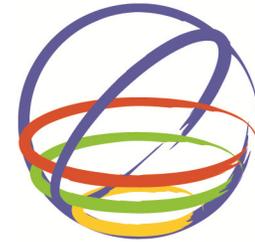


Initial Stiffness of Reinforced Concrete Columns and Walls

Bing Li

*School of Civil and Environmental Engineering, Nanyang Technological University,
Singapore 639798*



15 WCEE
LISBOA 2012

SUMMARY:

The estimation of the initial stiffness of columns and walls subjected to seismic loadings has long been a matter of considerable uncertainty. This paper reports a study that is devoted to addressing this uncertainty by developing a rational method to determine the initial stiffness of RC columns and walls when subjected to seismic loads. A comprehensive parametric study based on a proposed method is initially carried out to investigate the influences of several critical parameters. Two simple equations are then proposed to estimate the initial stiffness of RC columns and walls. The applicability and accuracy of the proposed method and equation are then verified with the experimental data obtained from literature studies.

Keywords: Reinforced Concrete; Column; Wall; Initial Stiffness

1. INTRODUCTION

The past 15 years have seen major developments in seismic design provisions, with a paradigm shift from a force-based approach to a displacement-based one and an increasing in focus on the deformation characteristics of structures. Stiffness properties of reinforced concrete (RC) column and wall structures can affect the estimation of the fundamental period, displacements and the distributions of internal force response. The initial stiffness depends on the intensity and distribution of stress on walls and columns cross-section, as well as the extent of flexural and shear cracks. Flexural cracking causes reduction in the net cross sectional area and moment of inertia, hence a reduction in initial flexural rigidity of the wall and column section. This leads to the increasing difficulty in making accurate predictions of the initial stiffness of RC members. Thus, stiffness reduction factor is employed in the analysis of RC members under lateral loads. In practice, the value of 0.35 and 0.70 the gross moment of inertia for cracked and un-cracked members, respectively is widely employed. However, this simplification may not be appropriate in many cases as the recommended moment of inertia for walls and columns is independent of the reinforcement content and axial load level.

2. REVIEW OF EXISTING INITIAL STIFFNESS MODELS

There are two methods as illustrated in Fig. 1 that are commonly utilized to determine the initial stiffness of RC columns (K_i). In the first method, the initial stiffness of RC columns are estimated by using the secant of the shear force versus lateral displacement relationship passing through the point at which the applied force reaches 75% of the flexural strength (Point A' in Fig.1). In the second method, the column is loaded until either the first yield occurs in the longitudinal reinforcement or the maximum compressive strain of concrete reaches 0.002 at a critical section of the column (point A in Fig.1). Generally, the two approaches give similar values. In this study, the later approach was adopted. Assuming the column is fixed against rotation at both ends and has a linear variation in curvature over the height of the column, the measured initial moment of inertia can be determined as:

$$I_e = \frac{L^3 K_i}{12E_c} \quad (2.1)$$

The stiffness ratio (κ) is defined as follows:

$$\kappa = \frac{I_e}{I_g} \times 100\% \quad (2.2)$$

where I_g is the moment of inertia of the gross section; K_i is the initial stiffness of columns and L is the height of columns and E_c is the elastic modulus of concrete.

ACI 318-08 (2008) recommends the following options for estimating member stiffness for the determination of lateral deflection of building systems subjected to factored lateral loads: (a) $0.35EI_g$ for members with an axial load ratio of less than 0.10 and $0.70EI_g$ for members with an axial load ratio of more than or equal to 0.10; or (b) $0.50EI_g$ for all members. FEMA 356 (2000) suggests the variation of initial stiffness values with the applied axial load ratio. The initial stiffness is taken as $0.50EI_g$ for members with an axial load ratio of less than 0.30, while a value of $0.7EI_g$ is adopted for members with an axial load ratio of more than 0.50. This value varies linearly for intermediate axial load ratios as illustrated in Fig. 2. As shown in Fig. 2, ASCE 41 (2007) recommends that the initial stiffness is taken as $0.30EI_g$ for members with an axial load ratio of less than 0.10, as $0.7EI_g$ for members with an axial load ratio of more than 0.50 and varies linearly for intermediate axial load ratios. According to Paulay and Priestley's recommendation (1992), the initial stiffness is taken as $0.40EI_g$ for members with an axial load ratio of less than 0.05, as $0.8EI_g$ for members with an axial load ratio of more than 0.50 and varies linearly for intermediate axial load ratios as illustrated in Fig. 2.

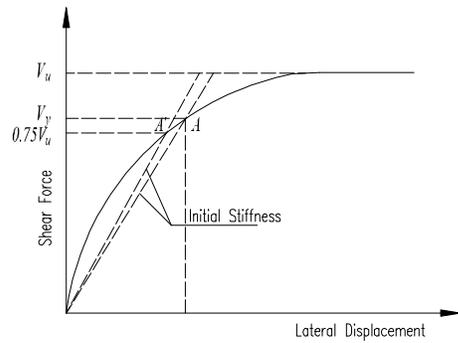


Figure 1. Methods to Determine Initial Stiffness

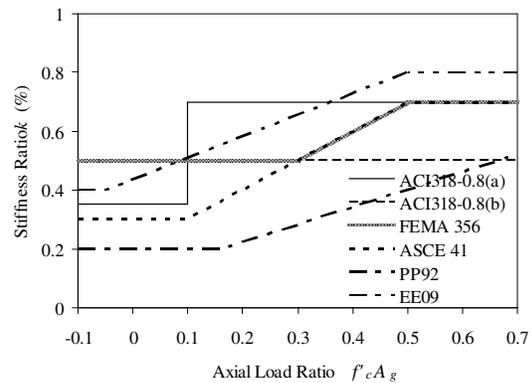


Figure 2. Relationships between Stiffness Ratio and Axial Load Ratio of Existing Models

3. INITIAL STIFFNESS OF RC COLUMNS

3.1. Proposed Method

Yield Force (V_y)

The initial stiffness of columns is determined by applying the second method as described in the previous section. The yield force (V_y) corresponding to point A in Fig. 1 is obtained from the yield moment (M_y) when the reinforcing bar closest to the tension edge of columns has reached its yield strain. Moment-curvature analysis is adopted to determine this moment.

Displacement at yield force (Δ'_y)

The displacement of a column at yield force (V_y) can be considered as the sum of the displacement due to flexure, bar slip and shear.

$$\Delta'_y = \Delta'_{flex} + \Delta'_{shear} \quad (3.1)$$

where Δ'_y is the displacement of a column at yield force; Δ'_{flex} is the displacement due to flexure and bar slip at yield force; and Δ'_{shear} is the displacement due to shear at yield force

Flexure Deformations (Δ'_{flex})

In this proposed method, the simplified concept of an initial length of the member suggested by Priestley *et al.* (1996) was used to account for the displacement due to bar slip in flexure deformations. Assuming a linear variation in curvature over the height of the column, the contribution of flexural deformations and bar slips to the displacement at the yield force for RC columns with a fixed condition at both ends can be estimated as follows:

$$\Delta'_{flex} = \frac{\phi'_y (L + 2L_{sp})^2}{6} \quad (3.2)$$

where ϕ'_y is the curvature at the yield force determined by using moment-curvature analysis and L is the clear height of columns.

The strain penetration length (L_{sp}) is given by:

$$L_{sp} = 0.022 f_{yl} d_b \quad (3.3)$$

where f_{yl} is the yield strength of longitudinal reinforcing bars; and d_b is the diameter of longitudinal reinforcing bars.

Shear Deformations (Δ'_{shear})

Park and Paulay (1975) derived a method to determine the shear stiffness by applying the truss analogy for short or deep rectangular beams of unit length. The shear stiffness is the magnitude of the shear force, when applied to a beam of unit length that will cause unit shear displacement at one end of the beam relative to the other. This model is reliable in estimating shear deformations of short or deep beams in which the influences of flexure are negligible. The behaviors of RC columns under seismic loading are much more complex because of the interaction between shear and flexure. The influences of axial strain due to flexure in estimating shear deformations of RC columns should be considered to accurately predict the initial stiffness of RC columns. By applying a method that is similar to Park and Paulay's analogous truss model (1975), the shear stiffness of RC columns is derived in this part of the paper. The effects of flexure in shear deformations are incorporated in the proposed model through the axial strains at the center of columns ($\epsilon_{y,CL}$).

Assuming that transverse reinforcing bars start resisting the applied shear force when the shear cracking starts occurring, the stress in transverse reinforcing bars at the yield force is calculated as:

$$f_{sy} = \frac{(V_y - V_{cr})s}{A_{st}d \tan \theta} \quad (3.4)$$

where d is the distance from the extreme compression fiber to centroid of tension reinforcement; s is the spacing of transverse reinforcement; A_{st} is the total transverse steel area within spacing s ; and θ is the angle of diagonal compression strut. Hence the strain in transverse reinforcing bars is:

$$\epsilon_x = \frac{f_{sy}}{E_s} \leq \epsilon_{yt} \quad (3.5)$$

where ϵ_{yt} is the yield strain of transverse reinforcing bars; E_s is the elastic modulus of steel.

Similar to Park and Paulay's model (1975), the concrete compression stress at the yield force is given as:

$$f_2 = \frac{V_y}{bL_{cs} \cos \theta} \quad (3.6)$$

where b is the width of columns; $L_{cs} = d \sin \theta$ is the initial depth of the diagonal strut as shown in Fig. 3a.

Hence the strain in the concrete compression strut is given as:

$$\epsilon_2 = \frac{f_2}{E_c} \quad (3.7)$$

where E_c is the elastic modulus of concrete given as:

$$E_c = 5000\sqrt{f_c} \quad (3.8)$$

Based on Vecchio and Collins's model (1986), the initial compressive strength of concrete is calculated as follows:

$$f_{ce} = \frac{f_c'}{0.8 + 170\epsilon_1} \leq f_c' \quad (3.9)$$

By applying Mohr's circle transformation for the mean strains at the center of Section C-C as shown in Fig. 3b, it gives:

$$\epsilon_1 = \frac{\epsilon_x + \epsilon_{y,CL}}{2} + \sqrt{\left(\frac{\epsilon_x - \epsilon_{y,CL}}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \quad (3.10)$$

$$\epsilon_2 = \frac{\epsilon_x + \epsilon_{y,CL}}{2} - \sqrt{\left(\frac{\epsilon_x - \epsilon_{y,CL}}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \quad (3.11)$$

$$\tan 2\theta = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_{y,CL}} \quad (3.12)$$

For the axial mean strains, compatibility requires that the plain sections remain plane. Hence the mean strain at the center of section C-C is given as:

$$\epsilon_{y,CL} = \frac{\epsilon_{y,top} + \epsilon_{y,bot}}{2} \quad (3.13)$$

where $\epsilon_{y,top}$, $\epsilon_{y,bot}$ are the axial strains at the extreme tension and compression fibers, respectively as shown in Fig. 3(d).

There are six variables, namely ϵ_x , $\epsilon_{y,CL}$, γ_{xy} , ϵ_1 , ϵ_2 and θ ; and six independent equations (3.5), (3.7), (3.10), (3.11), (3.12) and (3.13). By solving these six independent equations, the shear strain (γ_{xy}) at the center of section C-C could be determined.

The column is divided into several segments along its height of the column to determine the total shear deformation at the top of the column. The mean axial strain at the center of the section is determined based on the moment-curvature analysis. The shear strains at the lower and upper section of the segment are calculated using the above equations. Hence, the total shear displacement caused by the yield force can be calculated as follows:

$$\Delta'_{shear} = \sum_{i=1}^n \left(\frac{\gamma_{xy}^i + \gamma_{xy}^{i+1}}{2} \right) h_i \quad (3.14)$$

where γ_{xy}^i and γ_{xy}^{i+1} are the shear strains at the lower and upper section of the segment i ; h_i is the height of segment i and n is the number of segments.

Initial Stiffness

Once the flexural and shear deformations at the top of columns under yield force are obtained, the initial stiffness of columns can be determined as:

$$K_i = \frac{V_y}{\Delta'_{flex} + \Delta'_{shear}} \quad (3.15)$$

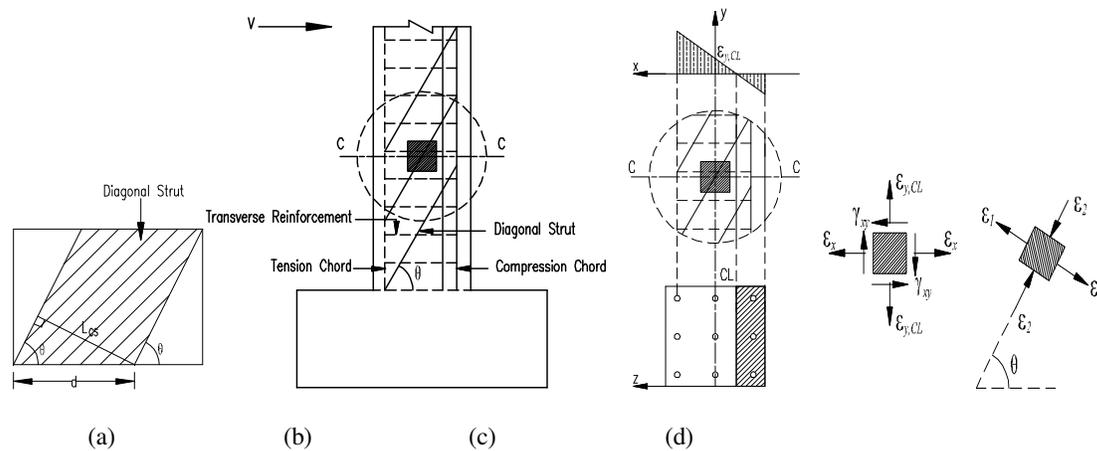


Figure 3. Influences of Flexure in Estimating Shear Deformations

3.2. Validation of the Proposed Method

The proposed method is validated by comparing its results to the initial stiffness of six columns obtained from the experimental study previously conducted by Tran and Li. (2012). It was found that the average ratio of experimental to predicted initial stiffness by the proposed method was 0.735 as tabulated in Table 1. It shows a relatively good correlation between the analytical and experimental results. The mean ratio of the experimental to predict initial stiffness and its coefficient of variation were 0.242 and 0.060, 0.301 and 0.076, 0.262 and 0.054, 0.312 and 0.084, 0.232 and 0.046, and 0.588 and 0.104 for ACI 318-2008(a) (2008), ACI 318-2008(b) (2008), FEMA 356 (2000), ASCE 41 (2007), Paulay and Priestley (1992) respectively. Comparison of available models with experimental data indicated that the proposed method produced a better mean ratio of the experimental to predicted

initial stiffness than other models. The proposed method may be suitable as an assessment tool to calculate the initial stiffness of RC columns.

Table 1. Experimental Verification of the Proposed Method

Specimen	K_{i-exp} (kN/mm)	$\frac{K_{i-exp}}{K_{i-p}}$	$\frac{K_{i-exp}}{K_{i-ACI(a)}}$	$\frac{K_{i-exp}}{K_{i-ACI(b)}}$	$\frac{K_{i-exp}}{K_{i-FEMA}}$	$\frac{K_{i-exp}}{K_{i-ASCE}}$	$\frac{K_{i-exp}}{K_{i-PP}}$	$\frac{K_{i-exp}}{K_{i-EE}}$
SC-2.4-	12.9	0.782	0.254	0.355	0.355	0.444	0.305	0.793
SC-2.4-	15.5	0.572	0.301	0.421	0.301	0.301	0.263	0.525
SC-1.7-	24.5	0.918	0.319	0.223	0.223	0.372	0.236	0.560
SC-1.7-	26.9	0.865	0.169	0.236	0.236	0.295	0.203	0.590
SC-1.7-	28.8	0.653	0.188	0.263	0.239	0.239	0.190	0.553
SC-1.7-	34.4	0.620	0.220	0.308	0.220	0.220	0.193	0.507
Mean		0.735	0.242	0.301	0.262	0.312	0.232	0.588
Coefficient of		0.141	0.060	0.076	0.054	0.084	0.046	0.104

3.3. Parametric Study for Initial Stiffness of Columns

A parametric study conducted to improve the understanding of the effects of various parameters on the initial stiffness of RC columns is presented within this section. The parameters investigated are concrete compressive strength (f'_c), aspect ratio (a/d) and axial load ratio ($P/f'_c A_g$). In the parametric study, the effects of the parameters that were investigated on the initial stiffness of RC columns are presented by the dimensionless stiffness ratio (k).

Specimen SC-2.4-0.20 with an aspect ratio of 2.4 (Tran & Li 2012) is considered as the reference specimen in the parametric study. An axial load of $0.2 f'_c A_g$ was applied to the specimen. The concrete compressive strength of the specimen (f'_c) at 28 days was 25.0 MPa. The longitudinal reinforcement consisted of 8-T20 (20 mm diameter). This resulted in the ratio of longitudinal steel area to the gross area of column to be 2.05%. The transverse reinforcement consisted of R6 bars (6 mm diameter) with 135° bent spaced at 125 mm, corresponding to a transverse reinforcement ratio of 0.129%.

Influence of Concrete Compressive Strength

Fig. 4a illustrates the influence of concrete compressive strength on stiffness ratios for two different axial loads of $0.05 f'_c A_g$ and $0.20 f'_c A_g$. The concrete compressive strengths investigated were 25MPa, 35MPa, 45MPa, and 55MPa. For both axial loads, with an increase in concrete compressive strength, no significant changes on stiffness ratios were observed.

Influence of Longitudinal Reinforcement Ratio

The influence of longitudinal reinforcement ratios on stiffness ratios is presented in Fig. 4b for two different column axial loads of $0.05 f'_c A_g$ and $0.20 f'_c A_g$. Four types of longitudinal reinforcement, 8T16, 8T20, 8T22 and 8T25 corresponding to longitudinal reinforcement ratios ρ_l of 1.66%, 2.05%, 2.48% and 3.21% respectively, were considered.

As shown in Fig. 4b, the stiffness ratios for columns under an axial load of $0.05 f'_c A_g$ were observed to rise slightly with an increase in longitudinal reinforcement ratio; while for columns under an axial load of $0.20 f'_c A_g$ the stiffness ratios almost remained the same. This suggested that for simplicity the influence of longitudinal reinforcement ratio on the initial stiffness of RC columns could be ignored.

Influence of Aspect Ratio

Fig. 4c show the influence of aspect ratio on stiffness ratios of RC columns. Six aspect ratios of 1.50, 1.80, 2.10, 2.43, 2.70, and 3.00 were investigated. In general, the stiffness ratio increased with an increase in aspect ratio. It can be seen that with an increase in aspect ratio from 1.50 to 1.80, 2.10, 2.43, 2.70, and 3.00; the stiffness ratios of columns without axial loads rose by approximately 18.5%, 39.8%, 62.8%, 83.6%, 109.4%, respectively. Similar trends were observed for the columns with an axial load ratio of 0.20. The stiffness ratios increased by approximately 15.6%, 27.4%, 37.8%, 45.2% and 52.3% for columns under an axial load of $0.60 f'_c A_g$ with an increase in aspect ratio from 1.50 to

1.80, 2.10, 2.43, 2.70, and 3.00, respectively. This suggested that the aspect ratio significantly influences the stiffness ratio.

Influence of Axial Load

It is generally recognized that the presence of column axial load can initially increase the flexural strength of columns and thus lead to larger initial flexural stiffness, which results in a higher stiffness ratio. The analyses as illustrated in Fig. 4d were carried out to assess the influence of axial load ratio on stiffness ratio. The axial load ratio was varied from 0 to 0.60. In general, the stiffness ratio increased with an increase in axial load ratio. Figure 4d showed that with an increase in axial load ratio from 0 to 0.20, 0.40, and 0.60; the stiffness ratios for specimens with an aspect ratio of 1.5 rose by approximately 35.2%, 98.7% and 167.9%, respectively. Similar trends were observed for other aspect ratios. It can thus be concluded that the axial load ratio significantly affects the stiffness ratio.

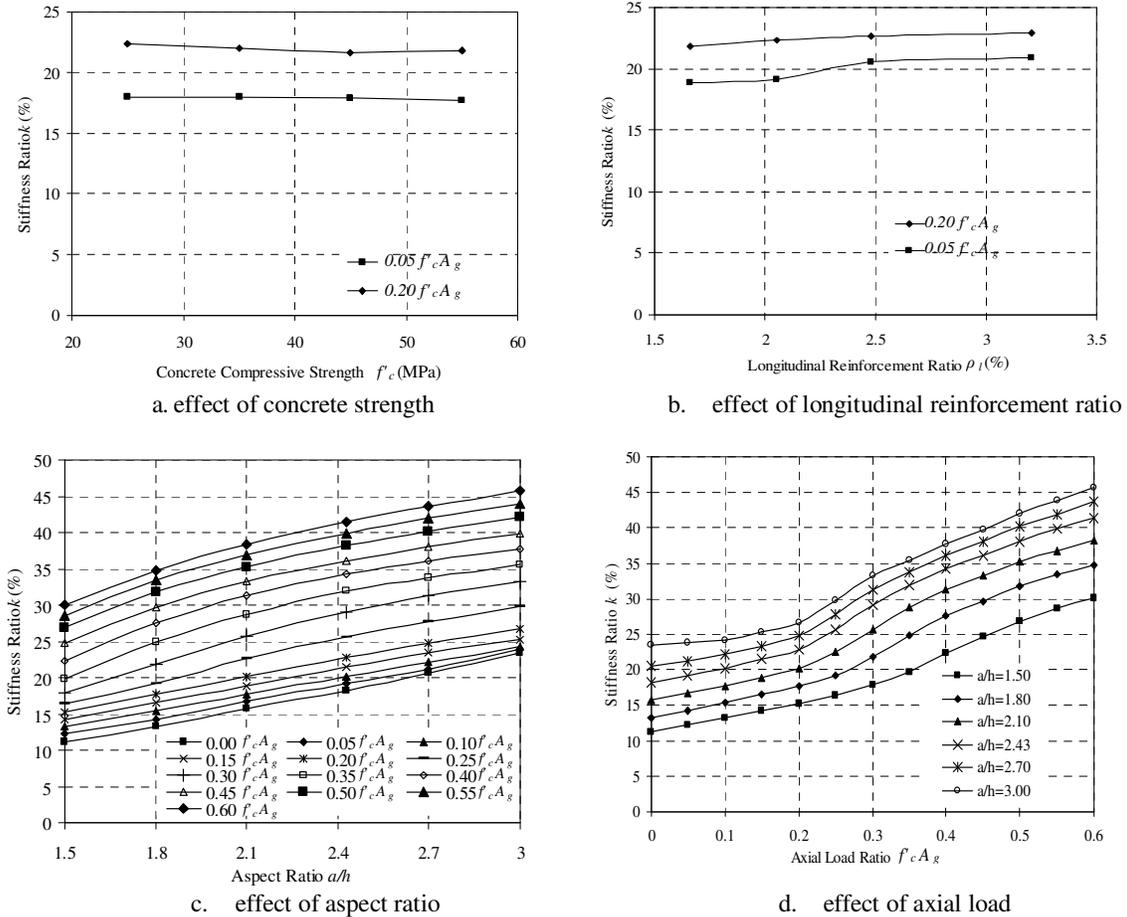


Figure 4. Parametric studies for Initial Stiffness of Columns

3.4. Proposed Equation for Effective Moment of Inertia of RC Columns

It is observed that the stiffness ratio apparently increased with an increase in aspect ratios (R_a) and axial load ratio (R_n). The transverse and longitudinal reinforcement ratios and concrete compressive strength insignificantly influenced the stiffness ratio of RC columns. For simplicity, the influences of these factors were ignored. Based on the results of the parametric study, the stiffness ratio (κ) is given by the following equation:

$$\kappa = (2.043R_n^2 + 2.961R_n + 1.739)(3.023R_a + 2.573) \quad (3.16)$$

Berry *et al.* (2004) collected a database of 400 tests of RC columns, which contained the hysteretic response, geometry, column axial load and material properties of test specimens. This database provided the data needed to evaluate the accuracy of the proposed equation for the stiffness ratio. The verification was limited to the range of the parametric study. The axial load was limited from 0 to $0.60f'_c A_g$, and the aspect ratio was limited from 1.5 to 3.0. Only rectangular columns tested in the double-curvature configuration under unidirectional quasi-static cyclic lateral loading were chosen. It was found that the average ratio of the experimental to predicted stiffness ratio by the proposed equation is 0.945 as shown in Fig. 5, showing a good correlation between the proposed equation and experimental data. Therefore, the proposed equation may be suitable as an assessment tool to calculate the stiffness ratio of RC columns within the range of the parametric study. Comparison of available models with experimental data (shown in Tran&Li 2012) indicated that the proposed equation produced a better mean ratio of the experimental to predicted stiffness ratio than other models

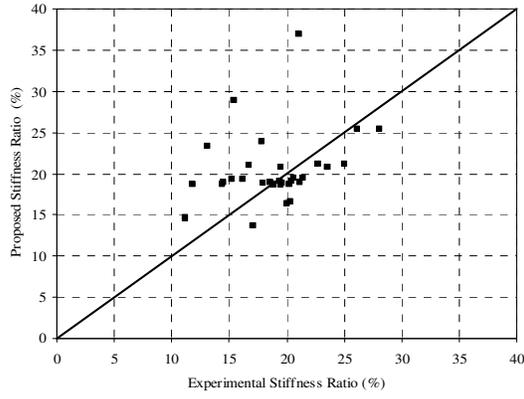


Figure 5. Comparisons between experimental and proposed stiffness ratio for columns

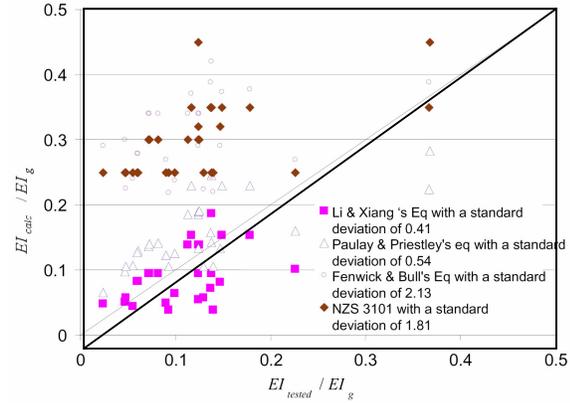


Figure 6. Comparison of effective stiffness between the analytical results and tested data for walls

4. INITIAL STIFFNESS OF RC WALL

4.1. Proposed Method

Another approach can be proposed to estimate initial stiffness of RC Walls by employing direct calculation of crack angle to calculate shear deformation.

Kim and Mander (2007) provided Eqn. 4.1 by considering the energy minimization on the virtual work done by the shear and flexural components.

$$\alpha = \tan^{-1} \left(\frac{\rho_h n + 1.57 \frac{\rho_h}{\rho_v} \times \frac{A_v}{A_g}}{1 + \rho_h n} \right)^{\frac{1}{4}} \quad (4.1)$$

The total shear distortion can be rewritten including two components: elongation of the horizontal reinforcements, Δ_S , and the shortening of the compression strut, Δ_C . The shear distortion, Δ_V can be defined by

$$\Delta_V = \Delta_S + \Delta_R = \Delta_S + \Delta_C / \sin \alpha \quad (4.2)$$

where α is the inclination of compression strut ($\alpha = 90 - \theta$ in Fig. 3)

Assuming that the shear force taken by the wall panel is V_S , the stress of horizontal reinforcement can be expressed as:

$$f_S = \frac{V_S \cdot s}{d \cot \alpha \cdot A_h} \quad (4.3)$$

where d is the length of wall panel, s is the distance between horizontal reinforcements, and A_h is the area of horizontal reinforcement spaced at a distance s . Hence the elongation of the horizontal reinforcement becomes

$$\Delta_s = \frac{f_s}{E_s} \cdot d = \frac{V_s \cdot s}{E_s \cot \alpha \cdot A_h} \quad (4.4)$$

The concrete compression stress is obtained

$$f_{cd} = \frac{V_s}{b_w L_{CS} \sin \alpha} \quad (4.5)$$

where b_w is the depth of wall panel and L_{CS} is initial depth of the compression strut as shown in Fig. 3.

Hence the shortening of the concrete strut is

$$\Delta_C = \frac{f_{cd}}{E_C} \cdot h_w / \cos \alpha = \frac{V_s \cdot h_w}{E_C b_w L_{CS} \sin \alpha \cos \alpha} \quad (4.6)$$

where h_w is the height of the wall panel. By making the appropriate substitution for web horizontal steel content, $\rho_h = A_h / sd$, and modular ratio, $n = E_s / E_C$, the shear distortion in the wall panel can be expressed as

$$\theta_v = \frac{\Delta_v}{h_w} = \frac{\Delta_s + \Delta_R}{h_w} = \frac{V_s}{h_w E_s b_w} \left(\frac{1}{\cot \alpha \cdot \rho_h} + \frac{h_w n}{L_{CS} \sin^2 \alpha \cos \alpha} \right) \quad (4.7)$$

when $\theta_v = 1$ and $L_{CS} = d \cdot \cos \alpha$, the shear stiffness of the wall panel can be defined by the following expression:

$$K_V = \frac{\rho_h \sin^2 \alpha \cdot \cos^2 \alpha}{\sin^4 \alpha + n \rho_h} E_s b_w d \quad (4.8)$$

Eqn. (4.8) indicates that the unit shear stiffness of the wall panel is mainly dependent on the extent of the crack angles

Hence the shear displacement caused by the yield lateral force F_y would be

$$\Delta_{yv} = \frac{F_y}{K_V} \cdot h_0 \quad (4.9)$$

Combination of Shear and Flexure Response

After the flexural and shear deformation at the top of wall under yield lateral load are obtained, the initial stiffness of walls can be determined as:

$$K_i = \frac{F_y}{\Delta_{yf} + \Delta_{yv}} \quad (4.10)$$

4.3. Proposed Equation for Moment of Inertia of Structural Walls

Based on the similar parametric studies conducted on RC columns, Eqn. (4.13) which considers three parameters investigated: yield tensile strength of steel bars in wall boundaries, axial loads, and aspect ratios is proposed to properly evaluate the effective stiffness of squat structural walls (details of those calculations and illustrations are presented in Li and Xiang 2011). For simplicity, the influence of longitudinal reinforcement content in wall boundaries on wall effective stiffness is conservatively disregarded.

$$\frac{I_e}{I_g} = 0.19 \left(\frac{100}{f_y} + \frac{N}{f_c A_g} \right) \left(0.53 + 0.37 \frac{h_w}{L_w} + 0.31 \frac{h_w^2}{L_w^2} \right) \quad (4.11)$$

4.4. Comparison of the Proposed Approach with other test results on RC Walls

Results from RC structural walls (Li and Wang 2011) are compared with analytical results using the proposed model, Eqn. (4.11) and other provisions previously reviewed. Experimental effective stiffness values, EI_e from the tests are back calculated by dividing the displacements at the yield point by the tested yield strength with an elastic model. All tested walls have aspect ratios not larger than two and axial load ratios which range from zero to 0.2, which covers almost all conditions likely to be encountered in practice. Yield strengths of outermost longitudinal bars for all specimens range from 300 MPa to 585 MPa. It is believed that the proposed stiffness model is applicable for all values of yield strengths studied. The longitudinal and transverse reinforcement content in the wall web is limited and remains at a low level for all walls selected. Fig 6 presents the comparison between the experimental and calculated stiffness (EI_e/EI_g) for the proposed model and that presented by other proposals. Of the three equations proposed, the currently proposed equation with a standard deviation of 0.41 appears to be more accurate in effective stiffness evaluation. This can be explained by the fact that the previous models (both Fenwick and Bull's design equation and the NZS 3101 code (1995)) ignored the shear deformation in calculating the effective stiffness of squat walls; whereas, the shear deformation was considered in the proposed model.

5. CONCLUSIONS

This paper presents an analytical method to estimate the initial stiffness of RC columns and walls. Comprehensive parametric studies are carried out based on the proposed method to investigate the influences of several critical parameters. Two simple equations to estimate the initial stiffness of RC columns and walls are also proposed. Verifications of the proposed models with experiment data on RC columns test and RC walls tests showing good agreement.

REFERENCES

- ACI Committee 318 (2008) "Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary (318R-08)," *American Concrete Institute*, Farmington Hills, Mich., 465 pp.
- ASCE 41 (2007) "Seismic Rehabilitation of Existing Buildings", *American Society of Civil Engineers*, Reston, VA.
- FEMA 356 (2000) "Prestandard and Commentary for the Seismic Rehabilitation of Buildings", *Federal Emergency Management Agency*, Washington D.C., USA.
- Li, B and Xiang, W.Z. (2011). Initial Stiffness of Squat Structural Walls. *ASCE Journal of Structural Engineering* 137:12,1470-1479.
- Tran C T N. and Li, B (2012). Initial Stiffness of Reinforced Concrete Columns with Moderate Aspect Ratios. *Advances in Structural Engineering* 15:2,265-276.
- Park, R., and Paulay, T. (1975) "Reinforced Concrete Structures," *John Wiley & Sons*, New York, 769 pp.