V_{S30} – A Site-Characterization Parameter for Use in Building Codes, Simplified Earthquake Resistant Design, GMPEs, and ShakeMaps

Roger D. Borcherdt U.S. Geological Survey, Menlo Park, CA USA



SUMMARY: (10 pt)

 V_{S30} , defined as the average seismic shear-wave velocity from the surface to a depth of 30 meters, has found wide-spread use as a parameter to characterize site response for simplified earthquake resistant design as implemented in building codes worldwide. V_{S30} , as initially introduced by the author for the US 1994 NEHRP Building Code, provides unambiguous definitions of site classes and site coefficients for site-dependent response spectra based on correlations derived from extensive borehole logging and comparative ground-motion measurement programs in California. Subsequent use of V_{S30} for development of strong ground motion prediction equations (GMPEs) and measurement of extensive sets of V_S borehole data have confirmed the previous empirical correlations and established correlations of V_{S30} with V_{S2} at other depths. These correlations provide closed form expressions to predict V_{S30} at a large number of additional sites and further justify V_{S30} as a parameter to characterize site response for simplified building codes, GMPEs, ShakeMap, and seismic hazard mapping.

Keywords: Shear velocity, V_{S30} , site coefficients, site response, site-specific design

1. INTRODUCTION

 V_{S30} , defined as the average seismic shear-wave velocity from the surface to a depth of 30 meters has found wide spread use as a parameter to characterize local site response for a wide variety of applications ranging from simplified earthquake resistant design procedures in building codes to regional and global seismic hazard mapping. Correlations with other local site characteristics including V_{Sz} measured to other depths have shown V_{S30} to be a robust parameter for characterizing local site response for many applications.

 V_{S30} is rigorously defined as the shear velocity to a depth of 30 m as inferred from the travel time (tt_{S30}) required for a shear wave to travel from the surface to a depth of 30 m or vice versa, namely

$$V_{S30} \equiv 30m/tt_{S30}.$$
 (1.1)

Equivalently, if the shear-wave velocities v_{s_i} are known for each of the intervening layers of thickness d_i between the surface and a depth of 30 m, V_{S30} may be defined as it is in section 20.4.1 of the ASCE 7-10 (2010) version of the current US building code, namely,

$$V_{S30} \equiv \bar{v}_s \equiv \sum_{i=1}^n d_i \left/ \sum_{i=1}^n \frac{d_i}{v_{si}} \right.$$
(1.2)

 V_{S30} was initially introduced (Borcherdt, 1992, 1994) to provide unambiguous definitions of site classes and site coefficients for the estimation of site-dependent response spectra for use in the 1994 edition of the "Recommended National (US) Earthquake Hazard Reduction Program (NEHRP) Building Code Provisions". These recommendations were based on correlations derived from extensive borehole logging and comparative ground-motion measurement programs in California (e.g., Borcherdt et al., 1976; Gibbs et al., 1975; Fumal 1978). Subsequently, shear-wave velocity measurements and inferences of V_{S30} at a large number of additional sites for which recent strong-motion data have been collected, especially in Japan and Turkey, have provided an extensive new data base for the development of strong ground-motion prediction equations (GMPEs) and comparative strong-motion studies for a large number of additional sites. These data provide a large additional data base to further evaluate the robustness of V_{S30} as a parameter to characterize site response for use in certain applications.

This paper reviews the theoretical and empirical basis for V_{S30} as a parameter to characterize site response for use in building codes and simplified earthquake resistant design procedures. It provides closed form expressions relating short- and mid-period amplification factors F_a and F_v to V_{S30} and V_{Sz} at other depths. It reviews correlations of V_{S30} with other local site characteristics such as physical properties, geologic age, and topographic slope for purposes of mapping geographic variations in ground response. It describes the option in building codes to use a detailed shear velocity profile to provide a detailed characterization of site response as an alternative to V_{S30} , when appropriate.

2. $V_{\rm S30}\,{\rm AS}\,{\rm A}\,{\rm BASIS}$ for characterization of site-specific site response

The theory for 2D and 3D wave propagation in layered viscoelastic media provides a general theoretical basis for site coefficients as proposed in current building code provisions. A simple theoretical model for site response is that of a homogeneous Type-II S or SH wave incident at the base of a viscoelastic soil layer. Closed form solutions for the problem (Borcherdt, 2009) permit the response to be calculated as a function of material parameters of the soil and underlying rock and parameters of the incident wave. The model, if only evaluated for a normally incident homogeneous Type-II S wave, provides results that correspond to the 1D response as calculated for an incident SH wave using the SHAKE program (Schnabel et al., 1972) with material parameters of damping ratio, shear wave velocity or shear modulus and density chosen to correspond to the appropriate input ground motion level. Corresponding layer responses, calculated as a function of period as normalized by the fundamental period of the site and the ratio of the soil and rock shear-wave velocities for damping ratios of 5 % and 20 % are shown in Fig. 2.1. The site-class boundaries as defined in terms of V_{S30} (Borcherdt, 1994) can be shown on the same plots by choosing the shear-wave velocity for rock equal to the empirically implied reference velocity of 1050 m/s used for the NEHRP site coefficients and an approximate density ratio. In addition, the site coefficients F_a and F_v may be illustrated on the same plots as a function of damping ratio or input ground acceleration level. These coefficients, defined as the average of the amplitude response over a short- (0.1-0.5 s) and a mid- (0.4-2.0 s) period band, respectively, are a function of V_{S30} as specified in eqns. 2.1 and 2.2. These simple theoretical estimates for F_a and F_v as a function of shear-wave velocity ratio and damping ratio or base acceleration level (see Fig. 2.1) are in general agreement with those in current building code

provisions as summarized in Fig. 2.2.

The estimates of F_a and F_v as adopted in the code provisions were initially derived empirically using comparative strong-motion recordings of the Loma Prieta earthquake and extrapolated to higher input ground motion levels using laboratory results and numerical SHAKE results (Borcherdt, 1994; Seed, 1994), then later adjusted slightly in committee based on engineering judgment. The equations describing the empirically derived dependence of amplification on V_{S30} are

$$F_a = V_{S30_{ref}} / V_{S30}$$
(2.1)

and

$$F_{v} = V_{S30_{ref}} / V_{S30}^{m_{v}}, \qquad (2.2)$$

where

1) V_{s30} is defined in 1.1 and 1.2,

2) $V_{S30_{ref}}$ is the empirically implied normalization V_{S30} of 1050 m/s at the mid-point of reference NEHRP site class B (see Fig. 2.2),

3) m_a and m_v are defined and depend on the input ground motion level as shown in the legends of Figs. 2.2a and 2.2b (Borcherdt, 1994).



Figure 2.1. Theoretical amplitude response of a soil layer to a vertically incident homogeneous Type II S wave calculated as a function of normalized period, homogeneous S velocity ratio, and damping ratio. The average short- and long-period theoretical response variations are consistent with variations in the F_a and F_v site coefficients implied empirically and incorporated in building code provisions (Borcherdt 1994; NEHRP Provisions, 1994; ASCE 7-10, 2010).

The theoretical results for F_a and F_v in Fig. 2 provide an exact model for the response of a soil layer of thickness 30m. They provide a simple model illustrating the role played by V_{S30} in providing unambiguous definitions of site classes and corresponding estimates of average short- and mid-period amplification factors, F_a and F_v .

Subsequent to the derivation of the empirical curves relating amplification to V_{S30} (Fig. 2.2), the amount of strong-motion data and corresponding set of measurements has increased significantly. This large data base and the resultant Ground-Motion Prediction Equations (GMPEs; Abrahamson, et al., 2008) provide an extensive empirical data base from which to examine the correlation of amplification and V_{S30} . Resultant estimates of F_a and F_v (short- and mid-period averages; SP Avg and MP Avg) derived from the GMPEs of (Abrahamson and Silva, AS; Boore and Atkinson, BA; Campbell and Bozorgnia, CB; Chiou and Youngs, CY) as compiled by Stewart and Seyhan, (2011; pers. commun.) are shown for two acceleration levels (0.1g, 0.4 g) in Fig 2.3. The GMPE results derived by the various investigators are consistent with those derived previously with the largest variations occurring for the softest soils. These differences can be attributed to the GMPE models chosen to account for nonlinear soil behavior by each of the investigators and the limited number of strong-motion recordings on soft soil deposits. Nevertheless, these results based on a large amount of additional data, confirm the previous empirical correlations and provide a large amount of additional empirical information to refine the mathematical description of the correlation between amplification and V_{S30} .



Figure 2.2 Short-period and mid-period site coefficients F_a (a) and F_v (b) expressed as a function of V_{S30} and input base acceleration as proposed for consideration in building code provisions (from Borcherdt, 1992, 1994; Dobry et al., 2000).



Figure 2.3. Short- and mid-period site coefficients F_a (a, b) and F_v (c, d) expressed as a function of V_{S30} and input base acceleration levels of 0.1g and 0.4g as adopted in the NEHRP code provisions and as inferred from the NGA GMPEs (see text for abbreviations, BA,CB,AS,CY) by Stewart and Seyhan (2011, personal commun).

3. V_{S30} CORRELATIONS WITH V_{Sz} AT OTHER DEPTHS

The wide spread use of V_{S30} to characterize site response for use in building codes, has led to the need to infer V_{S30} from measurements of V_{Sz} at other depths. Studies by Boore et al. (2011), Kanno et al. (2006), Cadet and Duval (2009) have shown the existence of well-defined empirical regression relationships between V_{S30} and V_{Sz} at other depths, with the correlation coefficients approaching unity as the depths approach 30m. A simple form of these relationships is

$$Log[V_{S30}] = c_{0z} + c_{1z} Log[V_{Sz}], \qquad (3.1)$$

where c_{0z} and c_{1z} are regression coefficients corresponding to the depth z for which V_{Sz} has been measured. Upon letting $Log[a_{0z}] \equiv c_{0z}$, the empirical dependence of V_{S30} on V_{Sz} also may be written as

$$V_{S30} = a_{0z} (V_{Sz})^{c_{1z}}.$$
(3.2)

Eqn. 3.2 permits the site class boundaries to be expressed in terms of V_{Sz} measured at other depths as shown by Cadet and Duval (2009).

Eqns. 3.1 and 3.2 permit the site coefficients as expressed by Eqns. 2.1 and 2.2 and derived for the NEHRP provisions (Borcherdt, 1994) to be expressed in terms of V_S at other depths z as well. Substitution of Eqn. 3.2 into Eqns. 2.1 and 2.2 and introduction of subscript "*ref*" to distinguish

reference site parameters implies the site coefficients may be written as

$$F_{a} = (a_{0z_{ref}} (V_{Sz_{ref}})^{c_{1z_{ref}}}) / (a_{0z} (V_{Sz})^{c_{1z}})^{m_{a}}$$
(3.3)

and

$$F_{v} = \left(a_{0z_{ref}} \left(V_{Sz_{ref}}\right)^{c_{1z_{ref}}}\right) / \left(a_{0z} \left(V_{Sz}\right)^{c_{1z}}\right)^{m_{v}}$$
(3.4)

Eqns. 3.3 and 3.4 provide a general representation of the short- and mid-period amplification factors as a function of $V_{Sz_{ref}}$ and V_{Sz} , where z_{ref} is not necessarily equal to z. If $z_{ref} = z$, then Eqns. 3.3 and 3.4 simplify to

$$F_{a} = V_{S_{z_{ref}}} / V_{S_{z}}^{c_{1_{z}}m_{a}}$$
(3.5)

and

$$F_{v} = V_{S_{z_{ref}}} / V_{S_{z}}^{c_{1z}m_{v}}.$$
(3.6)

Eqns. 3.5 and 3.6 permit site coefficients, F_a and F_v as specified in the codes as a function of site class and in turn V_{S30} , to also be expressed in terms of site class defined in terms of V_{Sz} at another depth z for which V_S information may be available.

The well-defined correlations of V_{S30} with V_{Sz} as described by Eqns. 3.1 and 3.2 help explain why V_{S30} has been found to be a rather robust parameter for characterizing site response for both purposes of predicting average site response for use in simplified code procedures as well as accounting for ground response in GMPEs (Abrahamson, 2011). For example, even though 30 m may be unequal to the thickness z of the deposit of interest, the correlations of V_{S30} with V_{Sz} implies that V_{S30} can be useful for the prediction of amplification at other depths, albeit with increased uncertainty.

Correlations of V_{S30} with V_{Sz} can be explained physically by the fact that shear-wave velocity profiles, in general, increase with depth due to consolidation of geologic materials associated with increasing over-burden pressure. As a consequence it is reasonable to expect that the time averaged shear velocity at one depth would be correlated with that at another depth and would be a useful parameter to characterize the near-surface seismic response of a site. However, for site response estimates, as might be needed for design of major structures, shear-wave velocity profiles measured at a depth resolution near 1 m throughout the soil layer should be expected to provide more accurate characterization of the site for purposes of developing detailed estimates of site response as a function of period.

4. $V_{\rm S30}\,$ AS A BASIS FOR MAPPING SITE RESPONSE

Maps depicting potential variations in earthquake shaking due to variations in local site conditions are needed for a variety of applications, including seismic zonation maps, seismic hazard and risk maps, and ShakeMaps. V_{S30} was first introduced for mapping regional variations in ground response (Borcherdt, 1991a; Borcherdt et al., 1991b) and later as a basis for defining site classes, (Borcherdt, 1992, 1994), because of its correlation with observed ground-motion amplification and mapped physical properties of geologic units in the San Francisco Bay region as evidenced by both weak- and strong-motion observations from distant nuclear explosions and the Loma Prieta earthquake (Borcherdt and Glassmoyer, 1992). Correlations between V_{S30} and physical properties that can be mapped were developed based on an extensive shear-wave velocity measurement program initially developed and conducted under the supervision of the author by Gibbs and Fumal (see e.g. Gibbs et al., 1976) throughout California. A comprehensive effort to identify characteristics of geologic units that could be used to map units with distinct seismic response was conducted by Fumal (1978) as part of his thesis. Based on detailed shear velocity and physical property logs from a large set of boreholes, the physical properties of grain size and texture for soils and hardness and fracture spacing for rocks provided the strongest correlation with measured shear-wave velocities and hence amplification characteristics of the units that could be mapped in California.

Fig. 3.1 shows the correlation between V_{S30} and physical properties as inferred in about 200 boreholes in California together with the site class boundaries as adopted in the NEHRP and subsequent code provisions (ASCE 7-10). It shows that in general, distinct shear velocity categories exist, namely; 1) Soft soils- Site Class E with fine grained soft clays (Quaternary Holocene bay mud, Qhbm), 2) Stiff clays and Sandy soils – Site Class D with Quaternary younger fine and medium grained (Qyf, Qym) and older fine grained (Qof) soils, 3) Gravelly soils and Soft Rock – Site Class C with Quaternary younger and older coarse (Qyc, Qoc) and older medium grained (Qom) soils and Tertiary sedimentary (Ts) and volcanic (Tv) rock and Franciscan sandstone (KJs) and granite (gr) with close fracture spacing, and 4) Firm to Hard rock – Site Class B with granite and Franciscan sandstone with wide fracture spacing. These correlations were used to define the V_{S30} boundaries for site classes as adopted with slight modification in the 1994 edition of the NEHRP code provisions (Borcherdt, 1994). For mapping purposes the correlations provide a rigorous basis for mapping seismic response provided the physical property information is available on a regional scale as it is for portions of northern and southern California.



Figure 4.1. Physical property (texture, fracture spacing, and age) classification and measured V_{S30} values for USGS boreholes in California showing distinct seismic response units (see text for explanation of notation; Fumal, 1978; Borcherdt, 1994).

For mapping applications in areas for which detailed physical property information is not available more generalized information with less well-defined correlations, such as geologic age (Wills et al., 2000) have been used to define groupings of geologic units with average V_{S30} values with implications for inferences of average amplification. Fig. 4.2 shows V_{S30} values for the boreholes shown in Fig. 4.1 with the sites grouped according to the geologic based site categories as inferred via a GIS from a digital version of the map prepared by Wills et al. (2000). The compilation shows that that the geologic based site classification yields non-distinct site categories with considerable overlap

in V_{S30} values. Nevertheless, Fig. 4.2 shows that the mean V_{S30} value for each of geologic based site categories E, DE, D, CD, C, CB, and B monotonically increases with corresponding amplification values expected to decrease. Maps based on this geologic classification can provide a useful geographic depiction of potential variations in ground response with the understanding that some of the geologic categories overlap and that some sites will be in more than one category as the names imply.

For purposes of rapidly estimating earthquake ground motion in areas of the world, for which detailed physical-property and geologic information are not available, other information such as topographic slope (Wald, 1994; Wald and Allen, 2007) have been introduced as proxies for V_{S30} and site amplification. Topographic slope tends to correlate with V_{S30} , because more competent rock like materials with higher V_{S30} values tend to maintain steeper slopes, while softer, more fine grained and soil-like materials with lower V_{S30} values tend to be deposited on and maintain more gradual slopes.

Compilation of a large amount of V_{S30} and corresponding topographic slope data from different regions is shown in Fig. 4.3 as presented by Wald and Allen (2007). The compilation shows a strong correlation between V_{S30} and topographic slope with the correlation being region dependent. Subsequent efforts by Wald et al. (2011) have developed hybrid procedures that include actual V_{S30} measurements and geologic information where available. They have developed a compendium of V_{S30} maps available world wide http://earthquake.usgs.gov/hazards/apps/vs30/ for ShakeMap and PAGER (prompt Assessment of Global Earthquakes for Response) applications.



Figure 4.2. Geologic age classification and measured Vs30 values for USGS boreholes in California showing non-distinct seismic response units (see text for explanation of notation; Wills et al., 2000).

5. DISCUSSION

Correlations of V_{S30} with: 1) strong-motion spectral amplification measurements, 2) empirical amplifications inferred from GMPE, and 3) V_{Sz} at other depths have established V_{S30} as robust single parameter for characterizing site response. Unfortunately, even though this is the case, some investigators (Castellaro et al., 2008) have challenged the use of V_{S30} based in part on their

application of an inappropriate regression model to infer the relationship between V_{S30} and amplification. It is well known (e.g. Leng et al., 2007) that the "Orthogonal Regression Model" yields a different regression relationship when applied to a set of data than does the "Ordinary Linear Regression Model" with the choice of the appropriate model determined by whether the uncertainty in the independent and dependent variables is comparable or whether the uncertainty in the independent variable (V_{S30}) is much less than that in the dependent variable (amplification, F_a and F_v). In the case of the empirical data used to derive the relationships between F_a and F_v as indicated in Fig. 2.2 for $I_a = 0.1g$, the V_{S30} measurements were inferred from travel-time measurements to a depth of 30 m in boreholes at or very near the sites (Borcherdt and Glassmoyer, 1992). The uncertainty in the V_{S30} measurements is within 1-3% (Moss, 2008) and much less than the uncertainty in the amplification variables (F_a , F_v). Hence, the incorrect application of the "Orthogonal Regression Model" by Castellaro, et al. (2008) yields an incorrect estimate of the regression relationship between the dependent variables F_a and F_v and the independent variable V_{S30} . Incorrect assumptions leading to misapplication of regression models is well known in the literature and has led to incorrect conclusions in other fields (Carroll and Rupert, 1994).



Figure 4.3. Correlations of measured V_{S30} (m/sec) versus topographic slope (m/m) for active tectonic (a) and (b) stable continental regions (b) (from Wald and Allen, 2007).

Judgments regarding the usefulness of V_{S30} can only be made in the context of its intended application. V_{S30} as initially introduced (Borcherdt, 1994) was intended to provide unambiguous definitions of site classes and site coefficients for a simplified procedure to estimate site-dependent response spectra for use in the 1994 edition of the NEHRP building code provisions. Realizing the limitations, however, of a single parameter to completely characterize the response of a site, it was recommended and code procedures provide the option to use complete and detailed V_S profiles with corresponding modulus degradation and damping ratio curves to develop improved estimates of site response as a function of period. Hence, for code related applications whenever there is a question regarding the adequacy of V_{S30} for purposes of developing a site-specific design spectrum, code procedures provide the option to use a complete V_S profile.

Correlations of V_{S30} with physical properties, geologic age, and topographic slope provide proxies useful in mapping V_{S30} as a parameter to distinguishing broad regional variations in site response. Such maps can be useful for preparing ShakeMaps and Probabilistic Seismic Hazard maps for certain applications.

REFERENCES

- Abrahamson, N., Atkinson, G., Boore, D., Bozorgnia, Y., Campbell, K., Chiou, B., Idriss, I.M., Silva, W., and Youngs, R. (2008). Comparisons of the NGA Ground-Motion Relations. *Earthquake Spectra*, **24:1**,45–66.
- ASCE 7-10 (2010). *Minimum Design Loads for Buildings and Other Structures*, American Society of Civil Engineers, 658 pp.
- Boore, D.M., Thompson, E.M., and Cadet, H. (2011). Regional correlations of Vs30 and velocities averaged over depths less than and greater than 30 m. *4th IASPEI/IAEE International Symposium: Effects of Surface Geology on Seismic Motion, Santa Barbara, USA, August 23-26.*
- Borcherdt, R. D., and Gibbs, J. F. (1976). Effects of local geological conditions in the San Francisco Bay region on ground motions and the intensities of the 1906 earthquake, *Bull. Seismol. Soc. Am.*, **66**, 467-500.
- Borcherdt, R. D. (1991a). On the observation, characterization, and predictive GIS mapping of strong ground shaking for seismic zonation - A case study for San Francisco Bay region, *Bull. New Zealand Nat'l. Soc. Earthquake Engineering*, 24,287-305
- Borcherdt, R. D., Wentworth, C. M., Janssen, A., Fumal, T., and Gibbs, J. (1991b). Methodology for predictive GIS mapping of special study zones for strong ground shaking in the San Francisco Bay region, California, *Fourth International Conference on Seismic Zonation*, Stanford, California, *Procs., Earth. Eng. Res. Inst. Report No. SZP-4*, III, 545-552.
- Borcherdt, R. D., and Glassmoyer, G. (1992). On the characteristics of local geology and their influence on ground motions generated by the Loma Prieta earthquake in the San Francisco Bay region, California, *Bull. Seismol. Soc. Am.*, **82**, 603-641.
- Borcherdt, R.D. (1992). Simplified site classes and empirical amplification factors for site-dependent code provisions. *NCEER, SEAOC, BSSC workshop on site response during earthquakes and seismic code provisions*, Univ. Southern California, Los Angeles, California, Nov. 1992.
- Borcherdt, R.D. (1994). Estimates of site-dependent response spectra for design (methodology and justification). *Earthquake Spectra* **10**,617–654.
- Borcherdt, R.D. (2009), *Viscoelastic Waves in Layered Media*, Cambridge University Press, Cambridge, New York.
- Borcherdt, R. and Fumal, T. (2002). Shear-wave compilation for Northridge strong-motion recording sites, U.S. *Geol. Survey OF Rept.* 02-107, 2 maps, 16 pp.
- Building Seismic Safety Council (BSSC). (2004). NEHRP Recommended provisions for seismic regulations for new buildings and other structures, 2003 edition (FEMA 450).
- Cadet, H. and Duval, A.M. (2009). A shear wave velocity study based on the KiK-net borehole data: A short note, Seismol. Res. Lett. 80, 440–445.
- Carroll, R.J. and Ruppert, D. (1996). The Use and Misuse of Orthogonal Regression in Linear Errors-in-Variables Models, *The American Statistician*, **50**, 1-6.
- Castellaro, S., Mulargia, F. and Rossi, P.L. (2008). Vs30: Proxy for Seismic Amplification?. *Seismol. Research Letters* **79:4**,540-543.
- Gibbs, J.F., Fumal, T.E. and Borcherdt, R.D. (1975). In-situ measurements of seismic velocities at twelve locations in the San Francisco Bay region. U.S. Geological Survey Open-File Report 75-564.
- Fumal, T.E. (1978). Correlations between seismic wave velocities and physical properties of geologic materials in the San Francisco Bay region, California. U.S. Geological Survey Open-File Report 78-1067.
- Leng, L., Zhang, T., Kleinman, L. and Zhu, W. (2007). Ordinary least square regression, orthogonal regression, geometric mean regression and their applications in aerosol science *J. Phys.: Conf. Ser.* **78**,1-5.
- Moss, R.E.S. (2008). Quantifying Measurement Uncertainty of Thirty-Meter Shear-Wave Velocity, *Bull. Seismol. Soc. Am.*, **98: 3**,1399-1411
- Schnabel, P.B., Lysmer, J. and Seed, H.B. (1972). SHAKE: A computer program for earthquake engineering analysis of horizontally layered sites. *Report no. UCB/EERC-72/12*, Earthquake Engineering Research Center, Univ. California, Berkeley, 102.
- Seed, R.B. (1994). NCEER, SEAOC, BSSC Workshop on Site-dependent Code Provisions, University Southern California, Los Angeles, Nov. 1992.
- Wald, D.J. and Allen, T.I. (2007). Topographic Slope as a Proxy for Seismic Site Conditions and Amplification. Bull. Seismol. Soc. Am. 97:5,1379-1395.
- Wald, D.J., McWhirter, L., Thompson, E.M. and Hering A.S. (2011). A new strategy for developing Vs30 maps, 4th IASPEI/IAEE International Symposium: Effects of Surface Geology on Seismic Motion, Santa Barbara, USA, August 23-26, 2011.
- Wills, C.J., Petersen, M.D., Bryant, W.A., Reichle, M.S., Saucedo, G.J., Tan, S.S., Taylor, G.C., and Treiman, J.A. (2000). A site-conditions map for California based on geology and shear wave velocity. *Bull. Seismol. Soc. Am.* **90**, 187–208.