Evaluation of maximum velocity for inelastic structures with supplementary dampers

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SUMMARY:
The maximum velocity is a significant structural parameter, especially for the seismic design of structures with supplementary dampers. This study proposes a simple and effective method to estimate the maximum velocity of inelastic single-degree-of-freedom (SDOF) systems subjected to strong earthquakes. The paper defines and computes the inelastic velocity ratio (IVR), i.e. the ratio of the maximum inelastic to the maximum elastic velocity of a SDOF system where its knowledge allows the computation of maximum inelastic velocity directly from the corresponding elastic counterpart. Extensive parametric studies are conducted to obtain expressions for IVR, in terms of period of vibration, viscous damping ratio, force reduction factor as well as of soil class.

Keywords: maximum velocity; inelastic structures; supplementary dampers

1. INTRODUCTION

An effective way to control the post-earthquake performance of structures is the installation of passive energy dissipation devices (*PEDD*). The role of adopting these devices is to provide supplemental damping and convert a part of the input seismic energy to heat. Typical examples of *PEDD* are the viscous fluid dampers, which is one of the most common types of dampers today and consist of a piston filled with viscous fluid where its movement causes energy dissipation [Makris and Constantinou 1991, Symans et al. 2008]. The output force of viscous damper is directly related to the velocity and usually possesses only viscosity. In this case, the structural response due to earthquake motions is reduced by the dissipation of the major part of earthquake input energy. Generally, the use of *PEDD* leads to reduced displacement structural response. Nevertheless, nonlinear time history analysis is also required for the majority of passively damped civil structures since their earthquake vibration induces inelastic deformations in one or more structural elements (Xu et al. 2003, Goel 2004).

One key parameter in the design of a structure with supplemental dampers is the design (maximum) velocity, which is generally based on elastic pseudo-velocity spectra as proposed by modern seismic codes (e.g., FEMA450, 2003). Thus, the actual velocity is assumed to be equal to the elastic counterpart, assuming an 'equal velocity rule', similar to the well-known 'equal displacement rule' which correlates the maximum elastic with the maximum inelastic displacement. However, taking into account that the inelastic behaviour appears to be unavoidable both for the structures with and without supplementary dampers, the elastic velocity spectra lead to different velocities in comparison with the actual ones, and therefore these spectra cannot be used. In order to confirm this discrepancy, one could define the inelastic velocity ratio (*IVR*) as the ratio of the maximum inelastic to maximum elastic velocity of a single-degree-of freedom (*SDOF*) system. Examining two typical strong ground motions, Fig. 1 shows the corresponding *IVR* spectra for force reduction factor R=4 and viscous damping ratio ξ=5%. This figure clearly shows that the assumption of 'equal velocity rule' generally overestimates the maximum velocity and therefore the design damping force, leading to overstated energy dissipation with fictitious seismic performance level and to overdesigned dampers.

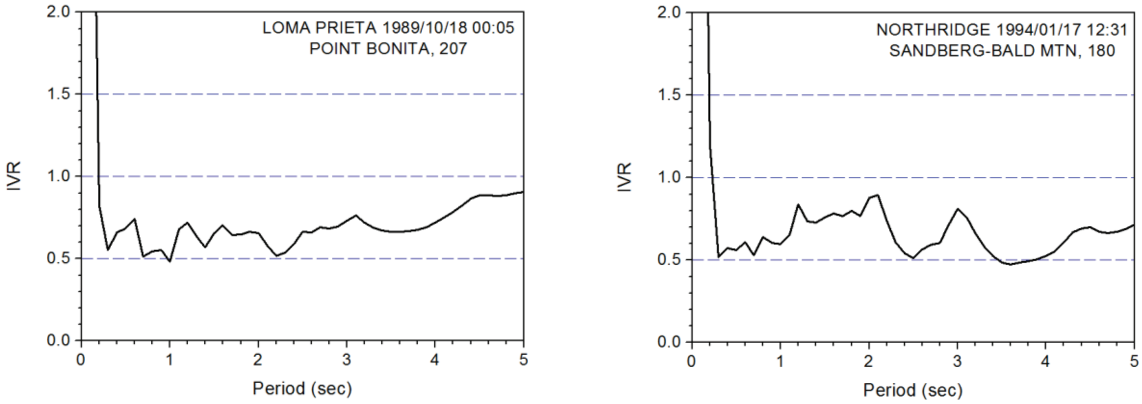

Figure1. Inelastic velocity ratio for characteristic strong ground motions and for R=4.

The assumption of *IVR* = 1.0, as usually held by modern seismic codes, is not necessarily a conservative assumption. This can be confirmed through the investigation of the total energy of a structural system. According to Uang and Bertero (1990), the absolute energy equation can be expressed by

$$E_I = E_k + E_s + E_h + E_d \tag{1.1}$$

where E_I is the earthquake input energy, i.e., the energy demand by the earthquake ground motion on the structure (Hwang, 2002). Furthermore, the right hand side of Eqn. 1.1 represents the energy capacity or supply of the structure, which has to do with the kinetic energy, E_k, the recoverable elastic strain energy, E_s, the irrecoverable hysteretic energy, E_h, and the energy dissipated by the inherent structural damping capability and/or the supplemental viscous dampers, E_d. It is apparent that the overestimation of actual velocity leads to overestimation of dissipated energy E_d. According to Eqn. 1.1 and for specific earthquake input energy, this overestimation causes underestimation, at least, for one of the other three energy terms on the right hand side of Eqn. 1.1. It is found that the aforementioned underestimation has mainly to do with the irrecoverable hysteretic energy, E_h, and therefore with the structural damage level. Therefore, the assumption of IVR = 1.0 creates a discrepancy that does not appear to be acceptable and safe for many structural systems.

The nonlinear time history analysis leads to reliable estimation of actual velocities reducing the aforementioned shortcoming. However, this approach appears to be complicated for the everyday engineering practice due to the increased computational effort. This paper proposes an alternative for SDOF systems to evaluate the actual velocity in a straightforward and effective manner. More specifically, this study constructs empirical expressions to estimate the *IVR*, where the knowledge of this ratio allows the computation of maximum inelastic velocity directly from the corresponding elastic one. This approach is quite similar to the estimation procedure and philosophy of 'inelastic displacement ratio', i.e., the ratio of the maximum inelastic to maximum elastic displacement for SDOF systems (Hatzigeorgiou and Beskos 2009). The proposed method is general and can be applied both to structures with or without supplementary dampers. Extensive parametric studies are conducted to obtain the empirical expressions for this ratio, in terms of the period of vibration, the viscous damping ratio, the force reduction factor and the soil class.

2. DESCRIPTION OF THE MODEL

An elastic-perfectly plastic (EPP) SDOF system with viscous damping is used to model the structural behaviour both for conventional structures and structures with supplementary dampers, where the effective (inherent + supplemental) damping can be used. According to Seleemah and Constantinou (1997), the behaviour of viscous fluid dampers can be appropriately expressed by the following relation

$$F_d(t) = c \cdot |\dot{u}(t)|^a \cdot sgn(\dot{u}(t)) \tag{2.1}$$

where $F_d(t)$ is the damping force, a an exponent whose value is determined experimentally and *sgn* the signum function. The physical model corresponding to Eqn. 2.1 is a nonlinear viscous dashpot. For earthquake engineering applications, the exponent a typically varies between 0.5-2.0 (FEMA 274, 1997) where the value a=1.0 corresponds to the case of a linear viscous dashpot. In preliminary analysis and design stages, the velocity exponent α=1.0 is recommended for simplicity. Moreover, according to Martinez-Rodrigo and Romero (2003), as the dampers behaviour approaches to the linear one ($\alpha = 1$), the structural performance improves, in terms of floor maximum accelerations, peak inter-story drifts and permanent story drifts. For these reasons, this paper focuses on the linear viscous dampers. The maximum damping force, $F_{d,max}$, is given by

$$F_{d,max} = c \cdot \dot{u}_{max} \tag{2.2}$$

where $\dot{u}_{\max}$ is the maximum velocity. Since the design of dampers of passively damped structures requires the knowledge of maximum damping force, it is important to reliably evaluate the maximum velocity. In the following, an *EPP*-SDOF system (see Fig. 2) with initial stiffness k_{el}=10000kN/m, period T=1.0sec and effective viscous damping ratio ξ=10.0% is seismically excited by the Imperial Valley earthquake (NS component, El Centro 1940). Considering various values of force reduction factors, Fig. 2a shows that the maximum displacement closely follows the 'equal displacement rule'. However, Fig. 2b demonstrates that the maximum velocity is drastically reduced as the R-factor increases, leading to the rejection of 'equal velocity rule'.

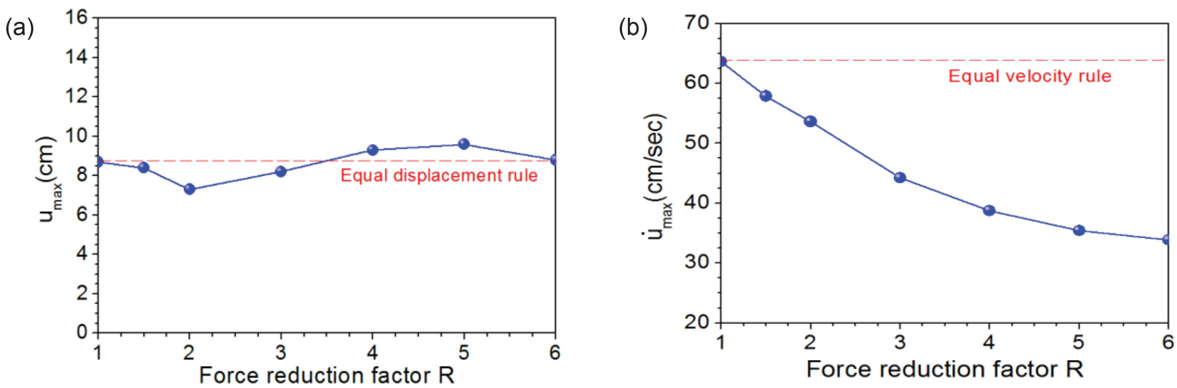

Figure 2. (a) Maximum displacement, and (b) maximum velocity

Furthermore, the maximum damping force, which is used to design the corresponding supplementary dampers or to evaluate the seismic performance of passively damped systems, is greatly influenced by the R-factor as clearly shown in Fig. 3.

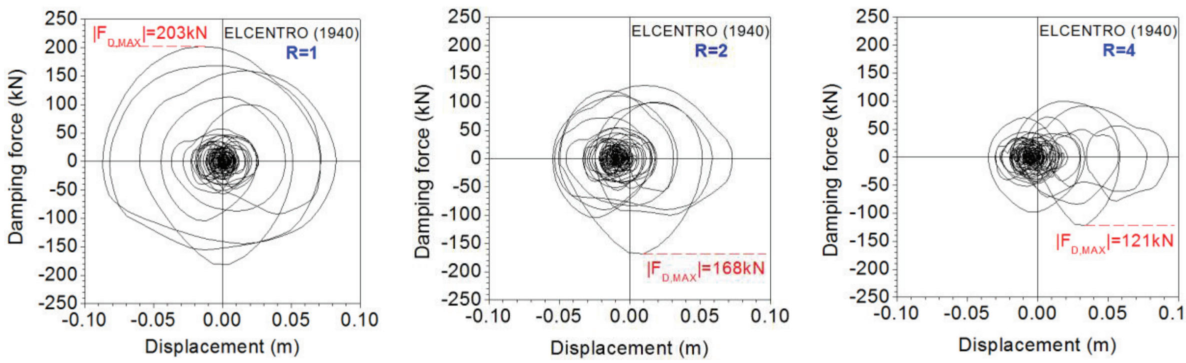

Figure 3. Damping force - displacement diagrams for R=1, 2 and 4

It should be noted that despite the apparent reduction of maximum damping force for R>1, the pertinent provisions of modern codes ignore this fact. One can mention here, among others, the provisions of FEMA-450 (2003) for supplementary dampers (Chapter 15) where the '*design earthquake story velocity*' is allowed to be evaluated by the '*design earthquake story displacement*', using the elastic pseudo-velocity spectrum.

3. SEISMIC INPUT

The strong ground motion database examined in this study constitutes a representative number of far-fault earthquakes from a variety of tectonic environments. Thus, a total of 400 records were selected to cover a range of frequency content, duration, and magnitude. The assembled database can be divided in 4 sub-datasets, which are recorded at sites ranging from hard rock to soft soil conditions according to the definitions of the Eurocode 8 (2005) site classification system [27], i.e., 4 groups of 100 accelerograms for soil type A, B, C and D. The examined 400 strong ground motions, which were downloaded from the strong motion database of the PEER Center (2012), have been recorded during the action of the following earthquakes: Parkfield (1966), Lytle Creek (1970), San Fernando (1971), Point Mugu (1973), Friuli, Italy (1976), Coyote Lake (1979), Imperial Valley (1979), Norcia, Italy (1979), Livermore (1980), Mammoth Lakes (1980), Victoria, Mexico (1980), Irpinia, Italy (1980), Coalinga (1983), Morgan Hill (1984), Mt. Lewis (1986), N. Palm Springs (1986), Chalfant Valley (1986), Whittier Narrows (1987), Loma Prieta (1989), Sierra Madre (1991), Cape Mendocino (1992), Big Bear (1992), Northridge (1994), Kobe, Japan (1995), Kocaeli, Turkey (1999), Chi-Chi, Taiwan (1999), Duzce, Turkey (1999), Yountville (2000) and Denali, Alaska (2002).

4. THE INELASTIC VELOCITY RATIO

4.1 Evaluation of Inelastic Velocity Ratio

This section examines an appropriate empirical expression for the IVR. Thus, for each earthquake record, the period of the SDOF system is increased from 0.1 to 10.0 sec with an increment of 0.1 sec (i.e., 100 values of period) and the force reduction factor is assumed to increase from 1.0 to 8.0 with an increment of 0.5 (i.e., 15 values of R factors). Thus initially, 600,000 analyses are examined: (400 ground motions) × (100 periods, T) × (15 force reduction factor, R). A comprehensive nonlinear regression analysis is then carried out on the basis of the data obtained by these analyses. The relation of inelastic velocity ratio versus the structural period and force reduction factor is regressed for the series of the aforementioned analyses and the following empirical expression for IVR–R–T is adopted

$$IVR(R,T) = 1 + (R-1)^{c_1}\left(\frac{c_2}{T^{0.9}} + \frac{c_3}{T}\right) \tag{4.1}$$

Eqn. 4.1 is one of the simplest equations that successfully described the numerical data following downward and upward concave curves, obtained by *Table Curve 3D* (2002) program after testing about 8000 mathematical equations. Examining the influence of soil types and viscous damping ratio on IVR, appropriate parameters c_1-c_3 can be adopted from Tables 1-6, where the Pearson's correlation coefficient, ρ^2, and the standard deviation, σ, are also given.

It is found that soil conditions and effective damping ratio affect the c_1-c_3 coefficients, and consequently the IVR. For example, Fig. 4 examines the IVR of an SDOF system with viscous damping ratio ξ=5% and force reduction factor R=4.0, both for the exact dynamic inelastic analyses and the proposed method. This system is subjected to the whole sample of earthquakes examining separately the groups of each soil type. It is evident that the soil type moderately affects the inelastic velocity ratio, where the soft soil (Soil D) generally leads to slightly lower IVR values in comparison with the other types of soils. Thus, for practical reasons, the influence of soil types on IVR can be ignored, taking into account results from the whole sample of records, i.e., for unacquainted site class.

Table 1. *IVR* parameters for ξ=5%

Parameter	c_1	c_2	c_3	σ	ρ^2
Soil A	0.32545	-1.33088	1.14553	0.150	0.978
Soil B	0.42522	-1.14146	0.97377	0.139	0.974
Soil C	0.38412	-1.28741	1.10136	0.152	0.976
Soil D	0.30898	-2.01978	1.75385	0.234	0.963
Total sample	0.35069	-1.43800	1.23579	0.163	0.987

Table 2. *IVR* parameters for ξ=10%

Parameter	c_1	c_2	c_3	σ	ρ^2
Soil A	0.41536	-1.20327	1.06330	0.184	0.984
Soil B	0.48799	-0.97654	0.84720	0.145	0.974
Soil C	0.47329	-1.07350	0.93697	0.164	0.971
Soil D	0.48748	-1.67075	1.49082	0.297	0.986
Total sample	0.45772	-1.23279	1.08484	0.192	0.987

Table 3. *IVR* parameters for ξ=20%

Parameter	c_1	c_2	c_3	σ	ρ^2
Soil A	0.54337	-0.86122	0.77105	0.171	0.978
Soil B	0.49700	-0.74667	0.65135	0.118	0.956
Soil C	0.63979	-0.65098	0.57221	0.130	0.965
Soil D	0.53087	-1.22179	1.09486	0.236	0.989
Total sample	0.54494	-0.87169	0.77337	0.161	0.986

Table 4. *IVR* parameters for ξ=30%

Parameter	c_1	c_2	c_3	σ	ρ^2
Soil A	0.51489	-0.69615	0.62262	0.133	0.974
Soil B	0.56408	-0.56396	0.49285	0.099	0.949
Soil C	0.52435	-0.58544	0.51202	0.098	0.959
Soil D	0.58609	-0.93579	0.84019	0.197	0.987
Total sample	0.55136	-0.69468	0.61634	0.130	0.984

Table 5. *IVR* parameters for ξ=40%

Parameter	c_1	c_2	c_3	σ	ρ^2
Soil A	0.58838	-0.54465	0.48927	0.119	0.961
Soil B	0.69231	-0.41918	0.36840	0.090	0.952
Soil C	0.52401	-0.49572	0.43442	0.084	0.956
Soil D	0.62073	-0.75412	0.67590	0.165	0.981
Total sample	0.60285	-0.55529	0.49353	0.112	0.981

Table 6. *IVR* parameters for ξ=50%

Parameter	c_1	c_2	c_3	σ	ρ^2
Soil A	0.55633	-0.51607	0.46563	0.112	0.958
Soil B	0.64797	-0.39350	0.34680	0.083	0.920
Soil C	0.61809	-0.39138	0.34410	0.077	0.945
Soil D	0.56488	-0.67865	0.60558	0.133	0.981
Total sample	0.59042	-0.49497	0.44038	0.100	0.978

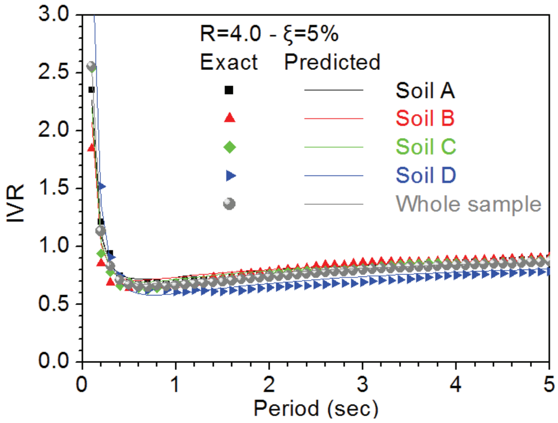

Figure 4. Influence of soil types on *IVR*

Additionally, the influence of viscous damping ratio on *IVR* is examined in Fig. 5, both for the exact dynamic inelastic analyses and the proposed method. It is obvious that the viscous damping ratio strongly affects the *IVR* where the lower the damping ratio, the higher the *IVR*.

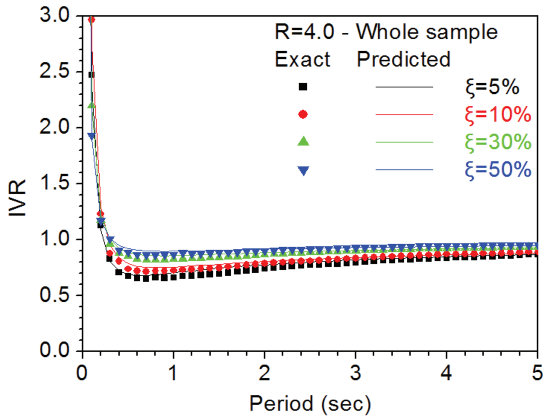

Figure 5. Influence of viscous damping on *IVR*

Finally, an EPP-SDOF system with viscous damping ratio ξ=5% is examined to evaluate the influence of force reduction factors on *IVR*. This system is subject to the whole sample of records corresponding to soil type A. Three specific values of force reduction factor are considered, i.e., *R*=2.0, 4.0 and 8.0. Figure 6 shows that the force reduction factor period strongly influences the *IVR*, where the higher the *R* factor, the smaller the *IVR*

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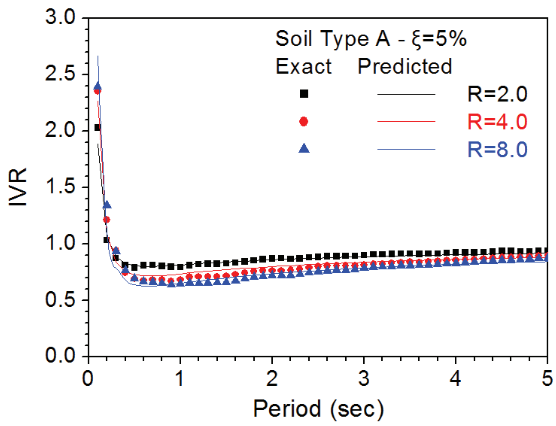

Figure 6. Influence of force reduction factor on *IVR*

4.2 Satisfaction of fundamental conditions

The effectiveness of the proposed method is examined in the previous section. It should be noted that in order to achieve an effective model, some rational fundamental conditions ought to be satisfied. Thus, the proposed empirical Eqn. 4.1 satisfies the following fundamental condition

$$IVR(R,T) \overset{T\to 0}{=} \infty \tag{4.2}$$

which means that, irrespectively of the value of *R*, very stiff structures should be designed elastically. This is because the yield displacement u_y tends to zero and even a small decrease of strength that ought to retain the structure in the elastic range, leads to a very large inelastic displacement and velocity. Therefore, very stiff structures should be designed as absolutely elastic systems. Furthermore, Eqn. 4.1 also satisfies the fundamental condition

$$IVR(R,T) \overset{R\to 1}{=} 1 \tag{4.3}$$

This obvious condition indicates that structures behaving almost elastically, their velocities should be identical with the counterpart of elastic structures. Finally, very flexible systems develop identical inelastic and elastic relative velocities where both of them are almost identical with the ground velocity. It should be noted that according to this thought, Eqn. 4.1 also satisfies the corresponding fundamental condition

$$IVR(R,T) \overset{T\to \infty}{=} 1 \tag{4.4}$$

7. CONCLUSIONS

This paper develops a new method for the reliable evaluation of actual velocities of elastic-perfectly plastic SDOF systems under strong earthquakes. The method is based on the computation of inelastic velocity ratio which is defined as the ratio of the maximum inelastic to the maximum elastic velocity of a SDOF system. Knowledge of this ratio allows the computation of maximum inelastic velocity directly from the corresponding elastic counterpart. This approach is simple, straightforward and leads to reliable results for SDOF systems without increased computational cost. The influence of period of vibration, of soil type, of force reduction factor and of viscous damping ratio on *IVR* is taken into account. A large number of inelastic time-history analyses were carried out to study these influences using many single-degree-of-freedom models excited by 400 far-fault records that have been recorded during numerous strong earthquakes. A detailed nonlinear regression analysis is carried out to provide simple empirical expressions for the inelastic velocity ratio, satisfying simultaneously fundamental conditions of dynamic inelastic analysis. It is found that the effective viscous damping ratio and the adopted forced reduction factors strongly affect the inelastic velocity ratio. On the other hand, local soil conditions moderately influence the *IVR* and for practical reasons, the influence of soil types on *IVR* can be ignored, taking into account results and findings from the whole sample of records, i.e., for unacquainted site class. The proposed approach appears to be useful both for traditional structures and structures with supplementary dampers.

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