Conditional simulation of spatially incoherent seismic ground motion using Gaussian process models.

I. Zentner
LAMSID UMR EDF-CNRS-CEA – EDF R&D, France

SUMMARY:
Spatial variability of ground motion is generally modelled by means of power spectral densities and frequency-domain coherency functions. This allows generating artificial time histories or to perform stochastic analysis by linear filtering. However, structural responses to a particular measured or synthetic accelerogram cannot be computed with these methods. The latter goal can be achieved with conditional simulation techniques. In this paper, we present a methodology capable of taking into account spatial incoherence of non stationary seismic ground motion based on conditional simulation using Gaussian process models. The quantities necessary for constructing this model are (time-domain) cross-correlation functions that are derived from commonly used evolutionary PSD. This is in contrast to other methods proposed in literature where conditional simulation techniques are applied in the frequency domain with respect to Fourier coefficients. The accuracy of the proposed methodology is demonstrated for the study case of El Centro earthquake NS record. The properties of the simulated ground motion fields are analysed and compared to the data and the theoretical ground motion model.

Keywords: spatial variability, conditional, Gaussian process, coherency, ground motion

1. INTRODUCTION

Recordings of strong motion arrays on test sites indicate that seismic free fields exhibit spatial variability over even short distances.

This is why, in the recent years, a considerable research effort has been employed to introduce spatial variability of free field motion in seismic response analysis. A review of the methods proposed in literature in the last years is given in Zerva (2009). Recent applications concern multi-supported long span structures such as bridges or lifelines as well as structures with extended foundations such as dams or nuclear power plants.

The spatially variable ground motion is generally described, in a statistical sense, by one-point power spectral densities (PSD) together with a coherency function. However, in design or verification studies it is often requested to work with natural accelerograms that have been selected by seismologists or other experts. The topic of computing (non linear) structural responses to a particular recorded ground motion time history can be tackled by the simulation of spatial ground motion fields conditioned on the given accelerogram. Several authors have applied conditional simulation techniques to generate incoherent ground motion. These procedures are generally based on stationary spectral densities and thus stationary simulation techniques (Kameda et al. 1992,Vanmarcke et al. 1993, Liao & Zerva 2006). In particular, some authors express the stochastic process modelling ground motion as a Fourier series. Ground motion fields are then generated by conditional simulation of the Fourier coefficients in the frequency domain. However, the non stationary nature of ground motion may not be well reproduced by this kind of approach. This is why some authors (Vanmarcke et al. 1993, Liao & Zerva 2006), proposed to divide the whole time series into a sequence of time windows to which different spectral properties are attributed. In Konakli & Der Kiureghian (2011), wave passage and site effects are modelled together with incoherence (of amplitude). Yet, expert judgement or further analyses are needed in order to choose appropriate time windows and their respective properties.

This drawback can be overcome by using time-domain Gaussian process models. In a first paper by the author (Zentner, 2007), this method together with heuristic correlation models from information

theory and statistical learning have been used (McKay 2003, Williams 1998). In this paper, a fully non stationary cross-correlation model based only on common ground motion models available in literature is constructed. Furthermore, a straightforward method for simulating the conditional ground motion field is proposed.

2. CONDITIONAL SIMULATION OF SPATIALLY INCOHERENT GROUND MOTION FIELDS

2.1. Gaussian process model for seismic ground motion prediction

Ground motion is modelled by a Gaussian stochastic process. Each earthquake acceleration record, observed at discrete times is treated as a realization of a set of Gaussian zero-mean random variables denoted here $\boldsymbol{a}_0 = (a_0^i, \dots a_0^N)$ (i refers to the recorded time instants t_i and the subscript to the location x_0). As it is common use, we will use capital letters to denote random variables and vectors and lower case letters to denote particular realizations. Then, the probability density function of the N-dimensional random vector $\boldsymbol{A}_0 \in \mathbb{R}^N$ is the multivariate Gaussian distribution with covariance matrix $\boldsymbol{K}_0 \in \mathbb{R}^{N \times N}$. The probability density function of random variables $\boldsymbol{A}_k \in \mathbb{R}^M$ modelling the ground motion at location x_k is, under condition that $\boldsymbol{A}_0 = \boldsymbol{a}_0$ has been observed, given by the conditional probability

$$f(\boldsymbol{a}_k|\mathbf{a_0}) = \frac{f(\boldsymbol{a}_k, \boldsymbol{a_0})}{f(\boldsymbol{a_0})}, \tag{2.1}$$

Let **K** be the covariance matrix of the joint distribution of $\boldsymbol{A}_0$ and $\boldsymbol{A}_k$:

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{K}_0 & \boldsymbol{K}_{0k} \\ \boldsymbol{K}_{0k}^T & \boldsymbol{K}_k \end{bmatrix} \in \mathbb{R}^{(N+M) \times (N+M)}, \tag{2.2}$$

where $\boldsymbol{K}_k \in \mathbb{R}^{M \times M}$ is the covariance matrix of vector $\boldsymbol{A}_k$ and $\boldsymbol{K}_{0k} \in \mathbb{R}^{N \times M}$ is the cross-covariance. It can be shown that $\boldsymbol{A}_k$ conditioned on $\boldsymbol{a}_0$ is a Gaussian random vector

$$(\boldsymbol{A}_k|\boldsymbol{a}_0) \sim \mathcal{N}\left(\boldsymbol{K}_{0k}^T \boldsymbol{K}_0^{-1} \boldsymbol{a}_0, \boldsymbol{K}_k - \boldsymbol{K}_{0k}^T \boldsymbol{K}_0^{-1} \boldsymbol{K}_{0k}\right). \tag{2.3}$$

Then we can use the classical formula for simulating a Gaussian random vector $\boldsymbol{Y} \sim (\boldsymbol{m}, \boldsymbol{\Sigma})$:

$$\boldsymbol{Y} = \boldsymbol{m} + \boldsymbol{L}\mathcal{X}, \tag{2.4}$$
$$\boldsymbol{\Sigma} = \boldsymbol{L}\boldsymbol{L}^T,$$

where **L** can be obtained by Cholesky decomposition and $\mathcal{X}$ is a vector of standardized independent Gaussian random variables.

2.2. Evaluation of the Covariance matrix from evolutionary Kanai-Tajimi model

In what follows, the practical construction of the covariances of the spatial ground motion field is described.
Dealing with Gaussian processes, the ground motion field can be described in the frequency domain by its power spectral density (PSD). It is acknowledged that seismic ground motion is non stationary both in amplitude and frequency content. This can be accounted for by choosing an evolutionary PSD model expressed as $S(\omega, t, t') = H(\omega, t)S_0(\omega)H^*(\omega, t')$ and introducing the coherency function $\gamma(\omega, d)$ with $d = |\boldsymbol{x} - \boldsymbol{x}'|$ such that:

$$S_d(\omega, t, t') = H(\omega, t)S_0(\omega)H^*(\omega, t')\gamma(\omega, \mathrm{d}), \tag{2.5}$$

A modulating function is generally applied to (2.5) in order to account for non stationary amplitudes. The autocorrelation function can be obtained Fourier Transform of the latter expression (Priestley, 1965):

$$R_d(t, t') = \text{h}(t) \int_\Omega e^{i\omega t(t-t')} H(\omega, t) S_0(\omega) \gamma(\omega, d) H^*(\omega, t') d\omega \; h(t'), \qquad (2.6)$$

where h(t) is the modulating function. This expression allows to evaluate the covariance matrices of equations (2.2),(2.3).

The evolutionary Kanai-Tajimi PSD (Lin & Yong 1987, Fan & Ahmadi 1990) is a very versatile evolutionary PSD model that accounts for the evolution of the frequency content of earthquakes:

$$S_{KT}(\omega, t) = \frac{\omega_g^4 + 4\xi_g^2 \omega_g(t)^2 \omega^2}{\left(\omega_g(t)^2 - \omega^2\right)^2 + \xi_g^2 \omega_g(t)^2 \omega^2} S_0, \qquad (2.7)$$

As can be seen from equation, the Kanai Tajimi (KT) model is essentially filtered white noise where the filter frequency ω_g depends on time. The latter can be estimated from the zero crossings of accelerograms (Fan & Ahmadi 1990, Rezaeian & Der Kiureghian 2010). Furthermore, the modified Kanai-Tajimi PSD (Clough& Penzien 1975) is generally preferred since it allows filtering the unwanted very low frequency content of the KT model. Such a filter has been added to the model (eq. 2.7) in the applications of §3.

Spatial coherence can be expressed by using coherency models available in literature, such as, for example, the semi-empirical Mita & Luco (1987) model:

$$\gamma(\omega, d) = exp\left[-\frac{(\alpha \omega d)^2}{v_S^2}\right]. \qquad (2.8)$$

2.3. Implementation

In summary, we have to perform the following steps in order to construct the spatially variable non stationary ground motion field:

- Select a pertinent accelerogram.
- Select an evolutionary PSD model and identify its parameters. For the applications presented in this paper, the modified evolutionary Kanai-Tajimi model has been chosen. This model is defined by a modulating function (in order to match the time-dependent amplitude) and the time varying fundamental ground frequency (in order to match the evolution of the frequency content). The latter function can be identified from the zero crossing rates.
- Choose the coherency model and evaluate the non stationary correlation matrix by virtue of equation (2.6)
- Compute the conditional ground motion field using equations (2.3) and (2.4).

The practical computation of these steps is illustrated in section 4 where conditional time histories are simulated for the El Centro 1940 earthquake NS record.

3. APPLICATION TO EL CENTRO RECORD

We adopt the modified evolutionary modified Kanai-Tajimi model in order to construct the non stationary correlation function for El Centro ground motion. The evolutionary frequency of El Centro 1940 earthquake is taken from the reference of (Fan & Ahmadi, 1990). Based on the knowledge of the free field ground motion in one point, we can now generate input ground motion for the supports of a long span structure. For our simulations we consider a highway bridge with 4 supports, denoted L0, L1, L2 and L3 each separated with distance *d=40m*. We suppose that the ground acceleration of the first support, L0, is specified by the El Centro accelerogram. We use the expression of evolutionary frequency developed in the reference of Fan & Ahmadi (1990) for of El Centro 1940 earthquake. In

the same reference, a modulating function has been fitted to match the time dependent standard deviation evaluated by averaging over windows of 2s. The estimated standard deviation and the modulating function are plotted on figure 3.1 together with the El Centro accelerogram. Using this data, we can generate conditional ground motion fields for the El Centro event. One ex   ample of a generated ground motion field is shown on figure 3.2. The conditional time histories as well as figure 3.1 suggest that the amplitude variation of El Centro is not accurately accounted for by standard modulating functions and in particular the one used here. This can be observed especially for the time interval [5s -10s], featuring very small amplitudes. An improvement of the ground motion prediction can be achieved by directly introducing the estimated standard deviation instead of the fitted modulating function. Figure 3.3 displays an example of predicted ground motion at stations L1, L2 and L3 for the modified model. One observes that the amplitudes of the simulated ground motion are in better agreement with the given ground motion when using the improved model (cf. figures 3.2 and 3.3). This model is in what follows called the "improved model".

In order to assess the accuracy of our model, we examine the response spectra of the simulated time histories and compare results to the El Centro response spectrum. The pseudo-acceleration response spectra of the conditionally simulated time histories are shown on figure 3.4. It can be concluded that the spectra of the simulated time histories are in good agreement with the El Centro response spectrum (red curve).

Finally, empirical coherencies are estimated and compared to the theoretical model. For this purpose, smoothed estimates of PSD and cross-PSD are used according to (Zerva 2009, Tseng et al 1997). The effects of time-varying amplitude are removed from the time histories for this purpose. The coherency functions, estimated from 20 realisations of spatial ground motion fields, are plotted on figure 3.5 together with the reference curves. The curves show the coherency for separation distances d=40m (stations L1-L2, L2-L3, L3-L4), d=80m (stations L1-L3, L2-L4) and d=120m (stations L1-L4). The red (dashed, dotted, dash-dotted) curves are the reference values from the Mita & Luco model while the solid lines are the estimated curves. The estimated coherencies are close to the reference. It can be concluded from these results that the studied features of El Centro ground motion can be reproduced by the model in a satisfactory way.
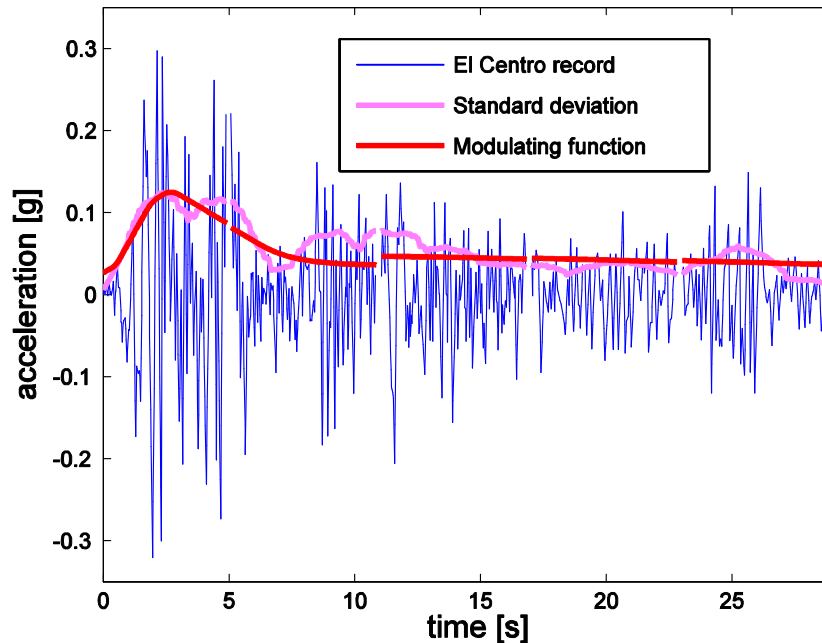

Figure 3.1. El Centro 1940 NS accelerogram, estimated standard deviation and modulating function of the evolutionary KT model according to Fan & Ahmadi (1990).
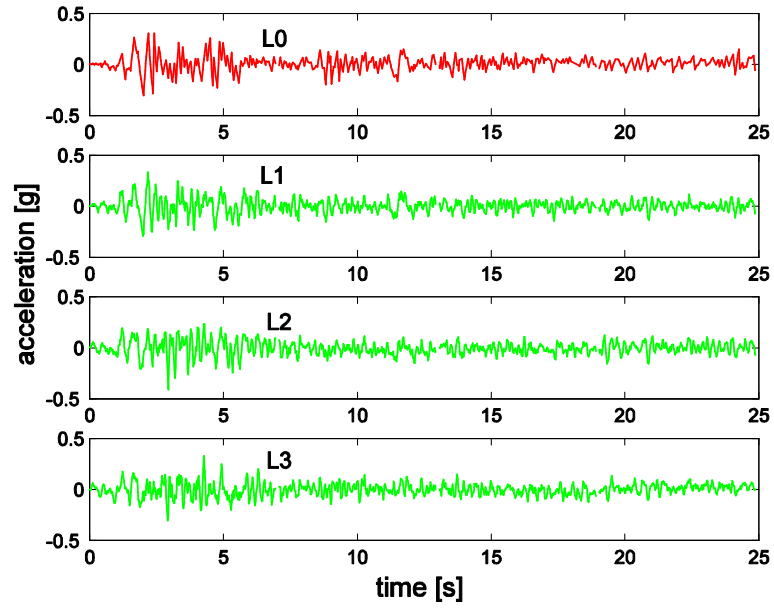
Figure 3.2. El Centro accelerogram and simulated time histories at locations L1, L2 and L3.
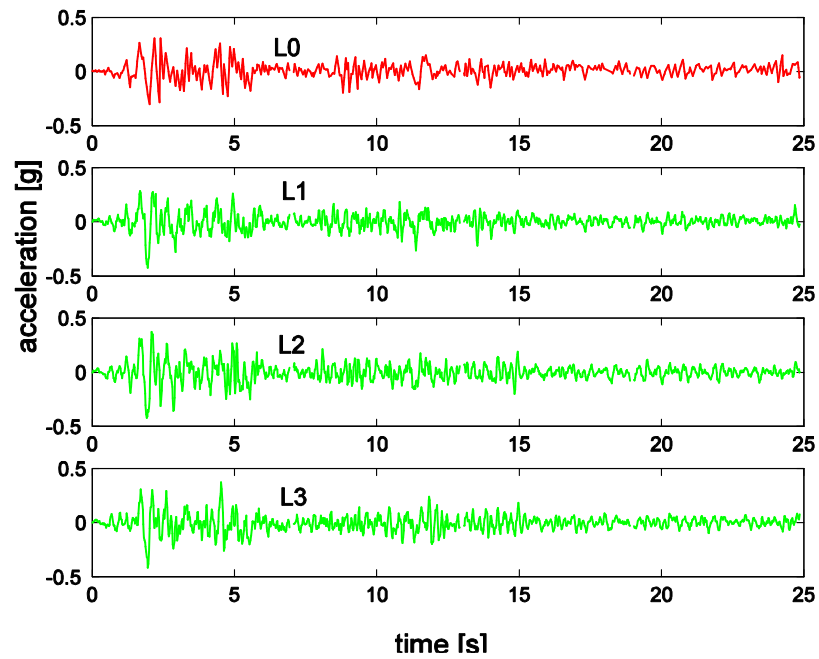

Figure 3.3. El Centro accelerogram and simulated time histories at locations L1, L2 and L3 using the estimated variances instead of the fitted modulating function.
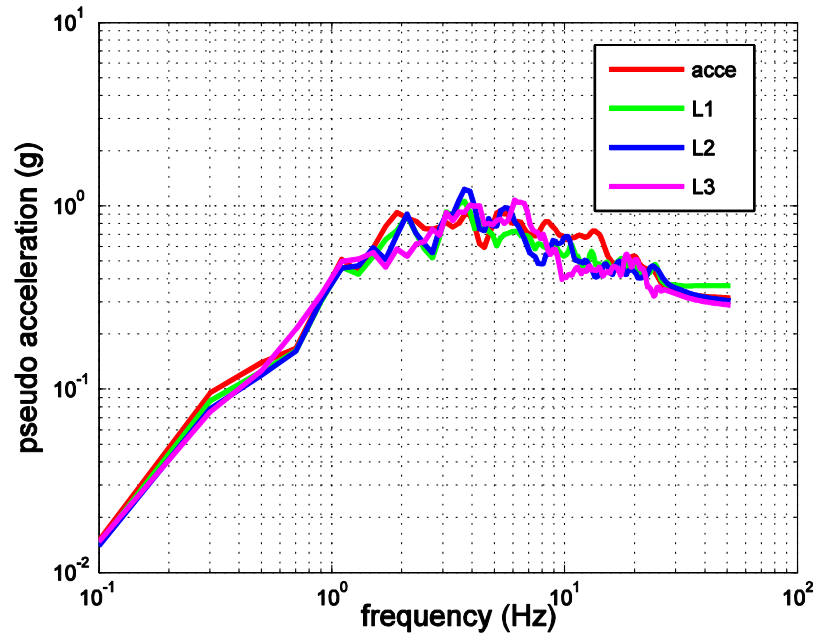
Figure 3.4. Pseudo-acceleration response spectra of El Centreo accelerogram (red) and of the simulated time histories (improved model).
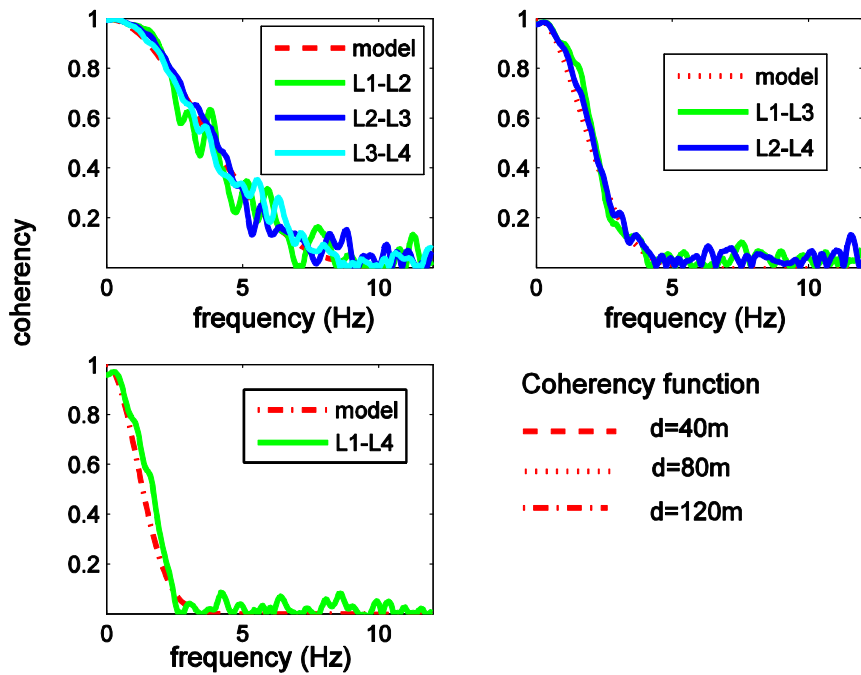

Figure 3.5. Estimated and theoretical coherency functions obtained after 20 simulations with the improved model.

5. CONCLUSION

A method for simulating seismic ground motion fields conditioned on a recorded accelerogram has been proposed. The non stationary correlation functions are determined from evolutionary PSD models, currently used for seismic analysis, and using a coherency function. The conditional ground motion field can be used for "best-estimate" transient seismic analysis required for seismic margin assessment or more generally in the framework of performance-based seismic analysis.

An application to the El Centro earthquake record is presented in order to illustrate the feasibility of the approach. The simulated ground motion fields show a good agreement with the theoretical coherency model.

REFERENCES

Clough RW., Penzien J. (1975), Dynamics of Structures. McGraw-Hill.

Fan FG., Ahmadi G. (1990), Nonstationary Kanai-Tajimi models for El Centro 1940 and New Mexico City 1985 earthquake. *Probabilistic Engineering Mechanics* **5**,171–181.

Kameda H., Morikawa H. (1992), An interpolating stochastic process for simulation of conditional random fields. *Probabilistic Engineering Mechanics* **7**, 243–254.

Konakli K., Der Kiureghian A. (2011), Simulation of spatially varying ground motions including incoherence, wave-passage and differential site-response effects. *Earth Engrg and Struct Dyn* DOI:1002/eqe.1141.

Liao S., Zerva A (2006), Physically compliant, conditionally simulated spatially variable seismic ground motions for performance-based design. *Earth Engrg and Struct Dyn* **35**,891-919.

Lin YK., Yong Y. (1987), Evolutionary Kanai-Tajimi earthquake models. *ASCE J Eng Mech* **113:8,**1119–1137.

MacKay (2003), Information Theory, Inference and Learning Algorithms, Cambridge University Press.

Mita A., Luco JE. (1987). Response of structures to a spatially random ground motion. *ASCE J Eng Mech* **113:1**,1--15.

Priestley MB. (1965), Evolutionary spectra and nonstationary processes. *J. Royal Statistical Society* 27:2,204–237.

Rezaeian S, Der Kiureghian A (2010), Simulation of synthetic ground motions for specified earthquake and site characteristics. *Earth Engrg and Struct Dyn* **39,**1155-1180.

Tseng WS., Lilhanand K. (1997), Soil structure interaction analysis incorporating spatial incoherence of ground motion. EPRI (Electric Power Research Institute), report TR-102631, Palo Alto.

Vanmarcke E., Heredi-Zavoni E., Fenton G (1993). Conditional simulation of spatially correlated earthquake ground motion. *ASCE J Eng Mech* **119:11**, 2333–2352.

Williams CKI. (1998), Prediction with Gaussian processes: from linear regression to linear prediction and beyond. M. Jordan, ed., Learning and Inference in Graphical Models, Kluwer Academic Press.

Zentner I., Devesa G. (2011), A Methodology for soil-structure interaction analysis accounting for spatially incoherent seismic free field motion. *Proceedings of EURODYN*, Leuven, Belgium.

Zentner I (2007). Simulation of spatially incoherent, non stationary seismic free field motion and its impact on response in SSI analysis. *Proceedings of Compdyn*, Rethymno, Greece, Papadrakakis DC. et al. (eds.).

Zerva A. (2009), Spatial Variation of Seismic Ground motion. Boca Raton, CRC Press.