# Methods for characterising effects of liquefaction in terms of damage severity

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### **ABSTRACT:**

This paper describes the development of a new method, the Liquefaction Severity Number (LSN), to assess vulnerability of land to liquefaction-induced damage by comparing measured damage attributes with parameters calculated from approximately 1500 Cone Penetration Tests (CPT's). The paper presents a review of the published literature relating to land vulnerability and the observed effects of strong earthquake shaking in Canterbury, New Zealand where various parameters have been calculated and correlated with the observed damage attributes. A sample CPT is presented, followed by preliminary results for two parameters plotted against land and foundation damage measured around Canterbury. The paper concludes that LSN is an alternative method for characterising vulnerability to liquefaction in residential areas.

Keywords: Liquefaction vulnerability, Liquefaction Severity Number, Liquefaction Potential Index.

## **1. INTRODUCTION**

The Canterbury region of New Zealand has been affected by a sequence of earthquakes and aftershocks since 4 September 2010. The four most significant quakes and aftershocks occurred on 4 September 2010, 22 February, 13 June and 23 December 2011. The shaking from these events triggered locally severe liquefaction. The incidence of subsidence associated with voluminous ejection of liquefied material, and other effects such as lateral spreading has been a strong determinant of foundation deformation damage to residential dwellings. However, while the liquefaction triggering was extensive in some areas, it was also observed to be of little to no consequence to the built environment in other areas, where more serious effects might have been anticipated from the soil properties alone. These variations have been observed in each of the damaging Canterbury earthquakes, suggesting that criteria other than soil properties influence the vulnerability of a given site to consequential effects of liquefaction.

There is extensive literature on the triggering of liquefaction within soil strata subject to seismic shaking, but substantially less on vulnerability indicators that address the consequences of liquefaction for residential dwellings on shallow foundation systems. This paper provides a review of published methods that predict whether liquefaction is likely to be consequential at the ground surface and compares the results obtained for these methodologies from the damage observations and soil test results obtained after each of the major earthquake events in Canterbury. A new vulnerability parameter, the Liquefaction Severity Number (LSN) is presented and compared to the measured damage datasets. Initial analyses indicate that it could provide a better fit to measured damage than the Liquefaction Potential Index (LPI) presented by Iwasaki (1982).

### 2. LIQUEFACTION DAMAGE VULNERABILITY INDICATORS

Ishihara (1985) published observations on the protective effect of an upper layer of non-liquefied material against the expression of liquefaction at the ground surface. The paper contained graphs that plotted thickness of the non liquefied upper layer ( $H_1$ ), referred to herein as the crust, and the

cumulative thickness of liquefied material  $(H_2)$ . The data points were divided into sites that did and did not express liquefaction at the ground surface. The paper was based on observations for two earthquakes with limited ranges of ground accelerations. Boundary curves were defined separating those sites which had expression of liquefaction at the ground surface, from those sites that did not.

Youd and Garris (1995) extended this concept by considering additional data and presented data and dividing curves for various ranges of peak ground acceleration. Both papers showed that, for sites with any substantial thickness of liquefied material, the crust typically had a critical thickness, beyond which surface expression of liquefaction became independent of the cumulative thickness of liquefied material ( $H_2$ ). The papers did not directly measure damage to structures, but instead considered only whether evidence of liquefaction was observed at the ground surface. The conclusion drawn from these papers is that an upper crust of non liquefiable material has a beneficial effect in mitigating the damaging effects of liquefaction at the ground surface.

The vulnerability of sites to liquefaction was also considered by Iwasaki (1980) and subsequently by Juang (2005). Iwasaki's Liquefaction Potential Index (LPI) describes a measure of the vulnerability of sites to liquefaction effects. The sum of liquefied layers is calculated, weighted for severity and a depth weighting factor linearly decreasing with depth to 20m. The resulting LPI varies between 0 and 100, with higher values indicating a higher vulnerability to liquefaction-induced ground damage. The LPI is a function of a liquefaction triggering methodology (discussed below), which incorporates the soil density and composition profile (inferred from the CPT), depth to groundwater, and shaking severity represented by the Cyclic Stress Ratio (CSR), and represents a multi variate problem as a single value. LPI is discussed in more detail in Sections 4 to 6 in this paper.

We are not aware of any published information relating to the quality or performance of the crust in relation to damage of structures and foundations, apart from the papers presented by Cascone and Bouckavalas (1998) and Bouckovalas and Dakoulas (2007). Bouckavalas' work presents the results of modelling that considers the ability of an upper layer of fine grained soils behaving in an un-drained manner, overlying liquefied material, to support loads from shallow strip or pad foundations. A critical layer thickness is defined, where the theoretical foundation bearing capacity failure surface occurs completely within the upper non-liquefied material. Our observations in Canterbury indicate that the upper materials do not typically exhibit in an un-drained behaviour because the crust material comprises silty sand, so the results of Bouckavalas' work are not directly comparable in this case.

All of the relevant published assessments of liquefaction vulnerability rely on identifying which layers of soil are likely to liquefy within a soil profile under cyclic shearing. To assess this, a number of CPT-based triggering methods have been considered, all of which have evolved from Seed's (1971) Simplified Method which compare the Cyclic Stress Ratio (CSR) to the Cyclic Resistance Ratio (CRR) and obtain a factor of safety (FoS). The methods considered for our analysis included:

- (a) Robertson and Wride (1998) modified in Youd et al. (2001),
- (b) the NCEER consensus report of Seed et al. (2003) as presented by Moss et al. (2006)
- (c) Idriss and Boulanger (2008)

The Idriss and Boulanger method requires fines content to be input for each layer. For the general case in this paper, the apparent fines content calculated in accordance with Robertson and Wride (1998) is adopted. For the liquefaction vulnerability analyses presented in this paper, the Idriss and Boulanger method has been adopted. We note that the application of a Seed et al. (2003) method to the liquefaction vulnerability analyses produces very similar results for the Canterbury dataset.

# **3. DAMAGE MAPPING AND FIELD INVESTIGATIONS**

Following the major 4 September 2010, 23 February and 13 June 2011 earthquake events, a qualitative mapping exercise covering land and dwelling foundation damage was undertaken as part of the

coordinated response by agencies of the New Zealand Government. Land damage mapping of residential properties was carried out to assess the extent and severity of surface liquefaction manifestation. The mapping was carried out by a small team of senior engineers who cross-checked observations to ensure broad consistency across the assessments. A more detailed land damage inspection programme was undertaken after each of the rapid land inspections by a team of approximately 400 engineers for insurance assessment purposes. As part of the detailed land damage assessment on each residential property, damage to the foundations of homes was also recorded based on set criteria (refer to the right hand columns in Figure 1).

Land Damage Categories		Residential Dwelling Foundation Deformation Damage Categories			
Land	Criteria / Description	Type of Damage	Severity		
Damage			Minor	Moderate	Major
Category					
Blue	<ul> <li>No observed cracks, undulations / deformations at the ground surface, and,</li> <li>No signs of ejected liquefied material at the ground surface, and,</li> <li>No apparent lateral movement.</li> </ul>	Stretching	0 to 5mm	5 to 30mm	>30mm
Green	<ul> <li>Shaking-induced damage resulting from cyclic deformation and surface-waves causing ground surface damage. Ground surface damage likely limited to minor cracking (tension) and buckling (compression) and/or minor undulations at the ground surface, and,</li> <li>No signs of ejected liguefied material at the ground</li> </ul>	Hogging	0 to 20mm	20 to 50mm	>50mm
Light Orange	<ul> <li>No apparent lateral movement.</li> <li>Minor to moderate quantities of ejected liquefied material on ground surface (generally &lt;25% of site covered with ejected material), and/or,</li> <li>Small cracks from ground oscillations (&lt;50 mm) may be present, but little to no vertical displacement</li> </ul>	Dishing	0 to 20mm	20 to 50mm	>50mm
Dark Orange	<ul> <li>across cracks, and,</li> <li>No apparent lateral movement.</li> <li>Moderate to severe quantities of ejected liquefied material on ground surface (generally &gt;25% of the site covered with ejected material), and/or,</li> </ul>	Racking/Twisting	0 to 10mm	10 to 30mm	>30mm
Red	<ul> <li>Severe observed ground surface subsidence, and/or;</li> <li>Small cracks from ground oscillations (&lt;50 mm) may be present, but little to no vertical displacement across cracks, and,</li> <li>Limited evidence of lateral movement.</li> <li>Moderate to major lateral spreading</li> </ul>	Tilting	0 to 20mm	20 to 50mm	>50mm
	<ul> <li>(&lt;1 m cumulative), and/or,</li> <li>Large cracks extending across the ground surface, with horizontal and/or vertical displacement (&gt;50 mm, but generally &lt;200 mm).</li> <li>Ejection of liquefied material at the ground surface may also be observed</li> </ul>	Discontinuous Foundation	0 to 10mm	10 to 20mm	>20mm
Black	<ul> <li>Extensive lateral spreading (≥1 m cumulative), and/or,</li> <li>Large open cracks extending through the ground surface, with very severe horizontal and/or vertical displacements (≥200 mm), and,</li> <li>Ejection of liquefied material at the ground surface may also be observed.</li> </ul>	Global Settlement	0 to 50mm	50 to 100mm	>100mm

Figure 1. Land damage and residential dwelling foundation damage categories and criteria

The land damage mapping was based on the observed surface expression of liquefaction caused by each major earthquake. Between each of the events the ejected liquefied material was removed and major cracks filled. Therefore, the qualitative land damage mapping recorded the incremental effects of each earthquake. Conversely the residential foundation deformation dataset was based on

assessments of the cumulative observed foundation deformation, for all previous events prior to the inspection date. Figure 1 summarises the criteria for the different land damage and dwelling damage categories<sup>1</sup>.

The land damage mapping was supplemented by an extensive geotechnical site investigation programme comprising boreholes, cone penetration testing, geophysical testing and the construction and monitoring of piezometers<sup>2</sup>. The CPT soundings have been used as the primary tool to predict where liquefaction occurred in the soil profile and to derive parameters representing liquefaction vulnerability. The CPT locations in eastern Christchurch are shown in Figure 2, overlain on the worst land damage observed following the third of the destructive earthquakes in the sequence (13 June 2011). The colour categories are defined in Figure 1). Figure 2 also shows the location of piezometers used in the development of a regional upper unconfined groundwater model. The ground water model was used to determine the hydrostatic groundwater pressures which were used in the liquefaction triggering analyses from each CPT to calculate various vulnerability parameters. The parameters calculated from each CPT are discussed in Section 4 below.



Figure 2. Investigation locations in eastern Christchurch and the worst observed residential land damage after 13 June 2011

The foundation damage to dwellings has been compiled into a database and the worst overall severity from the seven assessed damage categories (defined in Figure 1) was plotted on a map (refer to Figure 3 below). Comparison of Figures 2 and 3 show visually that areas with a high density of major foundation damage coincide with areas of more severe observed land damage. By contrast, areas with

<sup>&</sup>lt;sup>1</sup>Following the September 2010 earthquake, the light and dark orange categories were mapped as a combined 'orange' category.

<sup>&</sup>lt;sup>2</sup> All geotechnical site investigation data is available through the Canterbury Earthquake Recovery Authority geotechnical database (<u>https://canterburygeotechnicaldatabase.projectorbit.com</u>). The site investigation data in the in the database were provided courtesy of the New Zealand Earthquake Commission and Christchurch City Council.

a low density of foundation damage coincide with areas with none or minor observed land damage. This shows, not surprisingly, that land damage resulting from the triggering of liquefaction and foundation deformation damage of residential dwellings are strongly correlated.



Figure 3. Assessed residential dwelling foundation deformation damage after 13 June 2011

# 4. VULNERABILITY PARAMETERS

For each CPT, the following three vulnerability parameters were calculated based on the Idriss and Boulanger (2008) liquefaction triggering method:

- 1. Cumulative thickness of liquefaction (CTL)
- 2. Liquefaction Potential Index (LPI)

Calculated in accordance with Iwasakai (1982), Where W(z) = 10 - 0.5z, and  $F_1 = 1$ -FS for FS<1.0 and  $F_1 = 0$  for FS>1.0. z is the depth below the ground surface.

$$LPI = \int_{0}^{20} F_1 W(z) \, dz \tag{4.1}$$

3. Liquefaction Severity Number (LSN)

A new parameter developed to represent vulnerability and discussed in more detail below, where  $\varepsilon_v$  is the calculated volumetric consolidation strain in the subject layer and z is the depth below the ground surface

$$LSN = \int \frac{\varepsilon_v}{z} dz \tag{4.2}$$

Iwasaki's LPI was the first published index for assessing the vulnerability of land subjected to liquefaction effects. LPI provides a simple method for assessing the vulnerability of sites, with published ranges indicating the severity of liquefaction. Sites with an LPI of more than 5 have a high liquefaction risk and more than 15 indicates very high risk (Iwasaki, 1982).

A sensitivity study of LPI as a function of PGA for the CPT soundings undertaken in Christchurch

indicates that LPI responds in a broadly linear manner as PGA increases, which is shown graphically for a single CPT in Section 5. The nature of the LPI calculation means that potentially liquefiable layers only contribute to the LPI when their calculated FoS falls below 1.0, and that the lower the FoS, the higher the contribution.

LPI values have been computed for each of the major earthquake events based on the interpolated seismic demand at each location. These computed LPI values have been plotted against the corresponding observed land damage and dwelling foundation damage datasets. The results are presented in Section 6. The results show that the observed land damage and foundation damage increases linearly with increasing LPI for each of the earthquake events. However, the slope is different for each event which indicates that correlation between LPI and land damage or foundation damage is event specific and produces an inconsistent response across the three events.

In this study we have attempted to provide a better indicator of vulnerability, the Liquefaction Severity Number. This is an extension of the philosophy of the LPI, and represents intensity of liquefaction using volumetric densification strain as a proxy, with depth weighting by a hyperbolic (1/z) rather than a linear reduction. The intent was that the LSN should more heavily weight the contribution to ground surface damage from soil layers which liquefy closer to the ground surface compared to those layers that liquefy at greater depths. This inference was supported by general observations during the rapid land damage mapping work, particularly the observation that ejection of liquefied material tended to result in significant differential settlements, and the results of the site investigation programme. The use of volumetric densification (Zhang, Robertson Brachman, 2002) as a proxy for prediction of *consequential* liquefaction has two benefits:

- 1. The contribution starts as excess pore pressures rise when FoS < 2.0, and include smooth transitions when FoS < 1.0, and,
- 2. The maximum consequence contribution of a soil layer which liquefies is limited by the calculated strain which trends to a limiting value depending on the initial state of the soil as represented by CPT tip resistance.

It should be noted, that both the Ishihara and Yoshimine (1992) and Zhang, Robertson Brachman (2002) volumetric densification methods were assessed. However, in this paper, we have only provided the results of the Zhang, Robertson Brachman (2002) method as it provides a better fit to the damage data sets discussed in Section 6.

# **5. SINGLE CPT**

A single CPT from Richmond, Christchurch (location shown in Figure 2) has been presented for illustrative purposes in this paper (Figure 4). This is a representative CPT that has been assessed using the seismic demand from the 4 September 2010, 22 February and 13 June 2011 earthquake events and uses the corresponding groundwater depth immediately prior to each event, based on the regional groundwater model. The results show the relationship between CSR and CRR with depth, the calculated FoS with depth and the calculated volumetric strains with depth. Figure 5 then shows the contribution of each layer to the selected calculated vulnerability parameters described in Section 4. Figure 5 also shows the difference in contribution from shallower layers for LSN compared to LPI.

Figures 4 and 5 show that the 23 February 2011 earthquake event caused the greatest amount of liquefaction triggering (and the highest CTL, LPI and LSN values) in the soil profile at the Richmond site, and the least amount of liquefaction triggering (and the lowest CTL, LPI and LSN values) from the 4 September 2010 event. This is consistent with the observed severity of land and dwelling damage caused by each event. The response of calculated parameters: crust thickness, CTL, LPI and LSN to variations in PGA are presented in Figure 6. It shows that there is a particular sensitivity in the PGA range of 0.15g to 0.25g, where layers within the CPT trace begin to drop below a FoS = 1.. Below 0.1g, LSN is insensitive to PGA and below 0.15g CTL and LPI are insensitive to the PGA. We

note that CTL and LPI continue to increase with increasing PGA, but the rate of increase in LSN steadily decreases with increasing PGA due to the strain limiting effect when FoS reaches a specified value.



**Figure 4.** Liquefaction triggering analyses from CPT-RCH-17 with depth for the seismic demand from the 4 September 2010, 22 February and 13 June 2011 earthquake events.



Figure 5. Site response at CPT-RCH-17 of calculated liquefaction vulnerability parameters with depth for the seismic demand from the 4 September 2010, 22 February and 13 June 2011 earthquake events, normalised to the calculated February 2011 event values



Figure 6. Site response at CPT-RCH-17 of calculated parameters to varying PGA for a magnitude 6.3 earthquake and pre 23 February 2011 ground water levels.

The LPI and LSN values have been compared to the nearby damage attributes measured around Canterbury, and a detailed assessment of these results will be the subject of future publications. However, preliminary results from these analyses are presented in Section 6.

### 6. CORRELATION OF DAMAGE DATA WITH LPI AND LSN

This section presents a brief introduction to the data analyses that are currently in progress and provides some selected preliminary results. The available CPT traces (around 1,500) have been analysed using inputs based on the interpolated seismic demand from the 4 September 2010, 22 February and 13 June 2011 strong motion records<sup>3</sup> and the pre-earthquake ground water levels. These inputs were used to calculate parameters for each CPT using the different triggering methods presented in Section 2, combined with deformation calculations from Zhang, Robertson and Brachman (2002, 2004) and Ishihara and Yoshimine (1992). The calculated parameters have been plotted against measured land damage, dwelling damage and measured settlement (derived from LiDAR survey data obtained after each event) for residential sites within 40 m of each CPT. This typically generated datasets of between 5,000 and 10,000 points, depending on the earthquake event considered and the number of available measured / observed damage attributes<sup>4</sup>.

The results were initially used to prepare a plot similar to that of Youd and Garris (1995), using a two layer system (H<sub>1</sub> plotted against H<sub>2</sub>). Practically this was difficult to apply because the crust thickness and the thickness of the underlying liquefying soil layer, H<sub>2</sub>, are difficult to define in a soil profile where there is often more than one liquefying soil layer sandwiched between non liquefying layers. By assuming  $CTL = H_2$ , the data indicated that almost all H<sub>1</sub> crust thicknesses were typically lower than Youd and Garris' required crust thickness to protect against surface liquefaction effects. Therefore, sites with both surface expression and no surface expression of liquefaction plot on top of each other with no clear dividing line for the Christchurch damage datasets.

<sup>&</sup>lt;sup>3</sup> All strong motion data utilised in this study were recorded by the New Zealand GeoNet (<u>www.geonet.org.nz</u>) <sup>4</sup> All land damage observations are available through the Canterbury Earthquake Recovery Authority geotechnical database (<u>https://canterburygeotechnicaldatabase.projectorbit.com</u>). The land damage observations in the database were provided courtesy of the New Zealand Earthquake Commission.

Figure 7 below shows preliminary comparison of the observed land damage and observed residential dwelling foundation deformation damage data, plotted against the calculated parameters LPI and LSN for the September 2010, February 2011 and June 2011 earthquakes as a series of box and whisker plots. The criteria for the damage categories are defined in Figure 1 and the geospatial distribution of the damage categories is shown in Figures 2 and 3. The results show that there are generally strong correlations with the two vulnerability parameters for the different damage categories.



**Figure 7.** Box and whisker plots of LPI and LSN based on observed land damage and foundation deformation damage for the 4 September 2010, 22 February and 13 June 2011 earthquake events. For the land damage categories, 1 = blue, 2 = green, 3 = light orange, 4 = dark orange, 5 = red and 6 = black (as defined in Figure 1). For the foundation deformation damage categories, refer to Figure 1.

A preliminary statistical data analysis of the results indicates that both LPI and LSN correlate with measured damage to land and house foundations. For the LPI parameter however, the slope is different for each event which indicates that the LPI correlation with land damage and foundation damage is event specific and produces inconsistent responses to the three events. The LSN parameter on the other hand appears to provide a more consistent fit to the measured land damage over the three different events considered, where a range of seismic shaking levels are experienced. This preliminary conclusion will be further tested and reported in more detail in subsequent publications.

Observation of crust quality has been observed to be other important variable influencing whether liquefaction is likely to be consequential. This variable has not been considered in both the existing published methods (e.g. LPI) and the proposed LSN vulnerability parameter. This variable may account for some of the observed variability in the datasets.

### 7. SUMMARY AND CONCLUSIONS

A preliminary statistical data analysis indicates that LPI correlates well with the observed damage to land and house foundations. However, the slope is different for each event which indicates that the LPI correlation with land damage and foundation damage is event specific and produces inconsistent

responses to the three events. It is also noted that LPI values greater than 5 were calculated for which land and dwelling damage was recorded, which is outside of the ranges published by Iwasaki (1982). This is potentially because the correlation of LPI to damage is event and location specific.

The LSN vulnerability parameter more heavily weights the contribution to ground surface damage from soil layers which liquefy closer to the ground surface compared to those layers that liquefy at greater depths. In addition the LSN vulnerability parameter becomes independent of PGA beyond a specific level depending on the initial density state of the soil. Preliminary analyses indicate that for the range of seismic demands imposed by the recent earthquakes, the LSN vulnerability parameter provides a more consistent correlation with the available damage datasets for all events. This preliminary conclusion will be further tested and reported in more detail in subsequent publications.

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