Influence of Jointed Rock Foundation on Propagation Behavior of Earthquake induced Crack in Concrete Gravity Dam

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SUMMARY:

Dynamic cracking analysis of concrete gravity dam, considering progressive failure of jointed rock foundation, has been carried out in order to study the propagation behavior of the crack induced by large scale earthquake. Firstly, a previously proposed approach of using tangent stiffness proportional damping to model the propagation behavior of crack is presented and its effectiveness is discussed. Then, in order to take into account the progressive failure of rock foundation, a constitutive model of jointed rock is assumed and its validity is evaluated by comparing the results of simulation analysis with those of the past experiment on a dam model. Finally, dynamic cracking analysis of 100-m high dam model is performed and the influence of jointed rock foundation on the propagation behavior of earthquake induced crack in the dam body is investigated.

Keywords: concrete gravity dam, dynamic cracking analysis, jointed rock foundation, earthquake-induced failure

1. INTRODUCTION

The failure of concrete gravity dam under a large scale earthquake is considered to result from the occurrence and propagation of cracks in the dam body (River Bureau, 2005). In order to study the behavior of the crack propagation, the authors have proposed an assessment method using tangent stiffness proportional damping and carried out analytical studies on the propagation behavior in various aspects in past researches (Kimata et al, 2003, 2005, 2008-2011; Niimi et al., 2004).

In 1959, the Malpasset dam, an arch concrete dam located in France, failed explosively due to the shift of its left abutment caused by a thin clay filled seam in the rock behind the abutment. Since then, the study and prediction of the behavior of rock foundations supporting concrete dams have gained much attention. However, most of the researches relating to the seismic safety assessment of dams under large scale earthquake are still focusing solely on the failure of dam body (River Bureau, 2005).

This paper presents a study on propagation bahivior of earthquake induced cracks of concrete gravity dam considering the progressive failure of jointed rock foundation. The paper starts from briefly describing tangent stiffness proportional damping which has proposed previously by the authors. To consider the progressive failure of the rock foundation, a constitutional model of jointed rock has been assumed and then verified by comparing the analytical results with the past experiment on a dam model. Finally, dynamic cracking analysis based on a 100-m high dam model is performed and the influence of jointed rock foundation on the propagation behavior of earthquake induced crack in the dam body is investigated.

2. EVALUATION OF DAMPING CHARACTERISTICS DURING THE OCCURRENCE AND PROPAGATION OF CRACKS IN DAMS

In order to properly assess the crack propagation behavior of the concrete gravity dam, the authors



have proposed the usage of tangent stiffness proportional damping in the previous paper (Kimata et al, 2005), the detail of which will be briefly described in this section. The equation of motion used to model dynamic crack propagation in concrete gravity dams is shown in Eqn. 2.1 below. In this equation, Rayleigh damping is used to deal with structural damping. The damping matrix of Rayleigh damping is constituted of a mass matrix and an initial stiffness matrix as shown in Eqn. 2.2.

$$[M]\ddot{\mathbf{u}} + [C]\dot{\mathbf{u}} + [K]\mathbf{u} = \{f(t)\}$$

$$(2.1)$$

$$[C] = a[M] + b[K_0]$$

$$(2.2)$$

Here, [M] is the mass matrix, [C] is the damping matrix, $[K_0]$ is the initial stiffness matrix, $\ddot{\mathbf{u}}$ is the acceleration vector, $\dot{\mathbf{u}}$ is the velocity vector, \mathbf{u} is the displacement vector, $\{f(t)\}$ is the external force vector, and the constants a and b are determined from the first and second circular frequency and mode damping constant.

For a plain concrete structure, according to an past study in which the Rayleigh damping shown in Eqn. 2.2 was used to study the occurrence and propagation of cracks in dams (METI, 2001), it has been shown that cracks become more dispersed and less liable to propagate as the damping increases, whereas cracks become localized and propagate more easily when the damping is small. This result can not properly describe localized behavior of crack propagation in plain concrete structures found in the past experimental results (Tinawi et al., 2000). Essentially, when cracks have occurred in plain concrete structures, unlike in reinforced concrete structures, one would expect that tensile stress will no longer propagate at the crack plane. However, in case the damping matrix in Eqn. 2.2 is employed in the equation of motion, when the crack opening velocity occurs in orthogonal direction to a crack, it would induce damping force which act as a resisting force that transmits the tensile stress so that the crack becomes liable to disperse and less likely to propagate.

Therefore, in order to represent the abovementioned characteristics as faithfully as possible, we set up a damping matrix as described below.

(i) With regard to the Rayleigh damping used for dynamic crack propagation analysis, since the existence of [M] results in increased damping at low frequencies and an inability for stress to be suitably released in the regions adjacent to cracks, the effects of the damping force are eliminated when cracks occur by ignoring a[M] in Eqn. 2.2 as shown by the following formula:

$$[C] = b[K_0] \tag{2.3}$$

(ii) Furthermore, $[K_0]$ is not treated as a matrix consisting of constant elements, but as a matrix that varies as a function of time *t* as follows:

$$[C(t)] = b[K(t)] \tag{2.4}$$

Here, [C(t)] is the damping matrix depended on time t, [K(t)] is the stiffness matrix at time t, and b is a constant determined by the first circular frequency and mode damping constant. In other words, by applying damping that varies according to the constantly varying stiffness [K(t)], the stiffness becomes zero and no damping forces occur at the crack plane when a crack has become fully open. In order to examine the validity of the proposed damping model, we compared the results of large-scale shaking table tests on plain concrete structure of dam shape (Tinawi et al., 2000) and the results of dynamic crack propagation analysis performed using Eqn. 2.3 and 2.4 (Kimata et al., 2005). It was found that using tangent stiffness proportional damping (i.e. [C(t)] in Eqn. 2.4) can model the release of tensile stress in normal direction to the crack surface and suitably reproduce the crack propagation phenomenon.

3. CONSTITUTIVE MODEL TO SIMULATE THE PROGRESSIVE FAILURE BEHAVIOR OF ROCK FOUNDATION

3.1. Constitutive model of joint interface

This section presents a constitutive model assumed for modeling joint interfaces in rock foundation. The constitutive model of joint interface contains large numbers of indeterminate factors, especially for its dynamic properties, including deformation and strength characteristic, surface roughness, length, and types of filled material. A simplified constitutive model proposed in the past literatures (Plesha, 1987; Sasaki et al., 1994) is therefore used in this study. The detail of the model is described as follows:

(i) The relationship between shear stress τ and shear strain γ is assumed to be elasto-perfectly plastic hysteresis as shown in Fig. 3.1(a). The shear strength τ_f is defined by Mohr-Coulomb's failure criteria, Eqn. 3.1. Tensile strength and shear stress at tensile normal stress are equal to zero as shown in Fig. 3.2, where τ_f shear strength, σ normal stress, *c*: cohesion, ϕ : internal friction angle.

$$\tau_f = \pm c \mp \sigma \tan \phi \tag{3.1}$$

- (ii) The relationship between normal stress σ and normal strain ε is elastic linear in compression side and the rigidity is zero in tension side as seen in Fig. 3.1(b).
- (iii) In the stress- strain relationship shown by Eqn. 3.2, matrix [D] is expressed by Eqn. 3.3 for the elastic strain component and Eqn. 3.4a–3.4c for the plastic strain component. As seen in the non diagonal term in Eqn. 3.4a, shear stress increment due to the confining pressure is taken into consideration when the value of σ is positive in the plastic region. On the other hand, the influence due to the roughness of the joints, namely dilatancy, is disregarded. An extreme small value is given to S_{ν} in order to avoid numerical ill-conditioning when the value of diagonal component is zero.

$$\begin{cases} \tau \\ \sigma \end{cases} = [D] \begin{cases} \gamma \\ \varepsilon \end{cases}$$
 (3.2)

where τ : shear stress, σ : normal stress, γ : shear strain, ε : normal stain, [D]: element stiffness matrix.

For elastic strain component:

$$\begin{bmatrix} D \end{bmatrix} = \begin{bmatrix} G & 0 \\ 0 & E \end{bmatrix}$$
(3.3)

For plastic strain component:

$$\begin{bmatrix} D \end{bmatrix} = \begin{bmatrix} S_h & -S_v \cdot \tan \phi \cdot Sign(\tau) \\ 0 & S_v \end{bmatrix}$$
(3.4a)

$$S_h = 0(= G/1 \times 10^4)$$
 (3.4b)

$$S_{\nu} = \begin{cases} E(\sigma < 0) \\ 0(=E/1 \times 10^{4})(\sigma \ge 0) \end{cases}$$
(3.4c)

where *E*: Young's modulus, *G*: shear modulus, $Sign(\tau)$: +1 ($\tau \ge 0$), -1 ($\tau \le 0$).





(b) Normal stress vs. normal strain





Figure 3.2. Mohr-Friction model

3.2. Validation of the constitutive model

To assess the validation of the assumed constitutional model of the joint interface, the simulation analysis of a past experiment on a dam model (Takano, 1962) has been performed. The scheme of the test apparatus and experiment setup are illustrated in Fig. 3.3 and 3.4. The abutment section of an arch concrete dam has been modeled in order to assess the seismic safety of structure.



Figure 3.3. The scheme of the test apparatus (Takano, 1962).



Figure 3.4. The experiment setup (Takano, 1962).

The thickness of the specimen which includes both dam body and rock foundation is 50 mm. The specimen is placed inside a loading frame with a dimension of $1000 \text{mm} \times 860 \text{mm}$ and loading is applied using 13 jacks on the dam body in order to represent the water pressure (including increased dynamic water pressure due to seismic motions). As for the rock foundation, it is constructed using square elements with a dimension of $20 \text{mm} \times 20 \text{mm}$ made by mortar. In order to reproduce the joints and discontinuity inside the rock, a thin strip with a thickness of 2 mm made by the mixture of polyester with a certain shear property and diatomite is located between each two rock elements. The joints in the abutment are generally arranged in the parallel and perpendicular directions with the resultant forces. Finally, the load is applied incrementally and the deflection of the model is measured.

The failure pattern of the experiment is shown in Fig. 3.5. As it can be observed from this figure, the compressive load is applied to the rock foundation through the arch of the dam based on arch action. This pushing force would cause shear damage to the jointed rock foundation parallel to the load direction and make some openings between the rock elements which penetrates deeply inside the rock foundation. The rock foundation swells towards the inner part of the specimen which is not restraint. Finally, the failure of the model takes place when the openings between the rock elements spread through the foundation and result in a great loss of contact force which supports the dam model. Therefore, it can be said that if the thickness of the rock foundation which supports the dam at its inner side is not sufficient, remarkable damage can happen which may lead to the collapse of the dam.



Figure 3.5. Failure pattern of the specimen (Takano, 1962).

Figure 3.6. Analytical model

A two dimensional plane strain Finite Element model shown in Fig. 3.6 is used to simulate the above-mentioned experiment. 4-node quadrilateral elements with linear elastic properties are utilized to model the rock foundation since they do not suffer any damage during the experiment. Due to the symmetry of the experiment model, only half of that is modeled. Boundary condition of the analytical model is considered to the same as that of the experiment, i.e. the nodes located next to the loading frame are fixed, leaving the other nodes free as shown in Fig. 3.6. The incremental loading is also applied at the location of jacks in the experiment as concentrated loads. The joint elements are modeled based on the assumed constitutive model. The material properties of rock foundation and interface elements used in the analysis are shown in Table 3.1.

Property list	Intact rock	Joint interface
Young Modulus E (GPa)	2.0	2.0
Poison ratio v	0.2	0.2
Density γ (kg/m ³)	2000	—
Cohesion c (MPa)	_	0.12
Internal friction angle ϕ (°)	_	18

Table 3.1. Material properties used in the analysis

The deflection pattern and minor principal stress of the rock foundation which are obtained by conducting two-dimensional nonlinear analysis are shown in Fig. 3.7 and 3.8, respectively. It was found that by increasing the applied load, the rock elements tend to deflect towards the inner part of the foundation in the same manner with that observed in the experiment. As shown in Fig. 3.7, this trend can be easily seen when the load becomes large enough. Furthermore, Fig. 3.8 shows that applied load would be transferred to the deep parts inside the rock foundations and finally both ends of the abutment would damage due to the incremental loading. Therefore, it can be said that the shear damage and openings at joints of the rock foundation are well simulated by the analysis.

In addition, swelling of the rock foundation in the analysis is picked up as a damage index and is compared to that of the experimental data in terms of relative displacement in 5 locations inside the rock foundation as shown in Fig. 3.9. At some certain level of loading, the relative displacement

sharply starts to increase in the analysis results while experimental data show a smoother incremental trend. When the opening failure occurs, in fact, abrupt increase of the relative displacement along the opening surface is expected. It can thus be said that the analysis can simulate more accurate failure bahavior in this case. However, the relative displacement of the model roughly demonstrates the similar trend in both experiment and analysis. Therefore, the constitutive model used for simulating the joint elements is considered to be accurate and acceptable.



Figure 3.7. Deflection pattern of rock foundation in the analysis.



Figure 3.8. Minor principal stress pattern of rock foundation in the analysis.



Figure 3.9. Comparison between the experimental and numerical results.

4. DYNAMIC CRACK PROPAGATION ANALYSIS CONSIDERING PROGRESSIVE FAILURE OF JOINTED ROCK FOUNDATION

Dynamic cracking analysis of concrete gravity dam, considering progressive failure of jointed rock foundation, has been carried out, the detail of which is presented in this section. To model the propagation behavior of crack, the tangent stiffness proportional damping is used, while the progressive failure of rock foundation is simulated by the proven constitutive model of jointed rock. Finally, the influence of jointed rock foundation on the propagation behavior of earthquake induced cracks in the dam body is investigated.

4.1. Analysis model and related parameters

A 100m-high dam is modeled as shown in Fig. 4.1. A plane strain condition in upstream-downstream directions is assumed and a coupled model built up between dam body, rock foundation, and storage water is created. The effect of the storage water is considered by Westergaard's equation (Westergaard, 1933) as additional mass attached on the upstream surface of the dam body.



Figure 4.1. Analysis model.

To eliminate the influence of mesh arrangement on the crack propagation direction, the model of the dam body is created by triangular meshes which basically are equilateral triangles. Length of one side of the equilateral triangles is set to 1.5m (crack band width of about 1.5×sin60° m (Uchida et al., 1993)) in order to prevent snap-back phenomenon. Cracking of concrete is expressed using a smeared crack model. The tensile stress - crack opening displacement relationship is formed in linear type with two different slopes based on tension softening constitutive model (1/4 model, Fig. 4.2). During unloading, the origine oriented hystereses considering phase property are used to describe the tensile stress - crack opening displacement relationship. Table 4.1 shows the model parameters of the main dam model chosen based on the past researches (METI, 2001; CEB, 1993; Ueda et al., 1997).



Figure 4.2. Tension softening constitutive model of concrete.

On the other hand, the discontinuities in the underlying rock foundation is assumed to consist of 2 groups lying perpendicularly to each other in -45° and $+45^{\circ}$ (positive: clockwise) to the vertical direction. Although randomly mixed directions of discontinuity surfaces are naturally found in the real rock mass, the three sets of discontinuity surface arranging perpendicularly to each other are often found in sedimentary rocks and granite rocks (e.g. plutonic rocks) which are basically used as dam foundations. Moreover, even though the spacing of the joints inherent in real rock is also varied largely, 6m spacing of the joints are chosen with reference to the rock mass classification by Kikuchi et al. (1982). In this classification, for relatively continuous rock mass (joint spacing are more than 2m), modeling joint spacing of 2 ~ 10m is recommended for generally fresh rock (joint surface may be little contaminated by weathering) while more than 10m is recommended for fresh rock (the joint surface has not been contaminated by weathering at all). As already shown in Table 4.1, the angle of internal friction and cohesion of joints are chosen to represent cracked hard rock which is assumed to behave like generally fresh rock (Tada et al., 2002).

Property list	Dam body	Foundation rock	
Toperty list	Daili body	Intact rock	Joint interface
Young Modulus E(GPa)	28	10	10
Poison ratio v	0.2	0.25	0.25
Density $\gamma(\text{kg/m}^3)$	2300	2500	_
Tensile strength f _t (MPa)	3.0	—	_
Fracture energy G _f (N/m)	400	—	_
Cohesion <i>c</i> (MPa)	-	—	0.16
Internal friction angle $\varphi(^{\circ})$	_	—	42
Damping ratio in the first mode h_1	0.07	0.02	0.02

Table 4.1. Model parameters

4.2. Input seismic motion

Input seismic motion was simulated by considering the verified lower limit acceleration response (River Bureau, 2005) as a target spectrum. As for the phase characteristics, the acceleration time-history waveform observed in the inspection gallery of Hitokura dam during the Southern Hyogo Prefecture earthquake in 1995 is used. The seismic motion at the bottom of the analysis model is calculate by performing back-analysis based on the simulated seismic motion at the rock surface (opening surface), and is then applied as the input seismic motion. Fig. 4.3 shows acceleration time-history waveform and the acceleration response spectrum of the input earthquake motion.



Figure 4.3. Input seismic motion (at model bottom).

4.3. Result of dynamic cracking analysis

By performing dynamic cracking analysis, time history plot of the differential displacement between horizontal movement at the top and the bottom of dam body, the opening and slip surface conditions of the jointed rock foundation, and propagation of cracks in the dam body are shown in Fig. 4.4 through Fig. 4.6, respectively.

It can be seen that relative displacement is rapidly increased about at the 8th second (Fig. 4.4). For the rock foundation, it is found that when the dam body moves to the downstream side, openings occur at 45° direction in upstream side, and -45° direction in downstream side. The openings in the upstream side is relatively smaller. This is because the deformation is restrained by the hydrostatic pressure of the storage water. When the dam body moves to the upstream side, the foundation rock in the upstream side is compressed by hydrostatic pressure and the dam body so that only small openings and slip surfaces are found, while the openings and slip surfaces in the direction of 45° and -45° largely appear on the downstream side (Fig. 4.5). As for the crack in the dam body, the analysis results show that, comparing with the case in which the discontinuity of rock foundation is not concerned, the propagation of cracks at the bottom of the dam body greatly reduce by taking into account the jointed rock foundation (Fig. 4.6). This is due to the fact that the seismic energy acting on the dam body are absorbed by the occurrence of the openings and slip surfaces in the rock foundation.



Figure 4.4. The differential displacement between horizontal movement of dam body (top – bottom).



Figure 4.5. The opening and slip surface conditions of the jointed rock foundation (Magnified deformation shape, in case the dam body moves to the upstream side).



Figure 4.6. Propagation of cracks in the dam body.

5. CONCLUSION

The dynamic cracking analysis of concrete gravity dam, with consideration of progressive failure of jointed rock foundation, has been performed and the effect of the jointed rock foundation in the propagation behavior of the cracks in the dam body under large scale earthquake has been studied. The results of the study can be concluded as follows:

- (i) To simulate the progressive failure of jointed rock foundation, joint interface elements with special constitutive model have been used. The results obtained from simulation analysis using proposed model are compared with those of a past experiment under similar conditions and good agreement is found between them. The validation of the proposed model is therefore proved.
- (ii) Using the assumed constitutive model and the previously proposed tangent stiffness proportional damping, the dynamic cracking analysis on a 100m-high dam model has been carried out. It was found that considering progressive failure of jointed rock foundation can greatly reduce the level of crack propagation in the dam body. This suggests that progressive failure of jointed rock foundation is an important issue in seismic safety evaluation of concrete dams.
- (iii) In the future research, further studies on the effects of the directions of discontinuity planes in jointed rock foundation, plastic behavior of dam body and rock foundation under compressive

loads, and seepage water in the openings and cracks on the seismic safety of concrete gravity dam are considered.

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