Nonlinear Soil - Pile - Structure Interaction in Dynamic or Seismic Response

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SUMMARY :
The stiffness and damping of pile foundations are proposed, which vary with the ratio of shear modulus G_i / G_o to indicate the nonlinear properties of soil. The range of G_i / G_o is defined with different vibration intensities for applications, and a computer program DYNAN is available based on the boundary zone model with non-reflective interface. Two engineering cases are studied to illustrate the nonlinear soil-pile-structure interaction. In the first case a piled foundation is examined under dynamic loads. The measurements of vibration were carried out in the field for a revamped project, and the measured data matched with the results calculated by the program. In the second case a piled foundation is examined under seismic loads. The seismic forces and response are calculated using the time history analysis and response spectrum analysis, and compared with those using the method of equivalent static loads.

Keywords: Soil-pile-structure interaction, nonlinear soil, soil dynamics, structural dynamics, seismic response

1. INTRODUCTION

Great advances have been developed on the study of dynamic behaviour of pile foundations. However, there are some problems remaining, and some parameters are uncertain in the soil-pile system that affect the dynamic response of piles. A comprehensive method should be not only advanced in theory but also verified by tests and applications. The consideration is how to determine the parameters of soil, and how to consider the nonlinearity of soil in strong vibration or an earthquake environment. The curves of stiffness and damping of piles are proposed in this study. The curves are varied with the ratio of G_i / G_o, where G_i and G_o are the shear modulus of soil in the boundary zone and out zone respectively. It is corresponding to the p-y curves used in static analysis. The soil-pile system is simulated by a boundary zone model with a non-reflective interface. The model is an approximate but simple and realistic method that accounts for the nonlinearity of a soil-pile system.

Two cases are studied to illustrate the soil-pile-structure interaction under dynamic or seismic loads. In the first case, a compressor foundation supported by 34 steel piles is examined under the revamped dynamic loads after the foundation was constructed ten years ago. Two reciprocating compressors were placed on the foundation. The original foundation was designed based on one machine operating and another spare. However, the load condition changed such that two compressor would be running simultaneously. Accompanying modifications include an increase in the power, speed and unbalanced forces. That is, the unbalanced forces which were produced from one compressor for the existing machines will be produced from two compressors simultaneously for the updated machines. The existing foundation has to be evaluated to see if a modification of the foundation is needed. The dynamic tests were done at the site to measure the vibration of the pile foundation. The validation of the boundary zone model is confirmed, and the prediction from the computer program is shown reliable, since the measured data agree with the theoretical results.

In the second case, a vacuum tower structure is examined in a seismic zone as a typical industrial structure supported on a pile foundation. The vacuum tower was installed on a steel frame with height of 20 m, and supported by 25 steel piles with lengths of 30 m. Three base conditions are considered: rigid base, (i.e. no deformation in the foundation), linear soil-pile system; and nonlinear soil-pile system. The case of liquefaction of the sand layer is discussed for the pile foundation. The seismic loads and response are calculated from the time history analysis and response spectrum analysis, and compared with those from the method of equivalent static loads.

2. NONLINEAR SOIL - PILE SYSTEM

Many authors have made contributions to the subject of soil-structure interaction, such as Dobry & Gazetas (1988), Roesset et al (1986), Luco (1982), Gazetas & Makris (1991), Benerjee & Sen (1987), Wolf (1988) and Finn et al (1997). Different approaches are available to account for dynamic soil-pile interaction but they are usually based on the assumptions that the soil behaviour is governed by the law of linear elasticity or visco-elasticity, and that the soil is perfectly bonded to a pile. In practice, however, the bonding between the soil and the pile is rarely perfect, and slippage or even separation often occurs in the contact area. Furthermore, the soil region immediately adjacent to the pile can undergo a large degree of straining, which would cause the soil-pile system to behave in a nonlinear manner. A lot of efforts have been made to model the soil-pile interaction using the 3D Finite Element Method (FEM). However, it is too complex, especially for pile groups in nonlinear soil. A rigorous approach to the nonlinearity of a soil-pile system is extremely difficult and time consuming.

As an approximate analysis, a procedure is developed using a combination of the analytical solution and the numerical solution, rather than using the general FEM. This procedure is considered as an efficient technique for solving the nonlinear soil-pile system. The relationship between the foundation vibration and the resistance of the side soil layers was derived using elastic theory by Baranov (1967). Both theoretical and experimental studies have shown that the dynamic response of piles is very sensitive to the properties of the soil in the vicinity of the pile shaft. Novak and Sheta (1980) proposed including a cylindrical annulus of softer soil (an inner weakened zone or so called boundary zone) around the pile in plane strain analysis. One of the simplifications involved in the original boundary zone concept was that the mass of the inner zone was neglected to avoid the wave reflections from the interface between the inner boundary zone and the outer zone. To overcome this problem, Velestsos and Dotson (1988) proposed a scheme that can account for the mass of the boundary zone. Some of the effects of the boundary zone mass were investigated by Novak and Han (1990), who found that a homogeneous boundary zone with a non-zero mass yields undulation impedance due to wave reflections from the fictitious interface between the two media.

The ideal model for the boundary zone should have properties smoothly approaching those of the outer zone to alleviate wave reflections from the interface. Consequently, Han and Sabin (1995) proposed a model for the boundary zone with a non-reflective interface. The complex shear modulus, G (r), varies parabolically, as expressed by the function f(r) shown in Equation 1. The properties of the soil medium in the boundary zone are defined by the complex-valued modulus.

$$G^*(r) = \begin{cases} G_i^* & r = r_o \\ G_o^*\, f(r) & r_o < r < R \\ G_o^* & r > R \end{cases} \tag{1}$$

and

$$\begin{aligned} G_i^* &= G_i\,(1 + i\,2\,\beta_i) \\ G_o^* &= G_o\,(1 + i\,2\,\beta_o) \end{aligned} \tag{2}$$

in which G_i and G_o= shear modulus of soil in the boundary zone and outer zone; r_o = radius of pile; R = radius of boundary zone; r = radial distance to an arbitrary point; β_i and β_o = damping ratio for the two zones; and i = root(-l).

Obviously, when the modulus ratio equals to one, the soil behaviour is linear. The shear modulus in the outer zone is a constant. As the modulus ratio G_i /G_o is less (or larger) than one, the soil behaviour is nonlinear. For applications, this concerns the determination of the parameters of the boundary zone, such as the thickness of zone, damping ratio in two zones， and the modulus ratio. In general, the thickness of boundary zone is assumed to be equal to the radius of pile, and damping ratio $\beta_i = 2\ \beta_o$.

Thus, the parabolic function can be written as

$$f(r) = 1 - (1 - G_i^{*} / G_o^{*})\ (r/r_o - 2)^2 \tag{3}$$

The modulus ratio G_i /G_o is an approximate indicator for the nonlinear behaviour of soil. The value of the modulus ratio depends on the method for pile installation, the density of excitation and vibration amplitudes. Further dynamic tests on piles are needed to determine the value of the modulus ratio. The model of the boundary zone with a non-reflective interface has been widely accepted to approximately solve the problem of nonlinear soil. However, it should be explained that the method described here is not a rigorous approach to modeling the nonlinearity of a soil-pile system. It is an equivalent linear method with a lower value of G_i and a higher value of damping β_i in the boundary zone. With such a model, the analytical solutions can be obtained for the impedance functions of a pile.

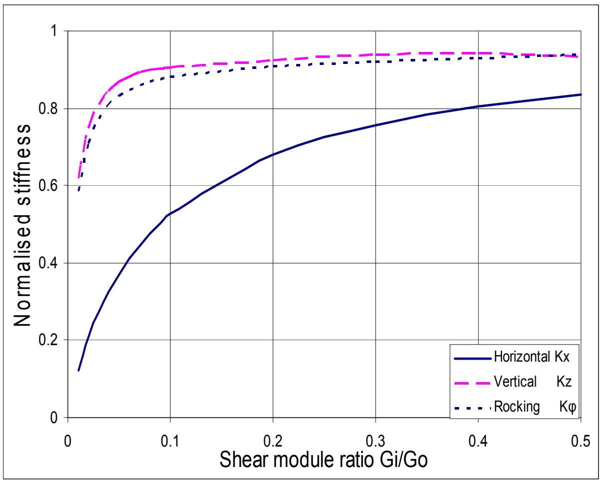
3. STIFFNESS AND DAMPING OF PILES

With the impedance of the soil layer, the element stiffness matrix of the soil-pile system can be formed in the same way as in the general finite element method. Then the overall stiffness matrix of a single pile can be assembled for different modes of vibration, including three translations and three rotations. The group effect of piles is accounted for using the method of interaction factors. The static interaction factors are based on Poulos and Davis (1980). The dynamic interaction factors are derived from the static interaction factors multiplied by a frequency variation, and the frequency variation of interaction factors is based on the charts of Kaynia and Kausel (1982).

There are six degrees of freedom for the rigid mat, and lateral vibration is coupled to rocking vibration. It should be explained that the foundations (or caps on piles) are assumed to be rigid. However, in most cases, the superstructures are flexible rather than rigid. The effects of soil-pile-structure interaction on dynamic response were discussed by Han, (2008). The dynamic response of the superstructure can be calculated using a finite element program, such as SAP2000.

For the pile foundation under static loads, the differential equation for a beam-column can be solved using nonlinear lateral load-transfer (*p-y*) curves. Nonlinear lateral load-transfer from the foundation to the soil is modeled using *p-y* curves generated by computer program LPILE for various types of soil.

Unfortunately, the dynamic equations of soil-pile system can not be solved analytically by using the *p-y* curves. An approximate analysis has to be used for the dynamic analysis of pile foundations. The dynamic equations have been solved using the ratio of shear modulus G_i /G_o to indicate the nonlinear properties of soil by Han & Sabin (1995). The plane-strain model is improved by the boundary zone model for the soil-pile system. The nonlinear variation curves of stiffness and damping and range of values for G_i /G_o are discussed in the following.



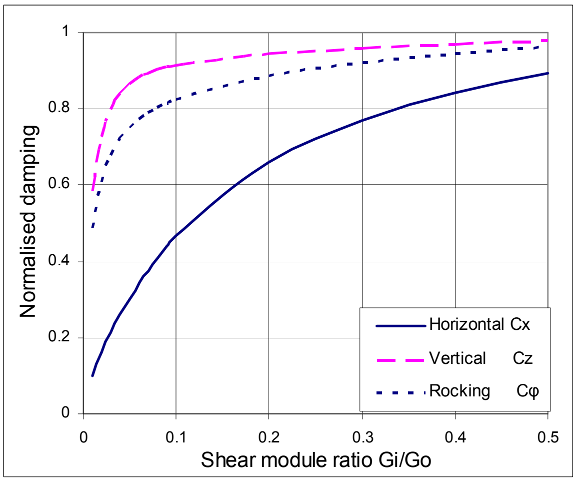


Figure 1. Normalised stiffness of piles *vs* G_i/G_o

Figure 2. Normalised damping of piles *vs* G_i/G_o

The normalised stiffness and damping of pile foundation varied with G_i /G_o as shown in Figure 1 and Figure 2 respectively. The values of stiffness and damping are generated using the program DYNAN, and applicable to general pile foundations no matter concrete piles or steel piles. The stiffness and damping are frequency dependent, varied with frequency. The values of stiffness and damping are normalised to show the effects of G_i /G_o. The normalised stiffness and damping are defined as the dynamic stiffness and damping to be normalised by static values. It should be explained that the static stiffness can not be generated directly from the program, and the values of stiffness and damping in very low frequency domain such as 0.01 Hz were assumed to be close to as static values.

From Figures 1 and 2, it can be seen that the variation of stiffness and damping is larger for horizontal vibration than those for vertical and rocking vibration. It is concluded that the effects of G_i /G_o are more significant on lateral impedances than those on vertical and rocking impedances. Also, it is noted that the stiffness and damping vary gently as G_i / G_o = 0.25 - 0.5, and vary sharply as G_i / G_o < 0.25. As G_i / G_o < 0.1, the stiffness and damping are reduced seriously for all of the vibration modes.

The value of G_i / G_o depends on the vibration intensity of pile, and the reduction increases with the vibration intensity. Based on dynamic tests of pile foundations (Han & Novak, 1988), it is suggested that G_i / G_o = 0.25 - 0.5 for design of machine foundations, and the value may be G_i / G_o < 0.25 for strong earthquake response.

For the soil medium, the relationship between the shear wave velocity V_s and modulus G can be expressed as $V_s^2 = G / \rho$ (ρ is the mass density). It can be understood that the shear wave velocity of soil V_s is reduced to about 50% to 70% corresponding to G_i / G_o = 0.25 - 0.5 in the boundary zone. V_s is reduced to less than 50% corresponding to G_i / G_o < 0.25, and V_s is reduced to about 1/3 corresponding to G_i / G_o = 0.1.

4. DYNAMIC RESPONSE OF COMPRESSOR FOUNDATION

Two reciprocating compressors (K42562 A/B) shared one foundation. The original foundation was designed based on one operating and another spare. Since the requirements of the compressor were modified, the operating conditions will be changed to two machines running simultaneously. This leads to an increase in the power, speed and unbalanced forces. The existing foundation has to be evaluated to find if the foundation needs to be modified.

4.1 Compressor foundation and soil condition at site

The soil properties were poor when the foundation was constructed, and the shear wave velocity of soft clay was 100 m/sec. The soil properties were measured at the site to check if the strength of the soft clay increased by consolidation. The dynamic response was measured at the foundation to see if the vibration was reduced by the engineering fill. In fact, the engineering fill was strengthened and the shear strength of the underlying clay increased due to the consolidation over the ten years since construction. The soil improvement turned out to be important in making the foundation meet the new requirement of vibration limitation.

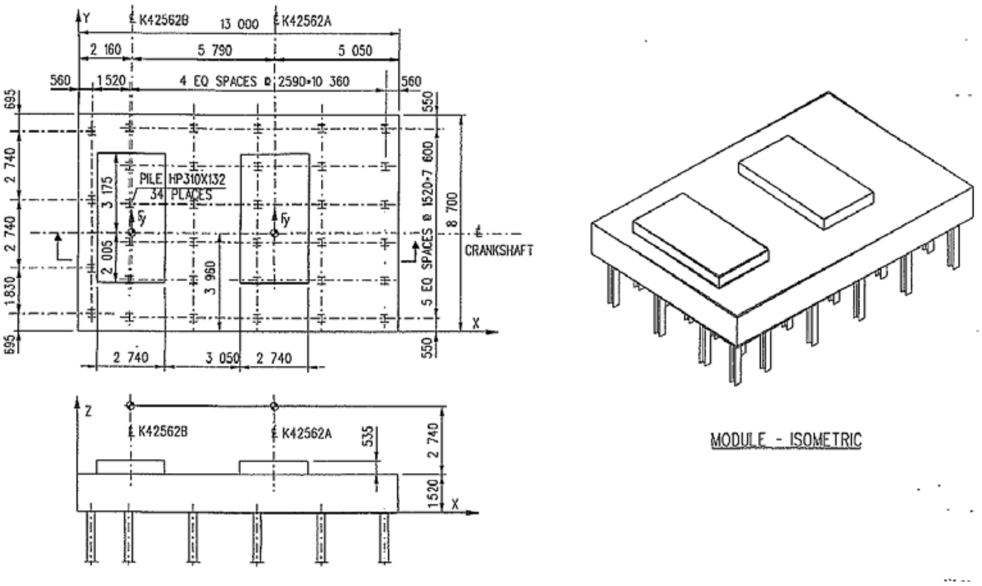


Figure 3. Foundation of reciprocating compressor

The compressor machine weight is 11.38 Mg for each. The center of gravity (C.G.) of the machines is located at 2.74 m above the top surface of foundation. The weight of the vertical vessel and platform are 4.54 Mg and 19.05 Mg respectively. The total mass of the concrete foundation and machines is 530 Mg. The primary operating speed is 390 rpm for the existing machine and 450 rpm for the updated machines. There is only one cylinder in the reciprocating compressor set, so the unbalanced forces produced from each machine are quite large. The original foundation dimension is 13 m by 8.7 m with thickness of 1.5 m supported on 34 HP 310 x 132 steel piles with length of 21 m, as shown in Figure 3. The soil profile is soft clay above glacial till. To improve the soil conditions, the soft soil was removed at the surface and the engineering fill was placed for about 5 m. From the site bore hole and CPT test, the main soil properties are reported as shown in Table 1. The unbalanced forces are shown in Table 2 for the existing and updated machines.

Table 1. Soil Properties at the Site

Depth (m)	Soil Type	Shear Wave Velocity (m/sec)	Unit Weight (kN/m^3)
0 – 3.5	Medium dense granular fill	270	20.4
3.5 – 5.0	Sandy silt	220	18.0
5.0 - 11	Medium clay	116	18.8
11 - 24	Glacial till	280	19.6

4.2 Dynamic response calculated and measured for existing machines

The horizontal vibration in the Y–direction is much larger than those in other directions from the dynamic analysis, so the horizontal amplitude governs the foundation design in this case. The amplitude A_y = 8.7 μm under the primary forces (390 rpm), and A_y = 2.8 μm under secondary forces (780 rpm). So, the maximum amplitude is calculated to be 11.5 μm. The vibration limit is 32 μm required by the vendor. In the calculation, the parameters of the side soil (depth of 1.5 m) were justified to lower values due to the confining pressure being reduced close to the ground surface. The buoyant density of the soil was used to consider the effect of groundwater. The nonlinear properties of soil was accounted for, and the modulus ratio is assumed, G_i /G_o = 0.5 in the calculation. To examine the theoretical prediction, the vibration measurements were done on the existing foundation in April, 2009 by Irving Oil project.

Table 2. Unbalanced Forces Produced from Existing and Updated Machines

Operating	Machine	Fx (kN)	Fy (kN)	Fz (kN)	Mx (kN-m)	My (kN-m)	Mz (kN-m)
Primary	Existing	0	23.8	4.58	- 80.4	19.4	-101
	Updated	0	65.7	12.6	-222	16.8	-87.5
Secondary	Existing	0	6.72	0	- 21.8	0	- 28.4
	Updated	0	18.5	0	-60.3	0	-24.7

Where Fx, Fy and Fz are forces in three directions and Mx, My and Mz are moments about the three axes.

The peak-to-peak value of 0.498 mil was measured in the horizontal direction at 390 rpm on the top corner of the concrete pedestal. Thus, the amplitude is approximately equal to 6.3 μm. Meanwhile, the overall velocity of vibration was measured directly at the same location. Overall Velocity V = 0.0181 inch/sec, and A = 11.3 μm were recorded. It can be seen that the theoretical results match very well the data of vibration measured on the foundation. At the speed of 390 rpm, the amplitude 8.7 μm was calculated and 6.3 μm was measured. It is close enough. As for the overall amplitudes it is almost the same, 11.5 μm calculated and 11.3 μm measured.

4.3 Dynamic response for updated machines

The primary operating speed is 450 rpm for the updated machines, and the secondary speed is 900 rpm. From Table 2, it can be seen that the horizontal force F_y produced from the updated machines is increased to more than two times than that from the existing machine (in-phase). The dynamic analysis was carried out using the computer program for the same foundation but different unbalanced forces. With the forces from the updated machines, the amplitude A_y = 25.2 μm under the primary forces, and A_y = 6.1 μm under secondary forces. The maximum amplitude was calculated to be 31.3 μm that is less than the allowable vibration limit of 32 μm. So, the original foundation is adequate for the revamped service.

To examine the effects of soil improvement on dynamic response, the dynamic analysis of foundation was carried out based on the soil properties of original clay. The amplitude was calculated to be 49.3 μm under the primary forces, and the amplitude 2.8 μm was calculated under the secondary forces. The maximum combined amplitude would be 52.1 μm. The vibration amplitude is much larger than the vibration limit of 32 μm. It is not allowable. It can be seen that the soil improvement of replacing the soft soil by granular fill effectively reduced the vibration. Another factor of soil improvement is that the strength of soft clay increased under the engineering fill layer, and measurement of soil shown the strength increased at the site.

5. SEISMIC RESPONSE OF VACUUM TOWER STRUCTURE

The vacuum tower structure was constructed in a seismically active area as shown in Figure 4. At the site, surface soil is soft clay with a depth of 2 m, underlain by a layer of saturate fine sand with a depth of 2 m, followed by some silty clay and dense sand layers with depths of 4 to 8 m in each layer, then bedrock. The depth to bedrock is about 30 m. Soil properties vary with depth and are characterized by the shear wave velocity and unit weight, as shown in Table 3.

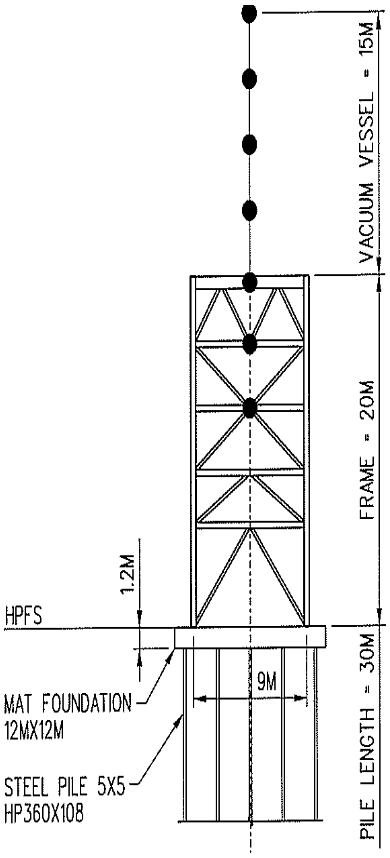


Figure 4. Vacuum tower structure

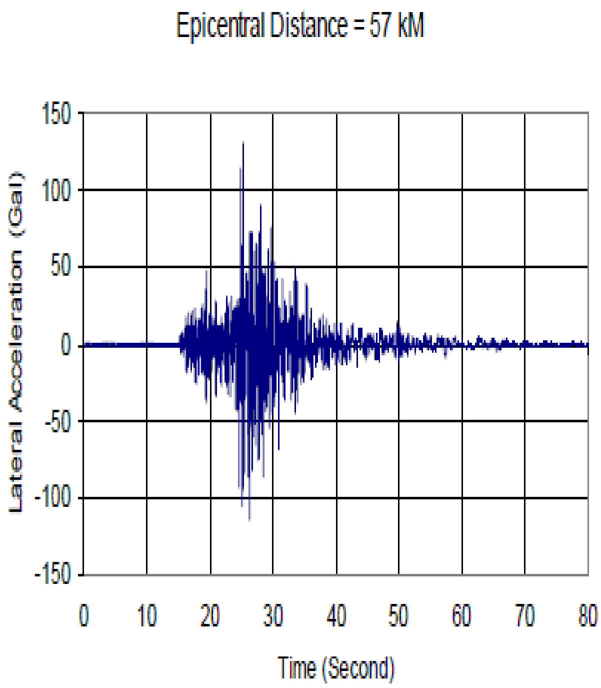


Figure 5. Horizontal ground acceleration from an earthquake record

Table 3. Soil Properties

Depth (m)	Soil	Unit Weight (kN / m^3)	Shear Wave Velocity (m / s)
0 -2	Soft Clay	18	130
2 - 4	Fine Sand	18	140
4 - 12	Stiff Clay	20	300
12 - 16	Silty Sand	19	240
16 - 20	Silty Clay	18	300
20 - 25	Weathered Shale	18	200
25 - 30	Dense Sand	20	300
Below 30	Bedrock	21.5	370

The concrete mat foundation is 12 x 12 m with a thickness of 1.2 m. The piles are steel HP 360 x 108 with length of 30 m driven to bedrock. Twenty-five piles in a square pattern were fixed to the mat foundation. The stiffness and damping of the pile foundation were calculated for different base conditions. In the first case a linear soil-pile system is assumed, that is, the soil layers are homogeneous, without the weakened zone. In the second case, a nonlinear soil-pile system is assumed, and the boundary zone is assumed around the piles. The parameters of the boundary zone were selected as: $G_i / G_o = 0.25$.

In the third case, liquefaction was assumed in the saturated fine sand layer, and the top layer of soft clay has not yielded. Both stiffness and damping are frequency dependent. Since the fundamental period of the structure is closed to 1.0 second, the stiffness and damping were calculated at a frequency of f = 1.0 Hz. The stiffness and damping calculated are shown in Table 4. Where, K_x, K_z, and K_φ are stiffness in the horizontal, vertical and rocking directions, and C_x, C_z, and C_φ are damping constants in the same directions. It can be seen that both stiffness and damping are lower in the nonlinear case than those in the linear case. For example, the horizontal stiffness in the nonlinear case is about half of that in linear case. In the case of liquefaction, the values of horizontal stiffness are reduced significantly, and great damage is possible.

Table 4. Stiffness and Damping of Pile Foundation (f = 1.0 Hz)

Soil status	Stiffness			Damping		
	K_x (kN/m)	K_z (kN/m)	K_φ (kN.m/ra)	C_x (kN/m/s)	C_z (kN/m/s)	C_φ (kN.m/rad/s)
Linear	$1.283\text{x}10^6$	$3.215\text{x}10^6$	$1.333\text{x}10^8$	$1.244\text{x}10^4$	$1.803\text{x}10^4$	$6.411\text{x}10^5$
Nonlinear	$0.646\text{x}10^6$	$2.877\text{x}10^6$	$1.160\text{x}10^8$	$0.998\text{x}10^4$	$1.005\text{x}10^4$	$3.171\text{x}10^5$
Liquefaction	$0.180\text{x}10^6$	$2.527\text{x}10^6$	$1.006\text{x}10^8$	$0.749\text{x}10^4$	$0.943\text{x}10^4$	$2.787\text{x}10^5$

5.1 Time history analysis

A record of horizontal ground acceleration from an earthquake was employed for the time history analysis. The peak value of acceleration is 0.13 g as shown in Figure 5. The time step is 0.005 second, and duration is 80 second in the earthquake record. To investigate the influence of foundation flexibility on the superstructure, the seismic analysis of the structure was conducted for three different base conditions: rigid base, linear and nonlinear soil-pile systems. The seismic response and forces of structure were calculated using a FEM model by the SAP 2000 program. The vacuum vessel was modeled as an elastic column with the mass distributed uniformly along its height. The steel structure was modeled using frame elements and the mat foundation was modeled using shell elements. The stiffness and damping of the pile foundation were generated from the program for the three base conditions. The deflection, base shear and overturning moment are shown in Table 5.

Table 5. Seismic Response and Seismic Forces of Tower Structure

Base Conditions	Amplitude at Top of Tower (mm)	Base Shear (kN)	Overturn Moment (kN-m)
Fixed Base	22.05	807	19,630
Linear Soil	26.30	598	14,980
Nonlinear Soil	26.05	545	14,120

From Table 5, it can be seen that the earthquake forces for the fixed base condition are larger than those for the cases with the soil-structure interaction. The theoretical prediction does not represent the real seismic response, since the stiffness is overestimated and the damping is underestimated for a structure fixed on a rigid base. From the comparison, it can be seen that the maximum values and time histories for the seismic forces and seismic response are different when the foundation is considered as a fixed base or a flexible base.

5.2 Response spectrum analysis and equivalent static forces

An elastic dynamic analysis of a structure can utilize the peak dynamic response of all modes, which have a significant contribution to total structural response. Peak modal responses are calculated using the ordinates of the appropriate response spectrum curve which correspond to the modal periods. Maximum modal contributions are combined in a statistical manner to obtain an approximate total structural response. The equivalent lateral seismic forces are calculated in accordance with the formula in ASCE / 7 -05.

$$V = (2/3)\ F_a\ S_s\ W\ /\ (R/\ I) \qquad \text{as} \qquad T <= T_s \tag{4}$$

and

$$V = (2/3)\ F_v\ S_1\ W\ /\ T\ (R/\ I) \qquad \text{as} \qquad T > T_s \tag{5}$$

where, the fundamental period of structure T = 0.75 second, and $T_s = F_v\ S_1 / F_a\ S_s$ = 0.53 second were calculated. For the location of the vacuum tower, the ground acceleration S_s (0.2) = 0.28 g, and S_1 (1.0) = 0.09 g. The site coefficient F_a = 1.7 and F_v = 2.8 based on the soil properties. The importance factor I = 1.0, and the ductility factor R = 2.0 for conventional construction of moment frames and braced frames. The total weight (including vessel) is W = 7,563 kN.

RSA (Response Spectrum Analysis) was done using the finite element program RISA-3D. The most difficult part of the entire procedure is calculating the scaling factor. The unscaled RSA base shear was calculated using the program. Thus, the scale factor is equal to V/Unscaled RSA base shear. The spectra were normalized using modal participation. In the calculation for the scale factor, 20 vibration modes are calculated making the modal participation to be over 90%. The response spectrum analysis was done for fixed base, and a local response spectrum was used in the analysis. The seismic response and seismic forces were calculated, and the comparison of results from the time history analysis, the response spectrum analysis and the method of equivalent static forces are shown in Table 6. It can be seen that the results calculated from different method are conformable.

Table 6. Comparison of Seismic Forces and Response by Different Analysis

Method of analysis	Amplitude at Top of Tower (mm)	Base Shear (kN)	Overturning Moment (kN-m)
Time history	22.05	807	19,630
Response spectrum	24.1	897	21,349
Equivalent static forces	20.9	862	20,516

6. CONCLUSIONS

The problem of nonlinear soil-pile-structure interaction is solved approximately using the model of boundary zone with non-reflective interface. The range of G_i/G_o is defined with different vibration intensity, and the computer program is available for applications. Two engineering cases are examined, and suggest the following conclusions.

1. The existing foundation in Case 1 is confirmed to be adequate in the project for the revamped service with the increase of dynamic forces and no modification to be needed to the foundation, since the soil strength has been improved over the past 10 years.

2. The measurements of vibration had been done on the compressor foundation and the measured data matched the theoretical results in Case 1. The validity of computer program is verified.

3. The soil-pile interaction is an important factor which affects the stiffness and damping of the foundation. With the ratio of shear modulus G_i/G_o, the nonlinear properties of soil can be accounted for in the seismic analysis.

4. The seismic response and earthquake forces calculated are conformable for the different analysis methods of the time history analysis, response spectrum analysis and equivalent static forces.

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