted pendulum structures during earthquakes. *BSSA*. **52:2**, 404-417.

Ishiyama, Y. (1982). Motion of rigid bodies and criteria for overturning by earthquake excitations. *Earthquake Engineering and Structural Dynamics.* **10**, 635-650.

Kirkpatrick, P. (1927). Seismic Measurements by the Overthrow of Columns. *BSSA*. **17:2**.

Milne, J., Omori, F. (1893). On the Overturning and Fracturing of Brick and Columns by Horizontally Applied Motion. *Seismological Journal of Japan*. **17**, 59-86.

Peña, F., Prieto, F., Lourenço, P.B., Campos Costa, A., Lemos, J.V. (2007). On the dynamics of rocking motion of single rigid-block structures. *Earthquake Engineering and Structural Dynamics*. **36:** 2383-2399.

Priestley, M.J.N., Evision, R.J., Carr, A.J. (1983). Seismic Response of Structures Free to Rock on their Foundations. *Bull. New Zealand Natl. Soc. Earthquake Engineering*. **11**, 141-150.

Psycharis, I.N., Jennings, P.C. (1983). Rocking of slender bodies allowed to uplift. *Earthquake Engineering and Structural Dynamics.* **11,** 57-76.

Shi, B., Anooshehpoor, A., Zheng, Y., Brune, J.N. (1996). Rocking and Overturning of Precariously Balanced Rocks by Earthquakes. *Bulletin of the Seismological Society of America*. **86:5**, 1364-1371.

Spanos, P.D., Koh, A.S. (1984). Rocking of rigid bodies due to harmonic shaking. *Journal of Engineering Mechanics, ASCE.* **110:11**, 1627-1642.

Tso, W.K., Wong, C.M. (1989b). Steady state rocking response of rigid block. Part II: Experiments. *Earthquake Engineering and Structural Dynamics.* **18**, 107-120.

Voyagaki, E., Mylonakis, G., and Psycharis, I. N. (2012) Rigid Block Sliding to Idealized Acceleration Pulses. *Journal of Engineering Mechanics ASCE* , (in press)

Voyagaki, E. (2012). *Dynamic response of non-linear systems to near-fault earthquake motions containing significant pulses.* Forthcoming Ph.D. Thesis, School of Civil Engineering, NTUA.

Yim, C.S., Chopra, A.K., Penzien, J. (1980). Rocking response of rigid bodies to earthquakes. *Earthquake Engineering and Structural Dynamics.* **8**, 565-587.

Zhang, J., Makris, N. (2001). Rocking response of a free-standing blocks under cycloidal pulses. *Journal of Engineering Mechanics, ASCE*. **127:5**, 473-483.