# An Exploration on Characteristics of Ground Motion Containing Harmonic Waves

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#### SUMMARY:

Long-period ground motion has become an important consideration because of the increasing number of large, long-period structures. However, the long-period effect has not been included in any available seismic design codes now. Aiming at the characteristics of long-period component, this work performed an analytical study to examine the effect of several parameters, as well as the combining mode for equivalent harmonic components on the dynamic response of systems. The results of the work show that the harmonic components in equivalent ground motion are evidently influenced by the intensity rise time, duration, phase and combining modes. Moreover, the long-period ground motions are simplified and simulated by separate harmonic components through proper combination. The findings of the work are believed to be useful in the selection of input ground motion in structural seismic analysis.

Key Words: harmonic earthquake wave; response spectrum; normalized response spectrum; component combining mode

### **1. INTRODUCTION**

An influential observation in recent major earthquakes is that severe damages took place not only in near-fault zones, but also in the far-source regions (Beck and Hall, 1986; Boore, 2001; Koketsu et al., 2005; Xu et al., 2009). Ground motion time histories during large earthquakes are able to feature considerable long-period cycles even at distant soil sites or basins, which has been identified to have close relationship with the destruction of long-period structures (Celebi, 1987; Jousset and Douglas, 2007; Xu et al., 2008; Ruiz and Saragoni, 2009). Such long-period effect can cause a localized peak within the earthquake response spectra that is currently not included in any available seismic design codes.

Considering the increasing number of large structures, such as high-rise buildings, suspension bridges, wind power supporting structures, oil storage tanks, off-shore oil drilling platforms, and base-isolated structures, long-period ground motions have become increasingly important (Bungum et al., 2003; Boore, 2004; Li and Zhang, 2006; Hatayama, 2008; Koketsu, 2008).

Different with near-fault pulse-type ground motion, there are almost harmonic damped vibrations during the strong motion part of time traces for such long-period ground motion. Four sets of long-period ground motions took place in the Chi-Chi and Wenchuan earthquakes have been given in Figure 1. It is evident that the long-period ground motions exhibit long duration, simple frequency content, but magnification in amplitudes even after distant propagation. The harmonic-like vibration acceleration (HA) of long-period ground motions is less than peak ground acceleration (PGA), while

velocity and displacement amplitudes in harmonic vibration portion (HV, HD) are capable of determining the peak ground amplitudes (PGV, PGD) of the velocity and displacement time histories. Response spectra (see Figure 2) of this kind of motion show that the harmonic-like components can cause a localized peak at the long period range, and match much of the spectral area of the full record. These observations need to be seriously considered in dynamic analysis for long period structures.



Figure 1. Ground motions containing harmonic wave components during the Chi-Chi and Wenchuan earthquake



Figure 2. Normalized response spectra of ground motions containing harmonic wave components

In this work, an idealized ground motion model, defined of several parameters is introduced to represent the long-period component; then, the major factors influence the response of SDF systems subjected to the modelled harmonic excitations are discussed; next, we explore the response spectral characteristics of all the possible synthetic ground motions generated by harmonic components by highlighting the effect of the combining mode (the arrangement in sequence); finally, examples are given that the long-period ground motion can be simplified and represented through combination of

separate harmonic components.

### 2. MATHEMATICAL MODEL FOR HARMONIC COMPONENT

For the sake of simplicity, a monochromatic waveform together with triangular envelope lines (also named Coulomb friction vibration) is used to approximate the harmonic-dominant motion acceleration a(t), that is:

$$a(t) = \begin{cases} X \frac{t}{t_1} \cos 2\pi \omega (t - \theta) & 0 \le t \le t_1 \\ X \frac{t - t_2}{t_1 - t_2} \cos 2\pi \omega (t - \theta) & t_1 < t \le t_2 \end{cases}$$
(1)

where X is the envelope curve amplitude;  $t_1$  is the time the envelope peak takes place, termed "intensity rise time";  $t_2$  represents the full duration;  $\omega$  is the natural frequency in unit of Hz, the wave vibration period  $T_p = 1/\omega$ ;  $\theta$  is the phase in second, and  $0 \le \theta \le T/2$ .

The corresponding velocity and displacement model can be written as:

$$v(t) = \begin{cases} \frac{X}{4\pi^{2}\omega^{2}t_{1}} [\cos 2\pi\omega(t-\theta) + 2\pi\omega t \sin 2\pi\omega(t-\theta) - \cos 2\pi\omega\theta] & 0 \le t \le t_{1} \\ \frac{X}{4\pi^{2}\omega^{2}t_{1}(t_{1}-t_{2})} [\cos 2\pi\omega(t-\theta) + 2\pi\omega t_{1}(t-t_{2}) \sin 2\pi\omega(t-\theta) - t_{1} \cos 2\pi\omega\theta - t_{2} \cos 2\pi\omega(t_{1}-\theta)] & t_{1} < t \le t_{2} \end{cases}$$

$$d(t) = \begin{cases} \frac{X}{4\pi^{2}\omega^{2}t_{1}} \{\frac{1}{\pi\omega} [\sin 2\pi\omega(t-\theta) + \sin 2\pi\omega\theta] - t[\cos 2\pi\omega(t-\theta) + \cos 2\pi\omega\theta] \} & 0 \le t \le t_{1} \\ \frac{X}{4\pi^{3}\omega^{3}t_{1}(t_{1}-t_{2})} [\pi\omega t_{1}(t_{2}-t) 2\pi\omega(t-\theta) + \pi\omega t_{2}(t_{1}-t) \cos 2\pi\omega(t_{1}-\theta) + \pi\omega t(t_{2}-t_{1}) \cos 2\pi\omega\theta \\ + t_{1} \sin 2\pi\omega(t-\theta) - t_{2} \sin 2\pi\omega(t_{1}-\theta) + (t_{1}-t_{2}) \sin 2\pi\omega\theta] \end{cases}$$

$$(2)$$

Considering the boundary conditions of ground motion:  $a(t_2) = 0$ ,  $v(t_2) = 0$ , then:

$$\left(\frac{1}{t_2 - t_1} + \frac{1}{t_1}\right) \cos 2\pi\omega(t_1 - \theta) = \frac{1}{t_2 - t_1} \cos 2\pi\omega(t_2 - \theta) + \frac{1}{t_1} \cos 2\pi\omega\theta$$
(4)

Thus,  $t_1 \neq 0, t_1 \neq t_2$ , and  $t_2 = mT_p, t_1 = lT_p$ , or  $t_1 = 2\theta + nT_p$  (where *m*, *l*, and *n* are integers); if

$$a(t_2) = 0, v(t_2) = 0, d(t_2) = 0$$
, then  $t_2 = mT_p, t_1 = lT_p$  (where *m* and *n* are integers, and  $m > l$ ).

In the following sections, we examine the response spectra of the harmonic component to account for the influence of control parameters on spectral shapes and amplifications. Besides, response spectra of the synthetic ground motions generated by harmonic components are explored by emphasizing effects of combining mode.

### **3. FACTORS AFFECTING HARMONIC COMPONENT**

Every parameter has a clear meaning in the mathematical description. Note that model envelop amplitude X has direct proportional relationship with amplitudes of harmonic component, so it can only influence the spectral ordinates and is independent on spectral shapes. While, the vibration frequency  $\omega$  has relations with both spectral ordinates and shapes, the influences of  $\omega$  can be eliminated by normalizing the spectral ordinate and abscissa simultaneously, which defined as the bi-normalized response spectra (BNRS, Xu and Xie, 2004; Xu et al., 2005). The following sections will be mainly concerned on the effects of intensity rise time  $t_1$ , full duration  $t_2$  and phase  $\theta$ .

### 3.1 Time parameter *t*<sub>1</sub>

Parameter  $t_1$  is defined as the time it takes for a site to reach its maximum vibration acceleration. We examine the bi-normalized acceleration, pseudo-velocity and displacement response spectra of five harmonic components with different  $t_1$  ( $t_1/T_p=1, 2, 3, 4$  and 5, respectively). Table 1 gives the values of other fixed parameters, together with the amplitude-period relations (HV/T<sub>p</sub>, HD/HV/T<sub>p</sub>) and peak spectral amplifications ( $\alpha_A$ ,  $\alpha_V$ ,  $\alpha_D$ ). It can be seen that the influence of  $t_1$  on motion amplitudes is negligible, whereas the spectral shapes are affected evidently. Spectral peak amplifications ( $\alpha_A$ ,  $\alpha_V$ ,  $\alpha_D$ ) increase with time  $t_1/T_p$  increases from 1 to 5. Spectral shapes of BNRS at are also significantly influenced, particular the spectral values at long periods (see Fig. 3), suggesting that the intensity rise time is one major factor that control the response of SDF systems to pulse excitations.

$X({\rm ms}^{-2})$	$t_1/T_p$	$t_2/T_p$	$\theta T_{\rm p}$	$HA(ms^{-2})$	$HV/T_{\rm p}({\rm ms}^{-2})$	$HD/HV/T_{\rm p}$	$lpha_{ m A}$	$lpha_{ m V}$	$lpha_{ m D}$
	1			1	0.156	0.325	4.640	4.661	2.335
	2			1	0.156	0.326	4.825	4.870	2.424
1	3	6	0	1	0.154	0.329	5.000	5.087	2.518
-	4			1	0.156	0.326	5.180	5.235	2.606
	5			1	0.156	0.325	5.340	5.383	2.684

**Table 1.** Comparison of some parameters for harmonic components with different  $t_1$ 

## 3.2 Duration parameter t<sub>2</sub>

Parameter  $t_2$  is the overall duration of the harmonic component. We examine the effect of  $t_2$  on BNRS (see Figure 4) by fixing the other parameters of the harmonic wave model. Model parameters, together with the amplitude-period relations and peak spectral amplifications have been given in Table 2.

$X(\text{ms}^{-2})$	$t_1/T_p$	$t_2/T_p$	$\theta T_{\rm p}$	$HA(ms^{-2})$	$HV/T_{\rm p}({\rm ms}^{-2})$	$HD/HV/T_{\rm p}$	$lpha_{ m A}$	$lpha_{ m V}$	$lpha_{ m D}$	
1		2	0	1	0.145	0.350	2.615	2.690	1.385	
		3		1	0.152 0.334		3.261	3.300	1.680	
	1	4		1	0.154	0.329	3.804	3.835	1.937	
		5		1	0.156	0.326	4.257	4.286	2.153	
		6		1	0.156	0.325	4.640	4.661	2.335	

**Table 2.** Comparison of some parameters for harmonic components with different  $t_2$ 

When the ratio of  $t_2/T_p$  increases from 2 to 6 exponentially, peak ground motions (HA, HV and HD) of the model wave change slightly. Note that, the peak spectral amplifications ( $\alpha_A$ ,  $\alpha_V$ ,  $\alpha_D$ ) increase evidently with the increasing number of cycles of ground motion. The discrepancies among BNRS for different  $t_2$  mainly emerge at periods in the vicinity of the vibration period, implying that duration is merely crucial to the resonant vibration of structures.

### 3.3 Phase parameter $\theta$

The phase angle  $\theta$ , defined as the time by which the vibration lags behind a complete cycle, is examined in terms of bi-normalized acceleration, pseudo-velocity and displacement response spectra. Firstly, a set of parameters of modeled components has been given and listed in Table 3. For different  $\theta$  (for example,  $T_p=1$ s, and  $\theta/T_p=0$ , 0.1, 0.2, 0.3 and 0.4), the amplitudes of the wave vary apparently. The effect of  $\theta$  on BNRS of acceleration and pseudo-velocity are relatively small, and the visible diversities mainly exhibit at the relative long periods (see Figure 5). On the contrary, the displacement BNRS reveals significant differences for different  $\theta$ . The spectrum of  $\theta/T_p=0.2$  is the highest of the plotted curves, because its displacement amplitude have a smallest value in that condition.

Table 3. Comparison of some parameters for harmonic components with different  $\theta$ 

$X(\text{ms}^{-2})$	$t_1/T_p$	$t_2/T_p$	$\theta T_{\rm p}$	$HA(ms^{-2})$	$HV/T_{\rm p}({\rm ms}^{-2})$	$HD/HV/T_{\rm p}$	$lpha_{ m A}$	$lpha_{ m V}$	$lpha_{ m D}$
			0	1	0.156	0.325	4.640	4.661	2.335
			0.1	0.980	0.156	0.294	4.726	4.668	2.571
1	1	6	0.2	0.960	0.159	0.210	4.806	4.557	3.538
			0.3	0.940	0.159	0.216	4.919	4.561	4.439
			0.4	0.920	0.158	0.290	5.039	4.592	2.575



Figure 3. Bi-normalized response spectra of harmonic component with different  $t_1$ 



Figure 4. Bi-normalized response spectra of harmonic component with different  $t_2$ 



Figure 5. Bi-normalized response spectra of harmonic component with different  $\theta$ 

### 4. CHARACTERISTICS OF SYNTHETIC GROUND MOTION

To generate synthetic ground motion, harmonic components are produced by using three sets of model parameters. Model parameters together with component amplitudes for three components have been listed in Table 4. For three sets of components, there are totally 61761 overlay wavelets to be obtained if a fixed time interval  $\Delta t$ =0.05s is given. The modes of the arrangement in sequence for three components can greatly affects the amplitudes of the overlapped motion. For example, if the three wavelets are combined one after another, the duration of the synthetic motion is 19s (3+6+10); thus, the PGA of the result motion equals to that of wavelet WI; if we overlay the three waves when their acceleration amplitudes occur at exactly the same time, the HA of the combined motion will be the summation of the HA for three individual wavelets, and the synthetic motion lasts only 10s.

Parameters	$X(\text{ms}^{-2})$	$t_1(s)$	<i>t</i> <sub>2</sub> (s)	$T_{\rm p}({\rm s})$	<i>Ө</i> / <i>Т</i> р	$HA(ms^{-2})$	$HV(ms^{-1})$	HD(m)
CI	1	1	3	0.25	0.20	0.9750	0.0395	0.0021
CII	0.5	2	6	0.5	0.25	0.4840	0.0397	0.0032
CIII	0.2	4	10	1	0.25	0.1917	0.0318	0.0052

Table 4. Parameters for three example components

To study the dynamic response of the SDF system to synthetic ground motion generated by harmonic component, we computed 5% damped acceleration, pseudo-velocity, displacement response spectra, and the corresponding normalized spectra. For all the synthetic motions, the mean spectra, mean $\pm 1$  standard deviation spectra, and the envelop curves of these spectra are displayed in Figure 6. The coefficient of variation (COV, defining as the ratio of standard deviation to the mean value), which reflect the dispersion of spectra curves is also calculated and plotted in Figure 9. The following observations can be made:

(1) For different synthetic motions, the acceleration, pseudo-velocity and displacement response spectra all vary evidently with the computed periods. However, it seems that differences in between peak values of the response spectra are quite small.

(2) The effect of combining mode on normalized spectra seems more significant; it becomes uniform only for periods approaching to zero in the normalized acceleration spectra. The disperse extent becomes smaller at long periods of normalized displacement spectra.



Figure 6. Response spectra comparison of some of the special curves



Figure 7. COV of the response spectra

### 5. EQUIVALENT LONG-PERIOD GROUND MOTION

The dominant status of long-period component in response spectral shapes makes it possible for the simulation of actual ground motion. Take the ground motions listed in Fig.1 for example, to implement the simulation, a computer program is developed to select separate harmonic component and calculate the response spectra of combined ground motion. Six harmonic components are finally picked out to execute the simulation, the component acceleration traces have been given in Figure 8(a).



Figure 8. Harmonic components and their response spectra

Normalized acceleration response spectra for individual component and target ground motion are plotted in Figure 8(b). Note that response spectra of each component can match a localized peak of target ground motion in the spectral area, so the spectra envelop of six components are able to cover most of the region below target spectrum, while discrepancies are still visible, particularly in between the component spectra peaks. Then, the combining technique is needed to minimize the discrimination.

The program performed calculations on each possible combining mode of six harmonic components with a time step of 1s. Eventually, three combined ground motions generated by the components are obtained to equal the target ground motion. The equivalent motions can match the target acceleration, pseudo velocity and displacement spectra to the largest extent among all combining motions, respectively. It is interesting that the equivalent motion (here marked with Eq1 in Figure 9(a)) to match acceleration spectrum has a same combining mode with that of the velocity spectrum, while the optimal one (Eq2 in Figure 9(b)) to match the displacement spectrum has a different combining mode.



Figure 9. Equivalent ground motion time traces

Comparisons of response spectra for equivalent motions and target motion have been given in Figure 10. Which shows that combined motion spectra accord well with the target spectra; they are capable of coving 95.43%, 96.66% and 97.16% percents of the area for acceleration, pseudo velocity and displacement spectra, respectively. Through good similarities in the response of SDOF systems to long-period ground motion and to equivalent motion derived by separate harmonic components are found, it should be noted that the actual ground motion and the synthetic motion have almost no comparability in time histories, in both aspects of wave profile and duration.



Figure 10. Comparison of target response spectra and equivalent ground motion spectra

#### 6. CONCLUSIONS

This study investigates the characteristics of long-period ground motion generated in recent large earthquakes in order to determine an estimate of the response spectra with simple measure by means of harmonic components. The component model is defined by harmonic wave shape and several control parameters, factors affecting the response spectra of modelled components and the synthetic ground motion is evaluated by highlighting the effect of combining mode. Specific findings and enlightenments are:

(1) Response spectra of the harmonic-like motions feature a localized high peak in the long period portion, thus causing huge threat to large and long-span structures. The long-period effects in large earthquakes need to be deliberated in future reformulations of structural seismic design codes.

(2) A complete description of a harmonic component needs several parameters by considering boundary conditions of actual ground motion. The intensity rise time, full duration and phase angle, as well as the amplitude, and vibration period are main factors that control the shapes of response spectra. It is essential to choose appropriate parameters for the simulation of long-period component involved in ground motions.

(3) The mode of arrangement in sequence influences the amplitudes, duration of the synthetic ground motion. Statistical results show that disperses in spectral peaks is negligible, while significance in spectral valleys for all the combined motions derived from the same set of components. In addition, different types of spectra may exhibit different statistical characteristics concerning on the variation of spectral periods.

(4) The response spectra determined for harmonic components matched well with those determined for long-period ground motions, which have waveforms of much greater complexity. However, the actual ground motion and the synthetic motion have almost no comparability in both wave profile and duration. The combing mode plays a definite role in tuning frequency content of synthetic ground motion, which should be considered in the representation of a ground motion.

It should be noted that the simplicity for long-period ground motion is based on a deterministic method by means of a few separate harmonic components. The accuracy of the estimate made with equivalent motion is dependent on proper selections of harmonic components and the modes the components arrange in sequence. This method is relatively simpler and easier compared with the stochastic modeling of strong ground motion.

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