

Analytical Study on the Effect of Boundary Element Characteristics on the Behavior of Low-Rise Concrete Shear Walls

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SUMMARY:

In this paper, the effect of variables such as wall aspect ratio, axial force, and boundary element characteristics including longitudinal reinforcement and horizontal reinforcement (concrete confinement) on the behavior of low-rise shear walls are investigated. This is done by analyzing 30 models of such walls. Response parameters include maximum lateral strength, lateral displacement at maximum strength and failure mode of walls. Results indicate that in walls with aspect ratio of 0.5, displacement at maximum strength found to increase with increasing longitudinal reinforcements of boundary elements and in walls with aspect ratio of 1.0 and 1.5 found to decrease. Accordingly, with increasing longitudinal reinforcement of boundary elements, ductility increases in models with diagonal tension failure mode and decreases in models with flexural failure mode. Furthermore, change in boundary element characteristics result in change in failure mode of some models.

Keywords: Low-rise Shear Wall, Maximum Strength, Displacement at maximum strength, Mode of Failure

1. INTRODUCTION

Low-rise shear walls are common in low-rise buildings, also for their seismic rehabilitation, lower stories of high-rise buildings, and nuclear power plants. Wall segments formed by openings also have the same behavior as low-rise walls. Usually, walls with aspect ratio less than 1.5 are classified as low-rise or squat shear walls (Elwood et al, 2007). Shear stresses have significant effect in lateral strength and ductility of such walls. Concrete structures with shear dominant behavior are more complex and may have poor seismic behavior. Also, low-rise shear walls have various failure modes under lateral loading.

Gulec and Whittaker (2009), by collecting results of previously tested 150 rectangular wall specimens and 284 wall specimens with boundary barbell and flanges, have created a comprehensive squat shear wall database. Assessing summarized experimental studies in the database shows that most of the researchers have tried to prevent flexural failure mode of specimens by selecting high ratios for longitudinal reinforcement of boundary elements or large dimensions for end regions. Because, they were tended to have more studies on shear stresses and shear dominant behaviour, and fewer attempts were made for the improvement of squat walls seismic response.

However, using diagonal reinforcements (paulay and priestly, 1992) as a method for response improvement of squat shear walls has limited usage because of practical problems. Furthermore, experimental studies by Kuang and Ho (2008) have shown that concrete confinement of boundary elements has relatively suitable improvements such as increasing energy dissipating ability and displacement ductility of specimens with the aspect ratio of 1.0 and 1.5, but not as effective as slender shear walls.

Accordingly, few researches can be found related to the effect of boundary elements characteristics on the behavior of squat shear walls. Therefore, in this paper, the effect of some variables such as wall

aspect ratio, amount of axial force, and specially effect of boundary elements and some of their characteristics such as longitudinal reinforcements and horizontal reinforcements (concrete confinement) on the behavior of squat shear walls are investigated, and this is done by analyzing 30 models of such walls. Models were analyzed using VecTor2 (Wong and Vecchio, 2002), which is a 2D finite-element nonlinear program designed to analyze concrete membranes.

2. COMPARISON OF SOFTWARE PREDICTIONS WITH TEST RESULTS

Modified Compression Field Theory (MCFT) (Vecchio and Collins, 1986) is implemented in VecTor2 for 2D analysis of concrete structures. MCFT is a rotating angle smeared cracking model that combines compatibility, inelastic constitutive relationship and equilibrium.

Selected test specimens are specimen WALL1 tested by Wiradinata and Saatciuglu (1986), specimen U1.0 tested by Kuang and Ho (2008), specimen SW26 tested by Lefas et al (1990) and specimens M3 and M4 tested by Greifenhagen (2006). In addition to the VecTor2 user's manual, recommendations by Palermo and Vecchio (2007) for finite element modeling of shear walls are used for modeling.

Fig. 2.1 presents experimental and analytical load-displacement curves of specimen U1.0. Analysis prediction is in a good agreement with test results. Reported failure mode of this specimen was yielding of vertical reinforcements. The point of strength loss in the analysis curve is near the point of yielding in test curve. The ratio of predicted to measured strength and the ratio of predicted to measured displacement at maximum strength for the specimen are 1.07 and 1.03 respectively.

Consequently average predicted maximum strength of all selected specimens is 96 percent of test results and average predicted displacement at maximum strength is 92 percent of test results. Accordingly, comparison of the experimental and analytical load-displacement responses indicates that the simulation has accurately predicted the behavior of all selected test specimens, including the ultimate load and corresponding displacement. Therefore, VecTor2 can be reliable for analyzing squat wall models as are defined in the next section.

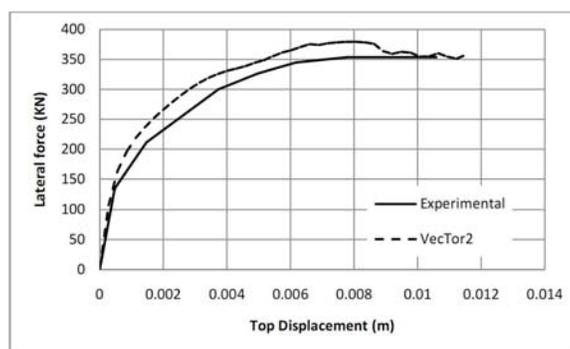


Figure 2.1. comparison of experimental force-displacement curve with analysis predictions using VecTor2 for specimen U1.0 tested by Kuang and Ho (2008)

3. PARAMETERES AND MODELS

Investigated variables are wall aspect ratio (h/l), existence of boundary elements, amount of axial force, longitudinal reinforcement of boundary elements, and horizontal reinforcement of boundary elements (concrete confinement). These variables and their quantities or conditions are explained in Table 3.1. In table 3.1; h is the wall height, l is the wall length, f_c is the concrete compressive strength and A_g is the gross area of wall section.

Table 3.1. Variables and their conditions

variable	quantities and conditions		
	0.5	1.0	1.5
existence of boundary elements (adding vertical reinforcements at the two ends of wall section)	Without longitudinal reinforcement of boundary elements		Boundary elements with 1 percent longitudinal reinforcement and unconfined concrete
axial force	0		$0.05f_c A_g$
longitudinal reinforcement of boundary elements	1 percent		3 percent
confinement of boundary elements	unconfined boundary elements		confined boundary elements

Some aspects of the variables are discussed in the following.

In order to study various aspects of wall failure, 3 quantities are selected for wall aspect ratios. These quantities are 0.5, 1.0 and 1.5. Wall aspect ratio is the principal variable for classification of models. Models are named m1 to m30. The aspect ratio of model m1 to model m10 is 0.5, model m11 to model m20 is 1.0 and model m21 to model m30 is 1.5.

For the assessment of the effect of boundary-element existence, models without boundary elements are compared with models with simplest case of boundary elements (without confined concrete and with 1 percent longitudinal reinforcement). The purpose of this comparison is to understand general effect of this variable on the behavior of such walls.

The load-displacement modeling parameters for walls with shear dominant behaviour depend on axial load acting on the wall (Elwood et al, 2007). If axial load increases from $0.05A_g f_c$, drift at the beginning of strength degradation will decrease from 1 percent to 0.75 percent. Therefore, by selecting 0 and $0.05A_g f_c$ for axial load, the effect of this parameter is assessed on the response of models.

To design a practical ductile shear wall, the amount of longitudinal reinforcement of boundary elements usually is about one percent. Correspondingly, 3 percent have been generally selected for the maximum of this parameter. Therefore, in this study 1 and 3 percent are used for the parameter.

Unconfined and confined are two selected cases for the confinement of boundary elements. In the unconfined situation, horizontal web reinforcements are continued to the boundary elements and there aren't any extra transverse reinforcements. In the confined situation, the amount of confining steel was calculated using special criteria (high ductility) of Iranian National Building Code (2006). VecTor2 automatically calculates confining stresses based on the assigned value for confining steel.

In addition to the defined variables, models have some similar and fix properties. These properties are presented in table 3.2.

Table 3.2. Fixed properties of models

model length (l)	3m
boundary element length	0.45m
web and boundary element thickness	0.2m
concrete compressive strength	26MPa
reinforcement yield stress	390MPa
reinforcement ultimate stress	590MPa
elastic modulus of reinforcements	2×10^5 MPa
horizontal web reinforcement	0.5 percent
vertical web reinforcement	0.5 percent

According to the above descriptions and table 3.1; table 3.3 presents model names and characteristics.

A parametric study is carried out to investigate the effect of each variable on the response parameters (maximum strength, its corresponding displacement and mode of failure). This is done by performing

a static nonlinear analysis on each model. Analysis results are presented in the following section.

Table 3.3. Properties of models according to their name

No	name			boundary elements	axial force	longitudinal reinforcements of boundary elements (percent)	concrete confinement of boundary elements
	h/l						
	0.5	1.0	1.5				
1	m1	m11	m21	without	0	----	----
2	m2	m12	m22	without	$0.05f_c A_g$	----	----
3	m3	m13	m23	with	0	1	unconfined
4	m4	m14	m24	with	0	1	confined
5	m5	m15	m25	with	0	3	unconfined
6	m6	m16	m26	with	0	3	confined
7	m7	m17	m27	with	$0.05f_c A_g$	1	unconfined
8	m8	m18	m28	with	$0.05f_c A_g$	1	confined
9	m9	m19	m29	with	$0.05f_c A_g$	3	unconfined
10	m10	m20	m30	with	$0.05f_c A_g$	3	confined

4. ANALYSIS RESULTS

Models were analyzed subjected to static monotonic loading. Lateral load was applied uniformly on the top side nodes of the models and increased with equal steps from zero to maximum strength during the analysis. Analysis was terminated when the iteration process didn't complete because of zero stiffness condition. Selected procedure can't give strength and displacement after maximum strength, but because top side nodes are allowed to displace from each other, the accuracy of analysis will increase. Axial load was applied before applying the lateral load in one step. Obtained response parameters for all models are shown in table 4.1. Calculated displacement is the lateral displacement of the topside midpoint of the models. In this paper, expressions "displacement" and "ductility" refer to lateral displacement at maximum strength. In this section, some interesting aspects of analysis results are explained. Statistical analysis on the results will be done in the next section.

In the case $h/l=0.5$, according to Fig. 4.1 it is obvious that increasing longitudinal reinforcement of boundary element from 1 percent to 3 percent results in a small increase in strength and ductility. This phenomenon happens because of stronger boundary elements. Stronger boundary elements limit distribution of diagonal tension cracks. Thus, web cracks cannot continue to boundary elements. In this case, wall can undergo lateral force and displacement as a truss structure and therefore, strength and ductility increase with increasing longitudinal reinforcement of boundary element (Fig. 4.2). It is known that in high-rise shear walls, ductility decreases with increasing longitudinal reinforcement of boundary element. However, this is different from obtained results in this study for models with aspect ratio of 0.5.

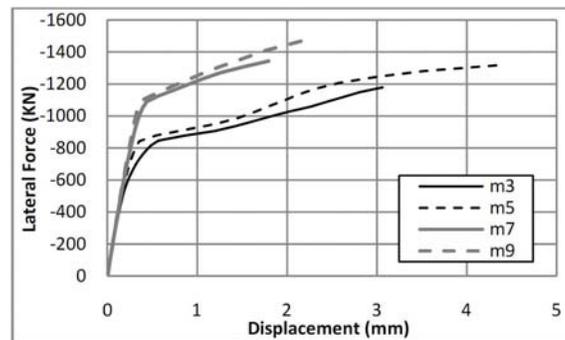
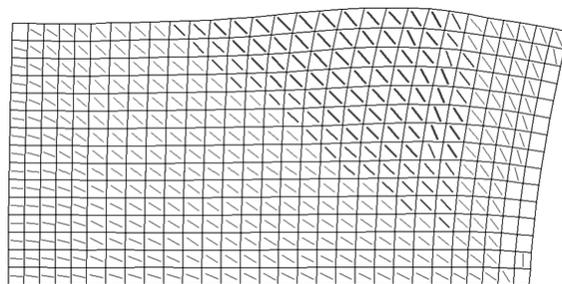


Figure 4.1. Comparison of force displacement curve of model m3 (1 percent longitudinal reinforcement of boundary element) with model m5 (3 percent longitudinal reinforcement of boundary element) and model m7 with model m9

Table 4.1. Failure mode, maximum strength and displacement at maximum strength obtained from analysis.

Model (h/l=0.5)	m1	m2	m3	m4	m5	m6	m7	m8	m9	m11
failure mode	Combination of Diagonal Tension and Flexure	Diagonal Tension								
maximum strength (KN)	1111	1377	1179	1225	1318	1325	1346	1439	1489	1522
Displacement at maximum strength (mm)	4.2	1.88	3.06	3.34	4.36	4.3	1.79	2.26	2.28	2.37
Model (h/l=1.0)	m11	m12	m13	m14	m15	m16	m17	m18	m19	m20
failure mode	Flexural	Flexural	Flexural	Flexural	Diagonal Tension	Diagonal Tension	Flexural	Flexural	Diagonal Tension	Diagonal Tension
maximum strength (KN)	722	976	846	906.9	1214.7	1205	1155.6	1244	1418	1427
Displacement at maximum strength (mm)	27.47	21.19	25.29	30.06	15.41	15.2	17.9	23.6	12.8	14.88
Model (h/l=1.5)	m21	m22	m23	m24	m25	m26	m27	m28	m29	m30
failure mode	Flexural	Flexural	Flexural	Flexural	Flexural	Flexural	Flexural	Flexural	Flexural	Flexural
maximum strength (KN)	458.2	661.1	553.3	567	951.8	987.3	757.4	781.9	1144	1179.9
Displacement at maximum strength (mm)	53.5	47.4	46.2	51.3	26.51	38.8	32.3	42.7	21.5	31.2

**Figure 4.2.** The view of wall model m5 (3 percent longitudinal reinforcement of boundary element) at its maximum strength

In the case $h/l=1.0$, with increasing of longitudinal reinforcement of boundary element from 1 to 3 percent, model failure-modes have been changed from flexural (Fig. 4.3) to diagonal tension (Fig. 4.4). Increasing the parameter, results in an increase in wall flexural strength. Therefore, under lateral loading, shear failure will occur before flexural failure if lateral force corresponding to flexural capacity is more than lateral force corresponding to shear capacity.

According to aforesaid issue, there are two debatable subjects; First, increasing longitudinal reinforcement leads to change in failure mode. According to Fig. 4.5 which shows the force-displacement curve of models m15 and m19, it is clearly observed that changing in failure mode leads to an increase in strength. In other words, the process of resisting lateral force is changed and curves have two separate parts. In the beginning of cracking, according to their angle (Fig. 4.4), they are flexural cracks in boundary element, dominant behavior of the wall is flexural behavior. However, with opening diagonal cracks in the wall web, horizontal reinforcements that were elastic before opening cracks, start yielding. This leads to increasing of wall ultimate strength with $h/l=1.0$, but leads

to decreased ductility.

Second; certainly, there is a balance state that flexural and diagonal tension failure occur simultaneously at a certain percentage of longitudinal reinforcements of boundary elements. In this research, for models with $h/l=1.0$, that certain percentage is between 1 and 3 percent. According to the fact that flexural failure mode (yielding of vertical reinforcements) is more desirable than shear failure modes, founding an accurate method for calculating the amount of longitudinal reinforcement of boundary element at balance state can be a suitable subject for further studies.

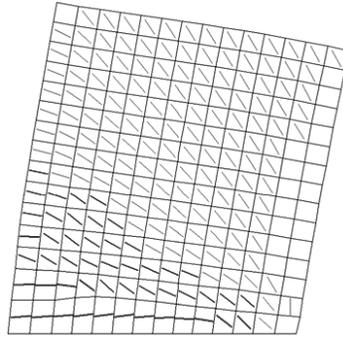


Figure 4.3. The view of wall model m13 (1 percent longitudinal reinforcement of boundary elements) at its maximum strength with flexural failure mode

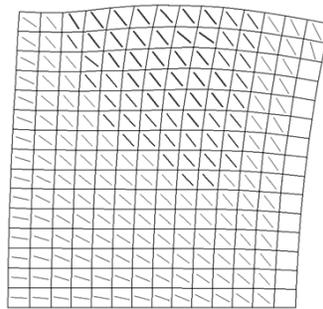


Figure 4.4. The view of wall model m15 (3 percent longitudinal reinforcement of boundary elements) at its maximum strength with diagonal tension failure mode

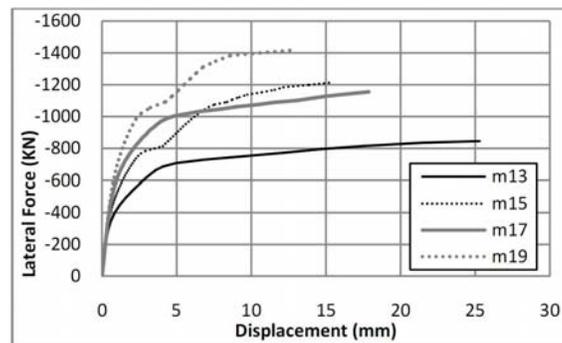


Figure 4.5. Comparison of the force-displacement curve of model m13 (1 percent longitudinal reinforcement of boundary elements) with model m15 (3 percent longitudinal reinforcement of boundary elements) and m17 with m19

5. COMPARISON OF ANALYSIS RESULTS

In this section, based on the analysis results of each model, the average effect of each variable in each

case of h/l on the maximum strength and its corresponding displacement have been compared with other cases.

5.1. Existence of Boundary Element

It must be remembered that for the assessment of the effect of this variable, models without boundary element are compared with models with one percent longitudinal reinforcement of boundary element and unconfined concrete. For example, in the case $h/l=0.5$ model m1 is compared with model m3 and model m2 is compared with model m7. In the case $h/l=0.5$ the existence of boundary elements leads to 2 percent increasing of strength. With an increase in h/l , the effect of this variable will increase. So, in the case $h/l=1$, 18 percent and in the case $h/l=1.5$, 16 percent are the average increases in the strength.

According to Fig. 5.1 it is interesting that existence of boundary elements in all cases results in decreased displacement at maximum strength. This reduction is 16 percent for $h/l=0.5$, 12 percent for $h/l=1.0$ and 23 percent for $h/l=1.5$. This phenomenon is due to the fact that by creation of boundary element, wall flexural strength and therefore the effect of shear stresses on the wall behaviour increase. Increasing shear stress results in flexural cracks change into flexure-shear cracks. Therefore, due to compression softening, concrete compressive strength decreases in diagonal direction. Furthermore, because of diagonal performance of shear force, compression in wall toe increases. These ingredients collected together result in wall-toe concrete to reach its ultimate strain when smaller lateral displacement is arrived, and consequently wall ductility decreases.

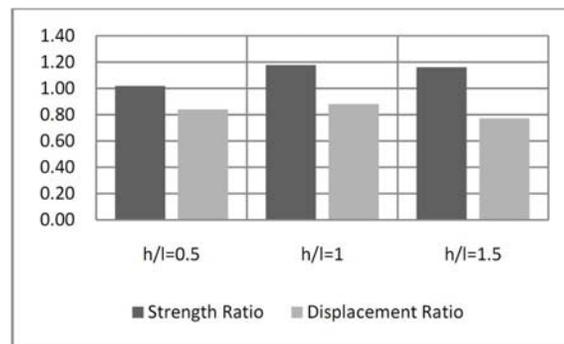


Figure 5.1. The average ratios of strength and its corresponding displacement in models with 1 percent longitudinal reinforcement boundary elements and unconfined concrete to models without boundary elements

5.2. Axial Force

Fig. 5.2 shows the average effect of axial force in strength and its corresponding displacement in each case of h/l . Axial force has the same effect in all cases. It means that increasing axial force in all cases, results in an increased strength and decreased ductility. It is interesting that with an increase in h/l , the effect of the axial force increases on the strength. Increasing of axial force from zero to $0.05A_g f_c$, leads to increased strength equals 17, 29 and 33 percent and decreased displacement at maximum strength equals 44, 15 and 19 percent for $h/l=0.5$, $h/l=1.0$ and $h/l=1.5$ respectively. It can be observed that the effect of axial force in decreasing of displacement at maximum strength in shear dominant behavior (diagonal tension failure) is higher than flexural dominant behavior.

5.3. Longitudinal Reinforcements of Boundary Element

This parameter has various effects on the behavior of models in different wall aspect ratios. According to Fig. 5.3 it is clearly observed that this parameter leads to increased strength in all cases. Average increase is 9 percent for $h/l=0.5$, 28 percent for $h/l=1.0$ and 64 percent for $h/l=1.5$. The effect of increasing the parameter on the strength increases with the aspect ratio of walls.

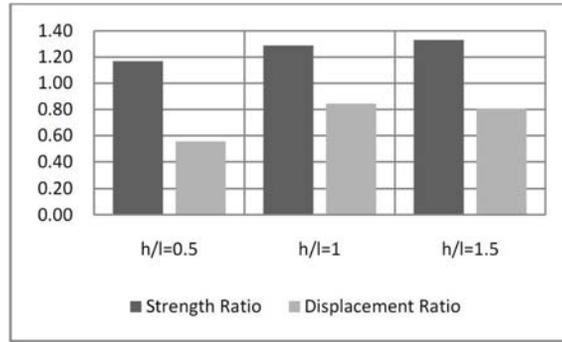


Figure 5.2. The ratios of strength and its equivalent displacement in models with axial force to models without axial force

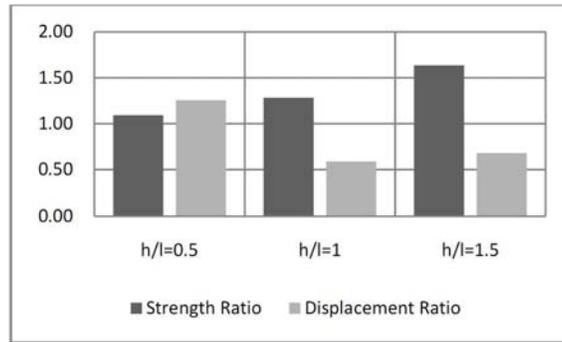


Figure 5.3. The ratios of strength and its equivalent displacement in models with 1 percent to models with 3 percent longitudinal reinforcements of boundary elements

According to section 4, failure mode of walls with $h/l=1$ changes by increasing longitudinal reinforcement of boundary element from 1 percent to 3 percent. Flexural failure mode has been changed to diagonal tension failure. This changing leads to a significant reduction of displacement at maximum strength equal to 41 percent. In the case $h/l=1.5$ this reduction equals 32 percent. But, in the case $h/l=0.5$ increasing longitudinal reinforcement of boundary element leads to 26 percent increase in displacement at maximum strength.

5.4. Concrete Confinement of Boundary Elements

This parameter has shown a little effect on the strength of models. Confinement of boundary elements results in 3, 4 and 4 percent increase in the model-strengths for $h/l=0.5$, 1.0 and 1.5 respectively.

The effect of this parameter on the displacement at maximum strength increases with the increasing of wall aspect ratios. Results have shown 9 percent reduction in the case $h/l=1.5$, 23 percent in the case $h/l=1.0$, and 34 percent in the case $h/l=0.5$. It was predictable that with increasing the effect of bending on the behavior of walls, the positive effect of concrete confinement of boundary elements also increases. This fact can be observed from Fig .5.4.

6. CONCLUSIONS

After the assessment and comparing of the analysis results, important conclusions can be explained as the following.

1. In walls with $h/l=0.5$ increasing longitudinal reinforcement of boundary elements from 1 percent to 3 percent leads to an increase in displacement at maximum strength (ductility) of the models. This

phenomenon is different from the observed behavior of slender walls. The reason of this phenomenon can be found in the wall mode of failure that is diagonal tension. With the increasing of longitudinal reinforcement of boundary elements and thus strengthening them, distribution of diagonal cracks is limited and ductility increases. Therefore, in diagonal tension failure mode, ductility can be increased by increasing longitudinal reinforcements of boundary elements.

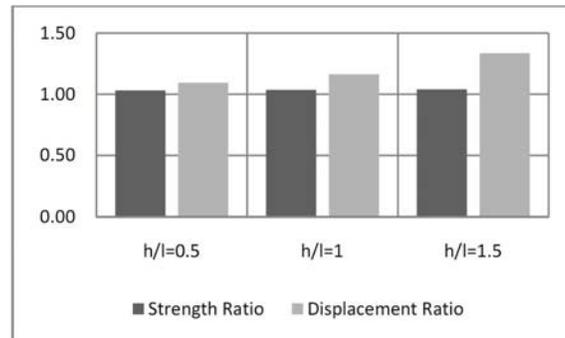


Figure 5.4. The ratios of strength and its equivalent displacement in models with confined boundary elements to models with unconfined boundary elements

2. In walls with $h/l=1.0$ increasing longitudinal reinforcements of boundary elements from 1 percent to 3 percent leads to a conversion in failure mode from flexural to diagonal tension. This phenomenon causes an increase in strength and a decrease of its equivalent displacement. Determining the amount of longitudinal reinforcements of boundary elements that leads to change in failure mode of walls can be a suitable subject for further studies.

3. Finally, it can be said about the effect of boundary elements on the behavior of models that in walls with $h/l=0.5$ (diagonal tension failure) using strong boundary elements (3 percent longitudinal reinforcement of boundary elements) can help to improve response with increasing ductility. In walls with $h/l=1.0$ boundary elements lead to decreased ductility and insignificant increase in the strength. Therefore, in such walls using boundary elements is unsuitable for response improvement. In walls with $h/l=1.5$ in spite of decreasing of ductility, boundary elements lead to a significant increase in strength of walls. So in such walls using boundary elements with increasing strength and thus area under the force-displacement curve can help to improve their seismic response.

4. Increasing axial force leads to significant reduction of ductility in diagonal tension failure mode ($h/l=0.5$), whereas, in vertical reinforcements yielding (flexural) failure mode, increasing axial force has smaller effect on ductility reduction

7. SUGGESTIONS

Based on the carried out studies in this research, and the requirement of more studies on some other aspects of squat shear walls, some suggestion are presented in the following.

1. According to Fig .4.2 and Fig .4.4, it is clearly observed that in walls with diagonal tension failure, diagonal cracks are distributed on the top side of the wall. Adding a beam on the top side of these walls limits crack distribution and limits crack opening. Therefore, this may leads to changes in strength, ductility and even failure mode of walls. In test specimens, a strong beam has been used to transfer loads from hydraulic jacks to the specimen. In real constructions, floor slab or reinforcement extension of the side beam can play the role of top beam, but certainly top beam in real construction is weaker than that used in the tests. This issue shows a difference between experimental and real conditions. Changes in the top beam properties such as dimensions and amount of reinforcements can affect on the behavior of squat shear walls with diagonal tension failure. The effect of this subject in

the available relationships for calculating shear strength of squat shear walls is not considered. Thus, assessment of the effect of the top beam on the behavior of squat shear walls can be a suitable subject for further studies. This suggestion has also been presented by some researchers (Esfandiari, 2009). According to the conclusions of this study about walls with diagonal tension failure mode and distribution of diagonal cracks on the top side of walls, importance of paying more attention to this issue is clear.

2. There are significant studies on the effect of foundation response on the response of slender or high-rise shear walls. Various failure modes can be happened in squat shear walls. Thus, response of the foundation of such walls is various too. For example, in the distribution of loads under the foundation, shear forces will have an effective role. The Effect of rocking on the seismic behavior, dissipating energy and failure mode of squat shear walls is different from slender shear walls. In this research, walls were modeled as a single structural element and their interaction with other elements specially foundation and soil are not considered. Accordingly, because of the lack of studies about this subject and its importance, doing more studies about the effect of soil-foundation-structure interaction and the effect of various types of foundation such as shallow foundation or using piles on the response of squat shear walls is suggested.

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REFERENCES

- Elwood, K.J., Matamoros, A., Wallace, J.W., Lehman, D., Heintz, J., Mitchell, A., Moore, M., Valley, M., Lowes, L.N., Comartin, C. and Moehle, J.P. (2007). Update to ASCE/SEI 41 Concrete Provisions, Pacific Earthquake Engineering Research Center (PEER Center).
- Esfandiari, A. (2009). Shear Strength of Structural Concrete Members Using a Uniform Shear Element Approach, PhD Thesis, University of British Columbia, Canada.
- Greifenhagen, C. (2006). Seismic Behavior of Lightly Reinforced Concrete Squat Shear Walls, PhD thesis, Structural Engineering Institute (IS-IMAC), Ecole polytechnique fédérale de Lausanne, Switzerland.
- Gulec, C. and Whittaker, A. (2009). Performance Based Assessment and Design of Squat Concrete Shear Walls, Technical Report MCEER-09-0010, University at Buffalo, State University of New York.
- Iranian National Building Code, Part: 9, Concrete Structures. (2006). Ministry of Housing and Urban Development.
- Kuang, J.S. and Ho, Y.B. (2008). Seismic Behavior and Ductility of Squat Reinforced Concrete Shear Walls with Nonseismic Detailing. *ACI Structural Journal* **105:2**, 225-231.
- Lefas, I.D., Kotsovos, M.D. and Ambraseys, N.N. (1990). Behavior of Reinforced Concrete Structural Walls: Strength, Deformation Characteristics and Failure Mechanism. *ACI Structural Journal* **87:1**, 23-31.
- Palermo, D. and Vecchio, F.J. (2007). Simulation of Cyclically Loaded Concrete Structures Based on the Finite-Element Method. *Journal of Structural Engineering* **133:5**, 728-738.
- Paulay, T. and Priestly, M.J.N. (1992). Seismic Design of Reinforced Concrete and Masonry Buildings. John Wiley, New York (NY).
- Vecchio, F.J. and Collins, M.D. (1986). The Modified Compression Field Theory for Reinforced Concrete Elements Subjected to Shear. *ACI Journal* **83:2**, 219-231.
- Wirandianta, S. and Saatcioglu, M. (1986). Tests of Squat Shear Walls under Lateral Load Reversal. *Proceedings of the 3rd US Conference on Earthquake Engineering*. Charleston, South Carolina, USA. **Vol II**: 1395-1406
- Wong, P.S. and Vecchio, F.J. (2002). VecTor2 and Formworks user's manual. Department of Civil Engineering, Univ. of Toronto, Canada.