Seismic Displacement Demands of Reinforced Concrete Structures by Cyclic Pushover Analysis

Phaiboon Panyakapo
Department of Civil Engineering, Faculty of Engineering,
Sripatum University, Thailand.

SUMMARY:
A Cyclic Pushover procedure is proposed to estimate the peak seismic displacement demands of reinforced concrete structures. The effects of stiffness degradation and strength deterioration of reinforced concrete members under cyclic loading were incorporated in the evaluation of seismic displacement demands. A 5-story reinforced concrete structure, which was not designed for earthquake resistance, was selected to estimate the seismic demands. These demands include the maximum roof displacement, the maximum floor displacement and the maximum inter-story drift ratio. The results were compared with the exact demands resulting from nonlinear time history analyses of multi-degree-of-freedom structure subjected to 20 ground motions, as well as the demands estimated from Modal Pushover analysis. It was found that the monotonic lateral load in the conventional pushover analysis provides an over-estimate in the lateral stiffness of the structure resulting in an under-estimate of displacement demands. Based on the proposed Cyclic Pushover procedure, the peak displacement demands are, on an average, more accurate than those of the Modal Pushover method.

Keywords: Seismic Displacement Demands, Cyclic Pushover Procedure, Nonlinear Time History Analysis, Modal Pushover.

1. INTRODUCTION

To evaluate the seismic displacement demands of a structure under earthquake loading, nonlinear time history analysis provides the solutions accepted as the exact demands. However, the accuracy of the solutions depends on the appropriate selection of ground motions as well as the modeling of structural behavior. This procedure requires computational efforts. In practice, nonlinear static analysis based on Pushover Analysis method has been widely employed to evaluate the seismic performance of structures. During the past decade, the Pushover Analysis procedures have been improved to estimate the more accurate displacement demands. The adaptive pushover method (Antoniou and Pinho, 2004; Papanikolaou et al., 2006) was proposed to capture the changes in the vibration properties of a structure. The lateral force distribution is evaluated and adjusted based on the nonlinear behavior of the structure for each step of an adaptive pushover analysis. Although this approach can provide good estimates of the displacement demands, however, the method is not simple for design practice. For the multi-mode pushover method, the Modal Pushover Analysis has been proposed by Chopra and Goel (2002, 2004) to allow for the influence of higher modes. This method uses invariant modal lateral force distribution to push the structure for each mode, and the results in each mode are combined with SRSS by assuming linear elastic behavior. The method is widely used to estimate seismic demands for tall buildings; however, some limitations have been reported (Chopra and Goel, 2005) regarding the reversal in the pushover curve under higher mode lateral force distribution, and the location of plastic hinges may not accurately predicted. Moreover, an over-estimate of the peak demands in the modal combination procedure has been pointed out.

The above mentioned methods typically employ monotonic lateral load in the pushover analysis. This is based on an assumption that the behavior of structural members under earthquake loading in the hysteretic model may be represented by a backbone curve or an envelope curve of cyclic hysteretic behavior. However, when the reinforced concrete members are subjected to cyclic loading, cumulative damage under several repeated loads resulting in stiffness degradation and strength deterioration. In addition, the effects of reinforcement slippage of reinforced concrete frame structure under cyclic loading cause a decrease in the lateral stiffness and an increase in the lateral displacement (Limkatanyu and Spacone, 2003; D'Ambrisi and Filippou, 1997). The monotonic lateral load in the pushover analysis may provide an over-estimate in the lateral stiffness of the structure, and this leads to an under-estimate of displacement demands.

The Cyclic Pushover procedure is proposed to capture these important characteristics of reinforced concrete members under cyclic loading. The seismic demands of a 5-story reinforced concrete structure were investigated by Cyclic Pushover procedure. The results were compared with the exact demands resulting from nonlinear time history analysis, as well as the demands estimated from Modal Pushover analysis. In this study, the cyclic pushover analysis procedure is currently limited to the structures, the responses of which are primarily governed by the fundamental mode of vibration.

2. Cyclic Pushover Procedure

The modal response analysis which was described in Modal Pushover Analysis procedure in Chopra and Goel (2002, 2004) is applied in this section. The differential equation governing the response of a multi-degree-of-freedom (MDOF) system subjected to earthquake ground motion is as follows:

$$[m]\{\ddot{u}\}+[c]\{\dot{u}\}+\{f_s\}\left(\{u\},sign\{\dot{u}\}\right)=P_{eff}(t) \tag{2.1}$$

Where $[m],[c]$ are the mass and damping matrices of the structure, $\{f_s\}$ is the internal resisting force vector, and $\{u\},\{\dot{u}\}$ are roof relative lateral displacement and velocity vectors, respectively.
The effective earthquake forces $P_{eff}(t)$ can be written as

$$P_{eff}(t)=-[m]\{i\}\ddot{u}_g(t) \tag{2.2}$$

Where $\{i\}$ is the unit vector, and $\ddot{u}_g(t)$ is acceleration of ground motion.
The term $[m]\{i\}$ represents the spatial distribution of the effective earthquake forces over the height of the building, and can be expressed as $\{s\}$ which can be expanded as a summation of modal inertia force distribution as follows:

$$[m]\{i\}=\{s\}=\sum_{n=1}^{N}s_n=\sum_{n=1}^{N}\Gamma_n[m]\{\phi_n\} \tag{2.3}$$

Where Γ_n is the modal participation factor of the nth mode and it can be determined from

$$\Gamma_n = \frac{L_n}{M_n} = \frac{\{\phi_n\}^T [m]\{i\}}{\{\phi_n\}^T [m]\{\phi_n\}} \tag{2.4}$$

and $\{\phi_n\}$ is the corresponding mode shape.
The governing equation of motion for the structure can then be written as:

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + \{f_s\}\left(\{u\}, sign\{\dot{u}\}\right) = -\sum_{n=1}^{N}\{s_n\}\ddot{u}_g(t) \tag{2.5}$$

Eqn. 2.5 can be uncoupled to the equivalent governing equation of motion for a SDOF system by introducing the lateral displacement vector, $\{u\}$ in the form

$$\{u\} = \sum_{n=1}^{N}\{u_n\} = \sum_{n=1}^{N}\{\phi_n\} q_n \tag{2.6}$$

Where q_n are the modal amplitudes.
Substituting Eqn. 2.6 and its derivatives into Eqn. 2.5, and using the mass and damping orthogonality of the mode shapes, the governing equation of motion for a SDOF system becomes

$$\ddot{D}_n + 2\xi_n\omega_n\dot{D}_n + \frac{F_{sn}}{L_n} = -\ddot{u}_g\left(t\right) \tag{2.7}$$

Where $F_{sn} = \{\phi_n\}^T \left\{ f_s\left(D_n, sign\,\dot{D}_n\right)\right\}$ is the modal internal resisting force, $D_n, \dot{D}_n, \ddot{D}_n$ are the modal displacement, velocity and acceleration, respectively, and ξ_n and ω_n are the modal damping and frequency, respectively. To solve Eqn. 2.7, it is generally to conduct a nonlinear SDOF dynamic time-history analysis. In the analysis, the relationship between F_{sn} and D_n is to be determined using pushover analysis. For Modal Pushover Analysis, the lateral force distribution for the pushover analysis in each mode is f_n, which can be expressed as follows:

$$f_n = \Gamma_n [m]\{\phi_n\} A_n \tag{2.8}$$

where f_n is the lateral force distribution in each mode, and $A_n = \omega_n^2 D_n$.
For cyclic pushover analysis, the lateral force distribution is proposed as follows:

$$f_n^* = \lambda_i \Gamma_n [m]\{\phi_n\} A_n \tag{2.9}$$

where f_n^* is the lateral force distribution for cyclic pushover in each mode.

λ_i is a variable factor which defines the direction of force, i is defined as the sequence numbers of peak displacement for the specified displacement history, when $\boldsymbol{i}$ is an odd number (1,3,5,…) λ_i = 1 , and for $\boldsymbol{i}$ is an even number (2,4,6,…..) λ_i = -1.

The structure is subjected to the force distribution in the positive direction until it reaches the first peak displacement, and then the force distribution is reversed to the negative direction aiming to the second peak displacement. This process repeats in cycles according to the specified displacement history. In this study, the displacement history pattern known as laboratory-test-like-displacement history, which is typically employed in the laboratory, is applied to control the displacement pattern in cyclic pushover analysis.

For the structures that significant participation from modes of vibration other than the fundamental mode is required, higher modes effects may be determined by conducting higher mode cyclic pushover analyses. That is, the lateral load distributions as shown in Eqn. 2.9 are applied for each mode. However, the cyclic pushover procedure in this study is presented for the structures, the responses of which are primarily governed by the first mode. For this purpose, Eqn. 2.9 can be simplified to

$$f_1 = \lambda_i \Gamma_1 [m]\{\phi_1\} A_1 \tag{2.10}$$

Where Γ_1 is the Participation factor of the first mode,

A_1 is the acceleration in the first mode $= \omega_1^2 D_1$,

ω_1, D_1 is the angular frequency and displacement in the first mode.

It is obvious that the base shear force and roof displacement relationship obtained from the cyclic pushover analysis is a cyclic loop reversal of force and displacement. To determine the pushover curve in the form similar to monotonic loading, an envelope of cyclic loop is normally used to represent the characteristic of cyclic reversal curve. The envelope of cyclic pushover curve is converted to an equivalent single degree of freedom (SDOF) pseudo-acceleration and displacement relationship. This relationship is developed to represent the first mode response of the structure based on the assumption that the fundamental mode of vibration is the predominant response of the structure. The pushover base shear force, V_b is converted to SDOF pseudo-acceleration, S_a or A by using the relation

$$S_a, A \quad = \quad \frac{(V_b / W)_{envelop}}{\alpha_1} \tag{2.11}$$

The pseudo-displacement, S_d or D is given by

$$S_d, D \quad = \quad \frac{u_{r,envelop}}{\Gamma_1 \times \phi_{1,roof}} \tag{2.12}$$

Where α_1 is the modal mass coefficient for the first mode,

$(V_b / W)_{envelop}$ is the envelope of base shear force normalized with building weight W ,

$u_{r,envelop}$ is the envelope of roof displacement,

$\phi_{1,roof}$ is the roof displacement for the first mode.

3. ANALYSIS OF 5-STORY RC BUILDING

3.1. Modeling of structure

In this study, a 5-storey reinforced concrete building was employed in the cyclic pushover analysis. This is a standard residential building according to the National Housing Authority, which is typically constructed throughout the Kingdom of Thailand. The details of building can be summarized as follows: a) The building is symmetric in both plan and vertical views with its floor dimension of 15.60x24.50 meters and overall height of 14.30 meters, b) the floor system is precast concrete plank supported by reinforced concrete beams, c) the structure is a beam-column reinforced concrete system with normal material strength, i.e., compressive strength of concrete is 32 Mpa, and the tensile strength of reinforcing steel is 400 Mpa. The reinforced concrete structure was designed primarily for gravity load according to EIT (2000). Since this is an old building that has been constructed before the seismic regulation is effective, it was not designed for seismic loading. The beam-column frame was modeled as a two-dimension moment resisting frame in the N-S direction as shown in Figure 1. The inelastic behavior of beam and column members is modeled according to the Giberson one-component concept (Sharpe, 1974), which has a plastic hinge possible at one or both ends of the elastic central length of the member. The hysteretic behavior of beam is Takeda with slip model (Kabeyasawa et. al., 1983).

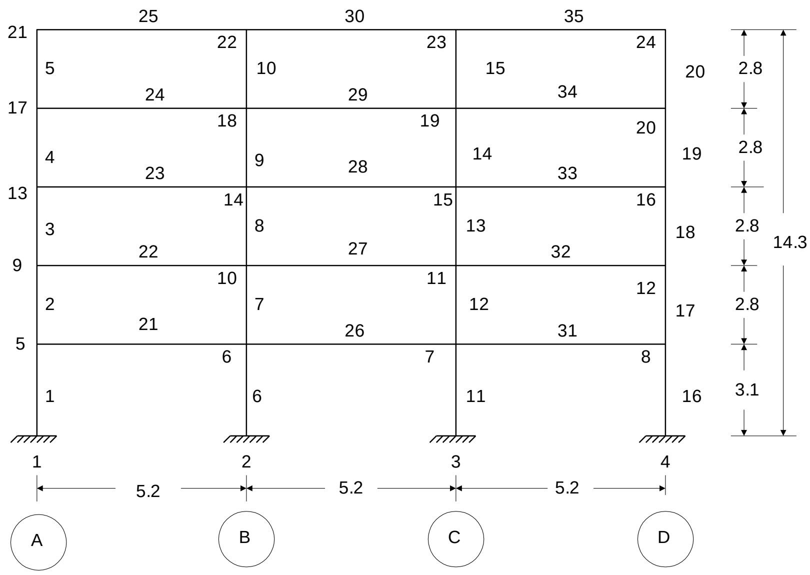

Figure 1. Two-Dimension model frame

For dynamic analysis, Nonlinear Time History Analysis (NTHA) was performed by RUAUMOKO (Carr, 2006) using a set of ground motion records. In this analysis, the mass of building was lumped to the nodes of beam-column joint for the horizontal degree of freedom. The initial stiffness Rayleigh damping was assumed with a damping ratio of 5%. The Newmark average acceleration method was employed in the dynamic time history analysis.

3.2. Ground motion records

For the ground motions, these were collected from moderately strong magnitude and near-fault earthquakes with magnitude between 6.1(M_L) and 7.1(M_s), and epicentral distance less than 40 km. They are represented for earthquake events that may occur in the northern part of Thailand. These ground motions were scaled to the Maximum Considered Earthquake (MCE) response spectrum, which corresponds to a 2% probability of exceedance in 50 years according to SPT 1302 (2009). Previous researches (Uang and Bertero, 1988; Kurama and Farrow, 2003, Spyracos et al, 2008, Climent et al., 2010) have studied reliable parameters to measure the damage potential of earthquake ground motion. Among many parameters, the input energy equivalent velocity V_I is an interesting parameter that closely relates to damage potential for moderate seismicity region. Therefore, this parameter was selected as an index to correlate with seismic demands of structure in this study.

4. Evaluation of Seismic Demands

This section presents the results of seismic displacement demands based on Cyclic Pushover Analysis (CPA) procedure for the 5-storey reinforced concrete building. The result of cyclic pushover curve under the cyclic distribution force and the controlled displacement history was evaluated. The envelope of cyclic pushover curve was transformed to the equivalent bilinear SDOF model. The seismic demands which were evaluated by nonlinear time history analyses of the idealized equivalent bilinear SDOF models are presented. The BISPEC program was employed to conduct the SDOF dynamic analysis for the equivalent bilinear SDOF models**.** The seismic demands are evaluated for the maximum roof displacements, the maximum floor displacement, and the maximum inter-story drift ratio. The results are compared with those from MDOF Nonlinear Time History Analysis (NTHA) and the Modal Pushover Analysis (MPA) procedure proposed by Chopra and Goel (2002, 2004).

4.1. Maximum roof displacement demands

The percentages of difference for Cyclic Pushover Analysis (CPA) procedure are plotted with the input energy equivalent velocity, V_I . The results for Modal Pushover Analysis (MPA) are also plotted for comparison, as shown in Figure 2. It was observed that the results of many ground motions were deviated from the exact demands. The positive difference means an over-estimate demand; on the other hand, the negative difference means an under-estimate demand. Both of these differences are considered as errors from the estimation. An average value of these differences which have similar percentage of difference in both positive and negative values may result in a low error percentage, which is an un-conservative result. Therefore, the percentages of difference are presented in three groups, i.e., a) the mean of difference percentages for over-estimate values; b) the mean of difference percentages for under-estimate values; and c) the mean absolute values of difference percentages.

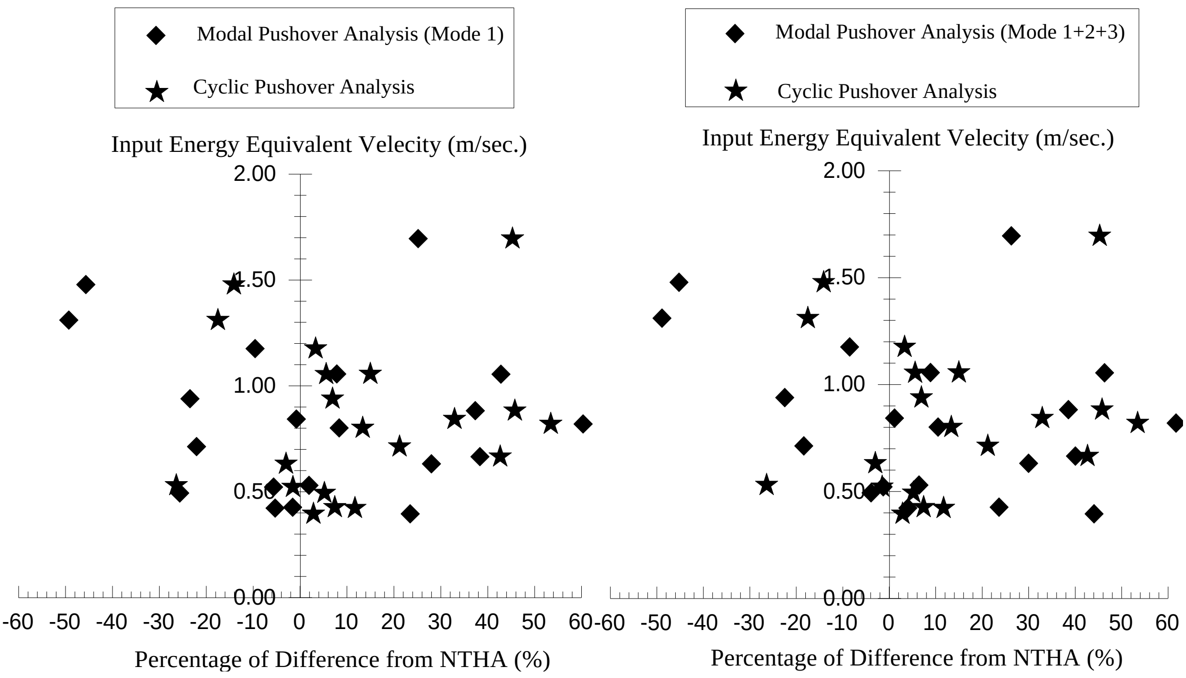

Figure 2. Difference of maximum roof displacement from NTHA for CPA and MPA

For an overview, the maximum roof displacement demands for both CPA and MPA provide are, on average, reasonable estimates. For the over-estimate group (a), the CPA procedure provides 20.83% over-estimate which is comparable to that of one mode MPA (20.44%). However, when the displacement demands of higher modes are considered by SRSS combination of the modal displacement, the mean percentages of difference in over-estimate tend to increase. For the under-estimate group (b), the CPA procedure provides 12.52% under-estimate, which is less than that of one mode MPA (25.92%). Although the percentages of difference resulting from higher modes effects tend to decrease, it is observed that these under-estimate displacement demands for MPA method are much lower than the exact demands when they are compared with that of CPA procedure. Finally, for the absolute difference group (c), the CPA procedure provides 18.75% difference which is less deviate than those of MPA, i.e., one mode (22.36%), two modes (24.26%), and three modes (24.51%). This indicates that the proposed CPA procedure gains more accurate results for the maximum roof displacement demands than the MPA method.

It can be observed that the combined modal response demands resulting from higher mode effects do not significantly reduce the errors for this 5-story building structure. Therefore, the Cyclic Pushover Analysis procedure using lateral force distribution in proportion to the fundamental mode in this study provides sufficient accuracy. The reason that the CPA procedure provides less percentages of difference than those of MPA method is due to the fact that the cyclic lateral force distribution leads to a reduction in the stiffness of structure resulting from the cracking of reinforce concrete members and reinforcement slippage under cyclic load reversal. This is consistent with the behavior of reinforced concrete building structures under earthquake loading. The roof displacement demands estimate is therefore close to the exact demands.

4.2 Maximum Floor Displacement Demand

To determine the maximum floor displacement of each story of the 5-story building, the floor displacement demands for a MDOF system can be obtained by multiplying the maximum roof displacement with the normalized mode shape. The results obtained from the 20 ground motions are presented in terms of a single value as the mean of the maximum floor displacement of each story. The mean values for the CPA procedure are plotted with the floor levels of building and compared with those of the exact Nonlinear Time History Analysis (NTHA) and those of the MPA method, as shown in Figure 3.

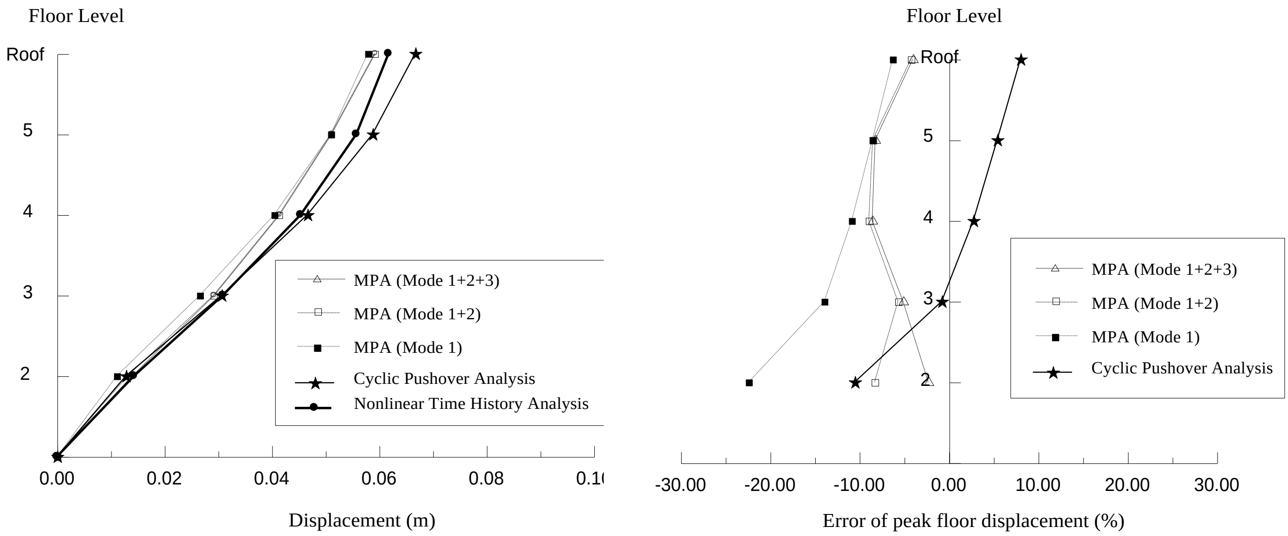

Figure 3. Mean maximum floor displacement under 20 ground motions

To determine the accuracy of the proposed procedure, the differences of the mean maximum floor displacement for each story between the CPA procedure and the exact solutions were evaluated. The results are also compared with those of the MPA method for one mode, two modes, and three modes. For an over-view consideration, the percentages of difference of the mean maximum floor displacement for each story were computed for the averages of all five stories. The results are also presented in three groups as described in the preceding section.

From Figure 3, it is observed that the mean maximum floor displacements for CPA procedure are close to the exact displacement demands for the 3rd floor to 4th floor. While the displacement demands of the other stories (2nd floor and 5th floor to roof level) relatively deviate from the exact values. Since the proposed CPA procedure employs the normalized first mode shape in the determination of the floor displacement, these errors are therefore caused by the higher mode effect that is not taken into account. For those of MPA method, the mean maximum floor displacements for MPA (mode 1) are under-estimate. The combined modal response demands resulting from higher mode effects tend to reduce the under-estimate errors, especially for the lower two stories (second floor to fourth floor). Considering for the whole building, the mean percentages of difference for the CPA procedure are 5.4% over-estimate, 5.66% under-estimate, and 5.5% deviation for absolute value. These errors are comparable with those of the MPA procedure (three modes). However, for seismic design consideration that is used in practice, the Cyclic Pushover Procedure provides better estimates for the MDOF maximum floor displacement demands than the Modal Pushover Analysis procedure. This is due to the conservative estimate of displacement demands resulting from the determination of a realistic lateral force distribution in pushover analysis.

4.3. Maximum inter-story drift ratio demand

The results obtained from the 20 ground motions are presented in terms of the mean of the maximum inter-story drift ratio of each story. The mean values for the CPA procedure are plotted with the floor levels of building and compared with those of the exact Nonlinear Time History Analysis (NTHA) and those of the MPA method, as shown in Figure 4. The differences of the mean maximum inter-story drift ratio for each story between the CPA procedure and the exact solutions were evaluated.

For an over-view, the mean maximum inter-story drift ratios for both of CPA and MPA procedures are consistent with the exact demands. It is observed that the maximum inter-story drift ratios for the CPA procedure are 16.84% over-estimate for the 3rd floor to roof levels, except the 2nd floor that is 10.53% under-estimate, while the absolute deviation is 15.58%. These errors are comparable to those of the MPA method, particularly for the absolute deviations which are 14.15%, 13.98%, and 13.55% for one mode, two modes, and three modes, respectively. The proposed CPA procedure provides, on average, over-estimate of mean maximum inter-story drift ratios, while the MPA method provides relatively under-estimate. In seismic design consideration, the proposed CPA procedure is more conservative than the MPA method.

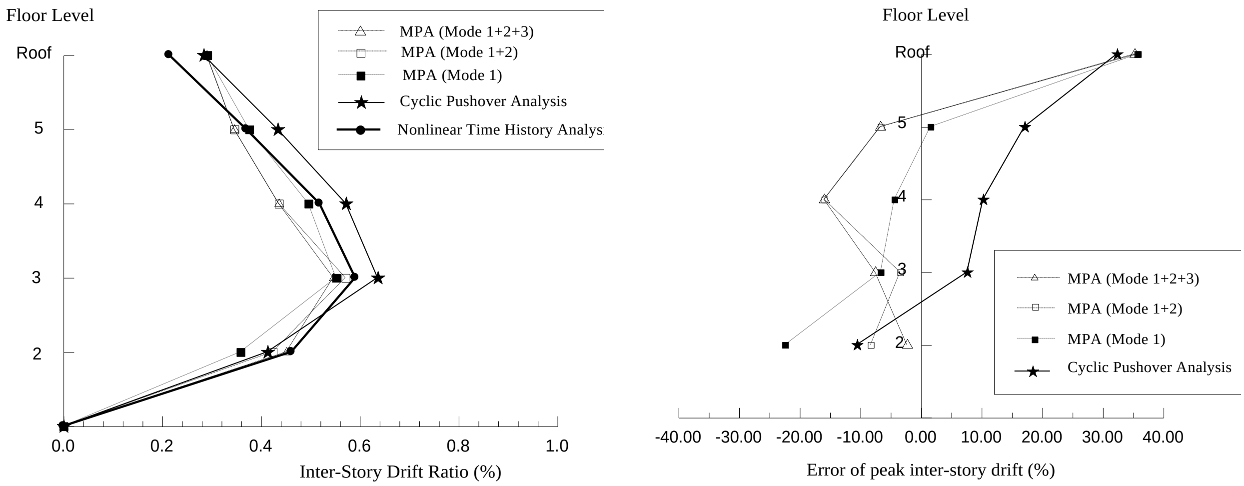

Figure 4. Mean maximum inter-story drift ratio under 20 ground motions

5. CONCLUSIONS

Based on the above results, the following conclusions can be drawn.

a) Cyclic Pushover Analysis procedure based on cyclic lateral load distribution provides larger displacement demands than those of monotonic lateral loads. This is due to the effect of cumulative damage resulting from cyclic load reversal, which cannot be account by monotonic pushover analysis; leads to a reduction in the stiffness of structure resulting from the cracking of reinforce concrete members and reinforcement slippage under cyclic load reversal. This is consistent with the behavior of reinforced concrete building structures under earthquake loading. The seismic displacement demands estimate is therefore close to the exact demands.

b) This approach relies on the selection of appropriate earthquake ground motions. The ground motion intensity is an important factor affecting on the evaluation of displacement demands.

c) For the structures that significant participation from modes of vibration other than the fundamental mode is required, higher modes effects may be determined by conducting higher mode cyclic pushover analyses. This requires further investigation particularly for tall buildings.

ACKNOWLEDGEMENT

This research was supported by the research fund of Sripatum University.

REFERENCES

American Society of Civil Engineers, ASCE. (2007). Seismic Rehabilitation of Existing Buildings, *ASCE Standard* No. ASCE/SEI 41-06.

Antoniou S., Pinho, R. (2004). Development and verification of a displacement-based adaptive pushover procedure. *Journal of Earthquake Engineering*; **8:5**, 643-661.

Carr A.J. (2006). RUAUMOKO computer program. *University of Canterbury*, Christchurch, New Zealand.

Chopra, A.K., Goel, R.K. (2002). A modal pushover analysis procedure for estimating seismic demand of buildings. *Earthquake Engineering and Structural Dynamics*, **31**:561-582.

Chopra, A.K., Goel, R.K. (2004). A modal pushover analysis procedure to estimate seismic demands for unsymmetrical-plan buildings. *Earthquake Engineering and Structural Dynamics*, **33:8**, 903-927.

Climent A. B., Almansa, F.L., Gonza, D.A.B. (2010). Design energy input spectra for moderate-to-high seismicity regions based on Colombian earthquakes. *Soil Dynamics and Earthquake Engineering*, **30**: 1129-1148.

D'Ambrisi A. and Filippou, FC. (1997). Correlation studies on an RC frame shaking-table specimen. *Earthquake Engineering and Structural Dynamics*, **26**: 1021-1040.

Engineering Institute of Thailand (2000). Standard for Reinforced Concrete Building (Strength Design Method). *EIT Standard 1008-38*, Bangkok, Thailand.

FEMA (2005). Improvement of Nonlinear Static Seismic Analysis Procedures. *Federal Emergence Management Agency, FEMA 440*, Washington D.C.

Kabeyasawa, T., Shiohara, H., Otani, S. and Aoyama, H. (1983). Analysis of the full-scale seven-storey reinforced concrete test structure. *Journal of the Faculty Engineering*, University of Tokyo, (B), **XXXVII:2**, 431-478.

Kappos, A.J. (1991). Analytical prediction of the collapse earthquake for R/C buildings: suggested methodology. *Earthquake Engineering and Structural Dynamics*, **20**: 167-176.

Krawinkler, H. (1992). Guidelines for cyclic seismic testing of components of steel structures. *ATC-24*, Redwood city, California.

Kurama, Y.C. and Farrow, K.T. (2003). Ground motion scaling methods for different site conditions and structure characteristics. *Earthquake Engineering and Structural Dynamics*, **32**: 2425-2450.

Limkatanyu, S. and Spacone, E. (2003). Effects of reinforcement slippage on the nonlinear response under cyclic loadings of RC frame structures. *Earthquake Engineering and Structural Dynamics*, **32**: 2407-2424.

Papanikolaou, V.K. Elnashai, A.S., Pareja, J.F. (2006). Evaluation of conventional and adaptive pushover analysis II: Comparative results. *Journal of Earthquake Engineering*, **10:1**, 127-151.

Park, Y. J. and Ang, A. H. (1985). Mechanistic seismic damage model for reinforced concrete. *Journal of Structural Engineering. ASCE*, **111:4**, 722-739.

Sharpe, R.D. (1974). The Seismic Response of inelastic structures, Ph.D. Thesis, Department of Civil Engineering, University of Canterbury, New Zealand.

SPT 1302 (2009). Standard of Earthquake Resistant Design of Building, Department of Public Works and Town & Country Planning, Bangkok, Thailand.

Spyracos C. C., Maniatakis, C.A., Taflambas, J. (2008). Evaluation of near-source seismic records based on damage potential parameters Case study: Greece. *Soil Dynamics and Earthquake Engineering*, **28**: 738-753.

Uang, C. M. and Bertero, V. V. (1988). Implication of recorded earthquake ground motions on seismic design of building structures. *UCB/EERC Report* **88:13**, University of California, Berkeley, California.