Influence of Superstructure Flexibility on Seismic Response Pile Foundation in Sand

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SUMMARY:
Piles are extensively used to support heavy elevated structure, thermal power plants, petrochemical complexes, offshore structures, nuclear power plants etc. They are usually taken up to hard stratum surpassing soft clays and weak deposits. Piles are commonly subjected to the vertical, lateral, inclined and uplift loads and moments, and seismic loads. Seismic load in particular is dynamic in nature which exerts a force in lateral direction. Generally the pile foundation is analyzed for earthquake loads considering the superstructure as a lumped mass. Nevertheless, it is imperative to predict the response of pile foundation under seismic loads considering the effect of superstructure flexibility. This paper presents the effect of flexibility of foundation as well as that of the superstructure subjected to lateral earthquake loads. The study has been carried out on a prototype model with the help of shake table experiments.

Keywords: Pile Foundation; Soil-Structure Interaction, Soil-Pile Interaction; Earthquake, Shake Table Test

1. INTRODUCTION

It is common practice to design the foundation for vibration control by increasing the mass of the foundation, but it may not be a correct analysis as the flexibility of the superstructure is not considered and the response is different due to the soil-structure interaction. Considering the frequent occurrence of earthquakes all over the world, studies on the behavior of structures under dynamic excitations are of great importance. There are many parameters affecting the dynamic response of structures, such as; the type of structure, type of foundation, soil characteristics etc. Observations from the earthquake damaged sites show that the local soil properties and the foundation geometry have great influence on the dynamic behavior of the structures. The local soil conditions and the interaction between soil and foundation affect the dynamic behavior of a structure in three different ways, such as, soil amplification effect, kinematics interaction effect and inertial interaction effect. The total interaction effect is generally termed as soil-structure interaction. Structures always interact with its surrounding soil and respond quite differently depending upon its own properties and that of the supporting soil. However, the seismic analysis of structures is often based on the assumption that the foundation soil is a rigid block. This assumption is approximately valid for average size structures founded on sound rock. When the structure is supported on soil deposits, this assumption leads to erroneous results since the motion at the soil surface gets significantly changed by the presence of the structure. The dynamic characteristics of the structure, such as vibration modes and frequencies are modified by the flexibility of the supports.

Considerable amount of research work has been done on various aspects of soil-structure interaction to simulate the real phenomenon in modeling as close as possible. Analytical models using spring-dashpot-mass were developed for foundation vibrations with frequency dependent/ independent coefficients. Wolf (1997) considered few internal degrees of freedom, Nakhaei et al. (2008) modeled bilinear- single degree of freedom -SDOF, single degree of freedom spring-dashpot system with nonlinear hysteresis in form of elasto-perfectly plastic behavior by Chatterjee et al. (2008). Pile bending due to laterally spreading forces and axial-load induced settlement interaction were studied by Dash et al. (2009). Seismic performance of moment-resisting frame (MRF) steel buildings with multiple underground stories resting on shallow foundations was investigated by Ganainy et al. (2009). Effect of soil-structure interaction on the response of base-isolated buildings founded on an elastic soil half space was demonstrated by Spyrakos et al. (2009). Dynamic instability of pile-supported structures founded on liquefiable soils subjected to transverse bending, dynamic buckling and resonance motion failure mechanism of piles during an earthquake and analytically model the pile-soil system was illustrated by Bhattacharya et al. (2009). Numerical studies on dynamic soil-structure interaction effects on the seismic response of asymmetric buildings were reported by Shakib et al. (2004). Coupled boundary element-finite element method for dynamic soil-structure interaction effects on un-retrofitted and seismically isolated typical bridge structure was developed by Stehmeyer et al. (2008) and Pitilakisa et al. (2008). Importance of bending-buckling interaction in seismic design of piles in liquefiable soils using numerical techniques was reported by Dash et al. (2009). Experimental studies on the response of low-rise buildings frames resting on shallow foundations under seismic ground excitation incorporating soil-structure interaction were performed by Dutta et al. (2004, 2009). Multi-tower reinforced concrete shear framed tall building for its structural complexity and irregularity on model structure was designed and tested on the shaking table under minor, moderate, and major earthquake levels by Ying et al. (2009). Low-rise steel buildings supported by shallow isolated foundations on dense silty sand were tested to demonstrate the effect of uncertainty in soil parameters on seismic response of structures by Raychowdhury (2009, 2011). Experimental and numerical studies on design, fabrication and commissioning of a single axis laminar shear box for use in seismic soil-structure interaction studies were presented by Turan et al. (2009) and Pitilakisa et al. (2008).

Based on the critical assessment of literature it can be concluded that soil-structure interaction (SSI) effects are significant for buildings. The soil in the shaking table exhibit nonlinear behavior. The effect of soil-structure interaction may increase response for low-rise stiff structural system. The SSI increases the damage index before a threshold period which is closely related to the predominant period of the ground motion. The uncertainty in soil parameters may result in significant response variability of the structures, especially when vertical factor of safety is low and the structure is relatively stiff. Uncertainty in friction angle results in significant variability of the peak base moment and base shear, while peak inter story drift ratio is found to be unaffected by uncertainty in soil parameters. From the critical review of the literature it is also found that in structural analysis invariably, the superstructure is considered as lumped mass (neglecting the flexibility of superstructure) at the foundation level while predicting the foundation response under earthquake loads. However, the flexibility of superstructure may alter the behavior of pile foundation substantially. Moreover, the superstructure and foundation is a single unit in reality. It is found from the literature that in most studies the superstructures were modeled as a lumped mass only except a few in analytical studies. There are limited experimental studies reported on the influence of superstructure flexibility on the response of pile foundations embedded in soils, which is the main aim of the present study. This paper discusses the results of shake table experiments carried out on pile foundations embedded in sand. The superstructure was modeled as lumped mass as well as framed structure for ascertaining the effect of flexibility of superstructure.

2. EXPERIMENTAL INVESTIGATION

In the present study, 3, 5, 7 and 10-storey prototype framed steel building of room size, 3 m × 3 m with concrete slabs is considered, which is assumed to be supported on pile group foundation. Pile foundation and superstructure were modeled with a scale factor of 1:30, considering their geometry and flexural properties of the system. Locally available sand (at Badarpur, Delhi) is used for preparing the soil bed with calibrated to a relative density of 95 %. Hence accordingly the length to diameter (L/d) ratio of pile of 15 is considered in this study. Free vibration and forced vibration tests (equivalent earthquake load) are carried out on the shake table test facility for all the cases. Predominant frequency and peak acceleration of Bhuj earthquake (January 26, 2001) was considered for earthquake input. The details of material used and their properties, physical modeling for scaling the prototype in to small-scale models, details of experimental setup, instrumentation, loading and data acquisition are discussed in the following sections.

2.1 Material properties

The sand collected from Badarpur, New Delhi was used in this study. The laboratory investigation was carried out on Badarpur sand to know its geotechnical properties, like grain size distribution, specific gravity, shear strength parameter in saturated condition at the maximum density and minimum density as per Indian Standard (IS) codes of practice. Density index (relative density) parameter was found out using dry method as per IS code of practice. The summary of material properties of Badarpur sand is shown in Table 2.1.

Table 2.1 Summary of Material Properties of Badarpur Sand

Sl. No.	Parameter	Description	Value
1	Gravel size analysis	Gravel Size (%)	0
		Sand Size (%)	98
		Silt Size (%)	2
		Clay Size (%)	0
		D_{60} , (mm)	0.5
		D_{30} , (mm)	0.29
		D_{10} , (mm)	0.12
		Coefficient of Uniformity, C_u	4.2
		Coefficient of Curvature, C_c	1.4
2	Specific gravity		2.7
3	Relative density test	Minimum Dry Density (g/ cc)	1.49
		Maximum Dry Density (g/ cc)	1.76
		e_{max}	0.78
		e_{min}	0.5
4	Shear strength at minimum density	Cohesion (c) kg/ cm^2	0.0
		Angle of Internal Friction (ϕ)	28.7°
5	Shear strength at maximum density	Cohesion (c) kg/ cm^2	0.0
		Angle of Internal Friction (ϕ)	44.1°

2.2 Physical modeling

Both the substructures as well as superstructure were modeled in the laboratory to a scale of 1:30. The details of modeling are presented hereunder.

2.2.1 Modeling of pile foundation

The substructure, i.e. pile foundation subjected to lateral loads and seismic load was modeled as per scaling law given by Woods et al. (2002), which is given below:

$$\frac{(EI)_p}{(EI)_m} = n^5 \tag{1}$$

where EI = flexural rigidity, p stands for prototype, m stands for model, n = scale factor. Aluminum solid rod of 8 mm diameter was used to simulate the prototype pile diameter of 732 mm having M25 grade of concrete having characteristic compressive strength of 25 N/ mm^2. Model pile for L/d ratio of 15 was fabricated to support the superstructure. The reinforced concrete pile cap of size 3.15 m × 3.15 m × 0.8 m was simulated using model pile cap of size 105 mm × 105 m × 25 mm size having weight of 735 g using 1:30 scale factor. The dimensions of model pile cap were calculated from Equation (2-4) and are presented below.

$$M_p = \rho_p \times V_p \quad (2)$$

where M_p = mass of prototype, ρ_p = density of reinforced concrete in prototype, V_p = volume of prototype

$$M_m = \frac{M_p}{n^3} \quad (3)$$

where M_m = mass of model, M_p = mass of prototype, n = scale factor

$$t_m = \frac{M_m}{A_m \times \rho_m} \quad (4)$$

where t_m = thickness of model pile cap, M_m = mass of model pile cap, A_m = area of model pile cap, ρ_m = density of model pile. The area of the model pile cap was arrived based on the linear scaling factor (n). Then the thickness of model pile cap was calculated based on Equation 4. The mode pile group is shown in Figure 1.

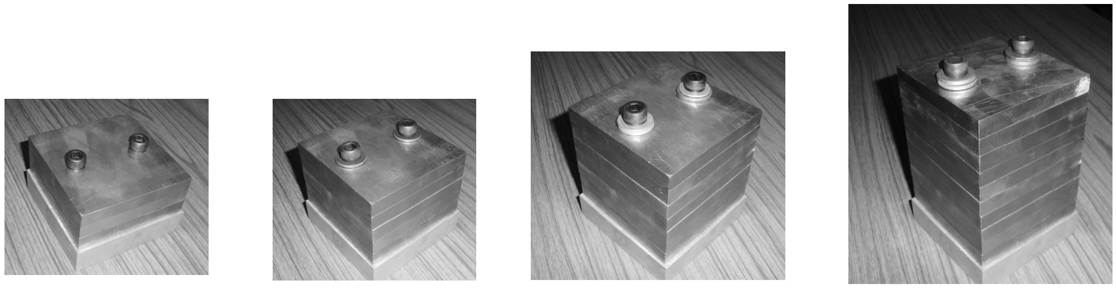
Figure 1. Model pile group with L/ d = 15.

2.2.2 Modeling of superstructure

The superstructure was considered as 3, 5, 7 and 10-storied steel framed structure of room size, 3 m × 3 m, each storey height of 3 m with steel column section ISLC 300 × 200 and steel beam ISLB 200 × 100 and for simulating the same damping ratio in prototype and model. Here, ISLC is Indian standard lightweight steel channel section, and ISLB is Indian standard lightweight steel beam section. The slab is made of reinforced concrete having 120 mm thickness in prototype. The dimensions of prototype and model are calculated by equation given by Woods (2004) and presented in Table 2.2.

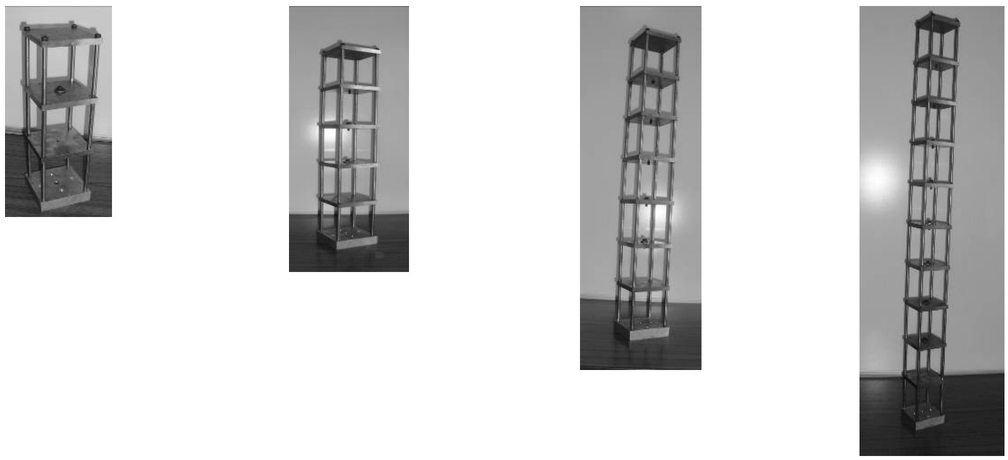
$$\frac{\omega_m}{\omega_p} = n^{1-\frac{\alpha}{2}} \quad (5)$$

where ω_m = frequency of model structure, ω_p = frequency of prototype structure, α = 0.5 for sand, n = scale factor. For a scale factor of 30, the value of this equation becomes 12.82. Based on the analysis in MATLAB using the model and prototype properties, frequency ratio obtained is 12.83. Hence, the modeling of superstructure is found in order. The superstructure is modeled with lumped mass as well as framed structure to investigate the effect of superstructure flexibility, which is shown in Figure 2.



3 Storey Model 5 Storey Model 7 Storey Model 10 Storey Model

Figure 2 (a). Superstructure as lumped mass model.



3 Storey Model 5 Storey Model 7 Storey Model 10 Storey Model

Figure 2 (b). Superstructure as framed structure model.

Table 2.2 Dimension of Superstructure (Prototype and Model)

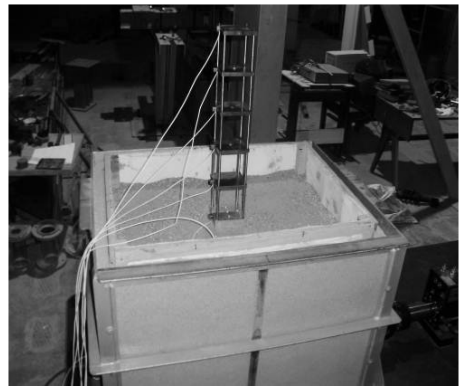
SL No.	Description	Prototype (Steel and R.C.C.)	Model (Aluminum)
1	Mass of 1st to 9th Floor	Mass of One floor 3m × 3m × 0.12 m (RCC), Steel Beam ISLB 200 × 100 and Four columns ISLC 300 × 200 (half length above and half length below the Slab) 8585 kg	Mass of 10 cm × 10 cm × 1.1 cm Steel plate with 2 Threaded Steel Rod of 8 mm diameter of total mass = 1.014 Kg
	Mass of 10th Floor	8216 kg	10 cm × 10 cm × 1.2 cm Steel plate with 2 Threaded Steel Rod of 8 mm diameter of total mass = 0.958 kg
2	Stiffness of Column	ISLC 300 x 200 Column section of Stiffness, K_p= 2.4870222 x 10^7 N/ m	8 mm diameter Steel Rod of Stiffness, K_m = 482743 N/ m
3	Frequency of 10-Storied Structure estimated using MATLAB	ω_p = 16.1546 radian/ sec	ω_m = 207.3 radian/ sec

2.3 Experimental setup

2.3.1 Test tank

The test tank fabricated to a size of 0.9 m length, 0.7 m width and 0.6 m height with 0.008 m thickness steel sheet from three sides and bottom of tank, one side of tank was made with Acrylic Perspex sheet of 10 mm thickness. The steel tank tightened using threaded bolts over the 1 m × 1 m shake table. The size of test tank is decided based on the basis of pay load capacity of shake table. The another flexible box of size of 780 mm length, 580 mm width and 610 mm height fabricated with steel wire mesh from all four sides wrapped with cotton cloth and pasted by Araldite solution, supported by wooden frame. The flexible box kept inside the steel box and the gap of 60 mm between steel tank and flexible box filled with saw dust to minimize the reflection of waves during the application predominant earthquake frequency and amplitude. The model test tank is shown in Figure 3.

(a) 5-storied lumped mass model



(b) 5-storied framed structure model

Figure 3. Model test tank.

2.3.2 Shake table testing facility and

The uniaxial shake table consists of Size of 1 m × 1 m, having payload capacity of 900 kg with the actuator capacity of 30 kN and 150 mm stroke length. Smaller-sized shake tables are better suited for small-scale model analysis. In addition, they avoid high operational and development costs, but are versatile enough in the case of dynamic experiments for instructional and research purposes. Accelerometers with sensitivity of 978, 991, 1020, 1036, 1050, 1075 and 1088 mv/ g were used to measure the acceleration response at pile cap level and at different storey levels directly. A data acquisition system (DAS) consisting of a multi-channel carrier frequency amplifier, data logger with LabVIEW software was used to create programs in block diagram form.

2.3.3 Sand bed and model preparation

The sand bed preparation means filling of sand to achieve uniform density in tank and proper placement of models on prepared sand bed. To achieve the uniform density, the sand loosely filled in the flexible box approximately 11 cm and vibrated using shake table for 6.7 minute (4000 cycle) at amplitude of 0.5 mm and 10 Hz frequency for each layer. The maximum relative density of 95 % (maximum dry density of 1.744 g/ cc) was achieved after completion of 50 cm fill, which simulate the very dense condition. Framed structure model and lumped mass model supported by piles with L/d = 15 were placed vertically over prepared sand bed and inserted gently by increasing weight over the model. The arrangement of accelerometer on framed structure and lumped mass models are shown in Figure 3.

2.3.4 Earthquake loading and test plan

Bhuj earthquake (January 26, 2001) is considered as an input for earthquake loading for all the model tests conducted. The magnitude of Bhuj earthquake is 7.9. Since the existing Shake Table facility can not apply random earthquake loading directly, an equivalent harmonic loading is applied by considering the predominant frequency of Bhuj earthquake and equivalent number of uniform stress cycles. Depending on Magnitude of Earthquakes, Equivalent number of uniform stress cycles calculated from Bhuj earthquake with predominant frequency of 1.2 Hz, Peak acceleration of 1.038 m/ s^2, magnitude of 7.9 on Modified Richter scale, the equivalent uniform stress cycles is calculated to be 30. The detailed test plan of the experiments on models is shown in Table 2.3. Free vibration tests were also carried out by giving a sudden jerk of shaking table for determining the natural frequencies of soil-foundation-structure system.

Table 2.3 Test Plan

L/ d ratio of pile foundation = 15	
Pile Cap with 3 Storey Lumped Mass	Pile Cap with 3 Storey Framed Structure
Pile Cap with 5 Storey Lumped Mass	Pile Cap with 5 Storey Framed Structure
Pile Cap with 7 Storey Lumped Mass	Pile Cap with 7 Storey Framed Structure
Pile Cap with 10 Storey Lumped Mass	Pile Cap with 10 Storey Framed Structure

3. RESULTS AND DISCUSSION

3.1 Free vibration test

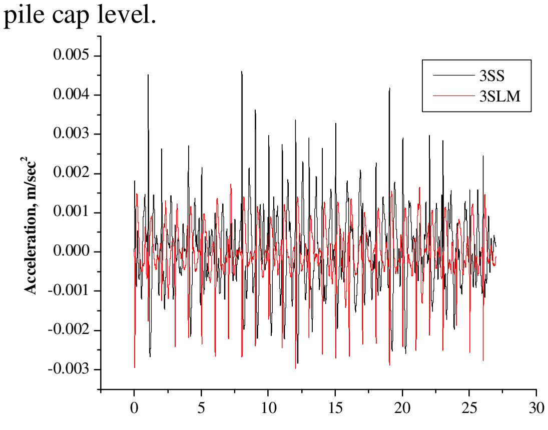
Free vibration tests were carried on 3, 5, 7 and 10-storied lumped mass models and framed structure models. The time history response of the lumped mass and framed structure models were measured using LabVIEW software. The time history response is converted into frequency domain by doing fast Fourier transform (FFT) analyses. The fundamental natural frequency obtained for different models from time history and Fourier spectrum are presented herein in Table 3.1. It is observed that fundamental natural frequency decreases as the number of storey increases. However, it is inferred that compared to lumped model, the fundamental natural frequency is much lower when we consider the framed structure. The reduction is the natural frequency is 40 to 65 %.

Table 3.1. Natural Frequency of Lumped Mass and Framed Structure Models

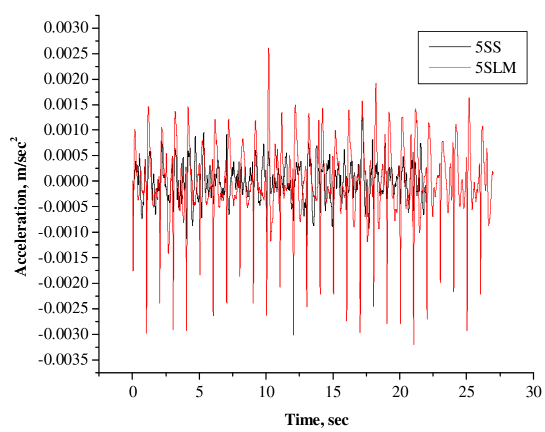
No. of Storey	Fundamental Natural Frequency (Hz)	
	Lumped mass model	Framed structure model
3-Storied	10.5	6.1
5-Storied	9.5	4.1
7-Storied	8.7	2.9
10-Storied	6.1	Model toppled down during test

3.2 Forced vibration test

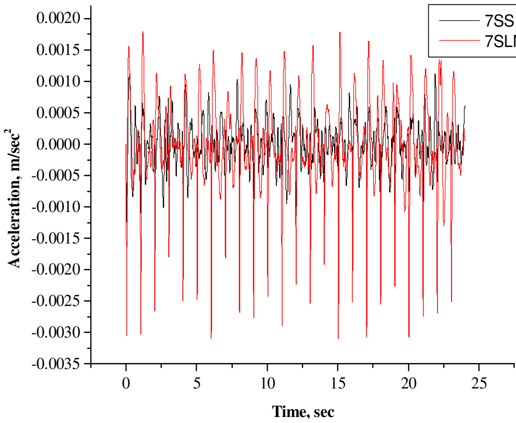
The models were tested under forced vibration considering the magnitude, acceleration and equivalent uniform stress cycles of 2001 Bhuj earthquake. Pile foundation with superstructure modeled as a lumped mass and framed structure were tested and the time histories of acceleration at the pile cap level, top storey level and at intermediate storey levels were monitored and measured online through a data acquisition system (DAS) consisting of a multi-channel carrier frequency amplifier and data logger and computer. The typical time histories of acceleration at pile cap level for different storey models are shown in Figure 4. It is observed from Figure 4(a) that the acceleration at pile cap level in 3-storied framed structure model is more than that of the 3-storied lumped mass model. This indicates that for short/ rigid building, consideration of superstructure flexibility may amplify the acceleration at pile cap level.



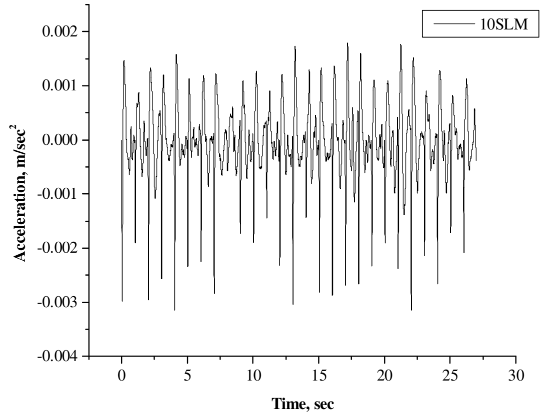
(a) 3-Storey



(b) 5-Storey



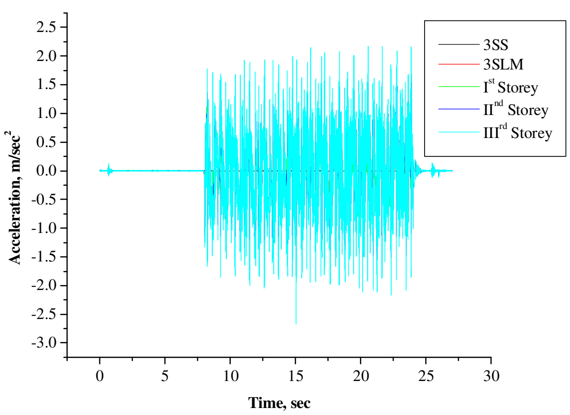
(c) 7-Storey



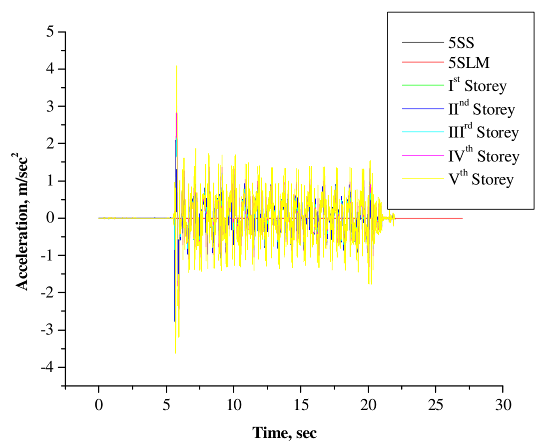
(d) 10-Storey

Figure 4. Time history plots of acceleration at pile cap level of lumped mass and framed structure models

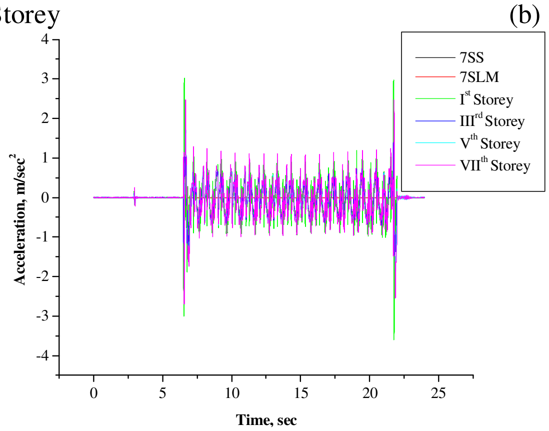
It is noticed from 4(b) to (d) that the acceleration at pile cap level in 5 to 10-storied framed structure model is lesser in comparison with their respective lumped mass models. This indicates that for flexible/ tall building, consideration of superstructure flexibility may reduce the acceleration response at pile cap level. The typical time histories of acceleration at different storey levels measured for different storey models are shown in Figure 5. Since the 10-storey framed structure model toppled down during the shaking, the time history of acceleration at pile cap and different storey levels were not measured.



(a) 3-Storey



(b) 5-Storey



(c) 7-Storey

Figure 5. Time history plot of acceleration at different storey levels of lumped mass and framed structure models

It is observed from Figure 5 that the acceleration at top storey level is always larger than the intermediate storey levels, which is obvious. Plots of peak acceleration measured at pile cap level versus number of storey is shown in Figure 6 for the responses measured from lumped mass models and framed structure models. From the acceleration response measured at pile cap level due to framed structure mass and lumped storey mass, it is observed that the acceleration increases for the framed structure model up to 4th storey level and then decreases beyond that. However, for the lumped mass model, the acceleration remains more or less constant. This indicates that for short/ rigid building, consideration of superstructure flexibility may increase the acceleration at pile cap level and for flexible/ tall building; consideration of superstructure flexibility may reduce the acceleration at pile cap level.

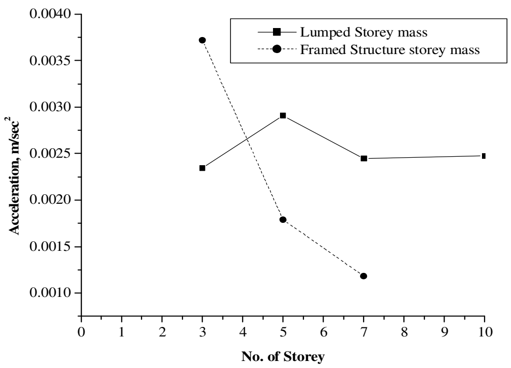


Figure 6. Maximum acceleration response at pile cap level of framed structure model and lumped mass models

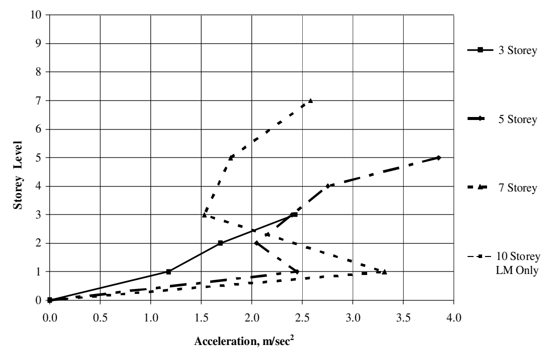


Figure 7. Acceleration response at storey levels measured from framed structure model

Plots of peak acceleration measured at different storey levels versus number of storey is shown in Figure 7 for the responses measured from lumped mass models and framed structure models. It is observed from the figure that the top floor always undergo increased acceleration. From the measured acceleration at 1st floor level for 3, 5 and 7-storey, it is observed that the acceleration increases at 1st floor level as number of storey increases in the structure. It is also noticed that the framed structure model of 10-storey model was toppled town during the shaking since the pile length is not adequate to support the 10-storey building under earthquake load although the soil is very dense. The FFT analysis was carried out for the measured time histories of response of all models and the Fourier amplitude of each model was calculated and plotted in Figure 8. It is observed from the figure that Fourier amplitude increases as the number of storey increases and remains constant.

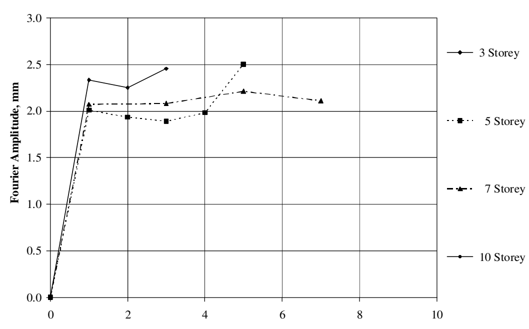


Figure 8. Fourier amplitude at pile cap level of framed structure model

4. CONCLUSIONS

The following conclusions have been arrived at based on the investigation reported:

1. Natural frequency decreases as the number of storey level increases for the framed structure model and lumped mass model. However, it is inferred that compared to lumped model, the fundamental

natural frequency is much lower when we consider the framed structure. The reduction is the natural frequency is 40 to 65 % for the models supported by piles with *L*/ d = 15.

2. It is observed that the superstructure flexibility, i.e. when the framed structure model is considered, the pile foundation response at pile cap level is significantly different, compared to lumped mass model. This clearly indicates that the superstructure flexibility has significant influence on the pile foundation response, which needs to be properly considered in the analysis and design. From the acceleration response measured at pile cap level for the framed structure model and lumped storey model, it is observed that the acceleration is high for 3 storey level in framed structure model compared to lumped mass model and then decreases, much below the values of lumped mass model for 5, 7 and 10 storey for the models supported by piles with *L*/ *d* = 15. This shows that typically for short/ rigid building, consideration of flexibility amplifies the acceleration and for long flexible buildings, it reduces the acceleration at pile cap level.

ACKNOWLEDGEMENT

The authors are thankful to Director, IIT Delhi and Director, CRRI, New Delhi for kind permission to publish this research paper.

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