Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.VIII)

SI-11

STABILITY OF SLENDER REINFORCED CONCRETE MEMBERS SUBJECTED TO STATIC AND DYNAMIC LOADS

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SUMMARY

The static and dynamic behavior of slender reinforced concrete members under uniaxial or biaxial bending is investigated. A total of 56 pin-ended column specimens, whose ratio of length to section depth ranged from 6 to 26, with square or rectangular cross section were tested. The static ultimate strength of slender columns is reduced by additional eccentricity due to lateral deflection even when the length to depth ratio is 15. Remarkable effects of biaxial bending on the ultimate load and the deformation behavior occur with the rectangular columns. In the dynamic loading test, increases in the buckling load of the slender columns with the increasing loading rate were precisely observed.

INTRODUCTION

Investigations on instability problems of relatively slender concrete columns have been extensively carried out in countries of no strong earthquakes. In recent years, as the material strengths of concrete and steel increased, the cross sections now can be designed rather compact and therefore members become slender. Moreover the reinforced concrete braces with slender proportions are used in frame structures to resist earthquake forces. Therefore, clarification of the elastic-plastic behavior of slender reinforced concrete members is very important. The slender member behavior becomes still more complex due to the biaxial bending and/or the dynamic loading. In this paper, fundamental data from centric and uniaxially or biaxially eccentric loading tests of slender columns (Ref.1) are presented and, in comparing with numerical analysis, general stability behavior of slender reinforced concrete members subjected to static and dynamic loads are discussed. The paper consists of two main parts. In the first part, the load-deformation behavior of slender columns under biaxial static loading is investigated experimentally, and the columns are analyzed by the numerical method taking into account second-order effect due to deflections. In the second part of the paper, the effects of dynamic loading on the elastic-plastic buckling behavior of concrete columns are discussed.

STATIC BEHAVIOR OF BIAXIALLY LOADED COLUMNS

<u>Test Program</u> A total of 44 column specimens were tested. The column length to minimum section depth ratios, ℓ/D or ℓ/b, ranged from 6 to 26. The geometry of test specimens is shown in **Fig. 1.** The columns had a 12cm(b) x 12cm(D) square cross section (Test Series A,B,C,E) or a 12cm(b) x 18cm(D) rectangular cross

section (Test Series R) with 8- or 6-D10 longitudinal bars. The mix proportion of concrete and the properties of materials are listed in **Table 1.** Both ends of the column were supported by universal bearings to simulate pin-ended conditions. Load was applied monotonically at each column end with equal eccentricities of the same sign (Series A,B,C,R) or opposite sign, or with unequal eccentricities (Series E), e, at various angles, θ, from principal axis of the cross section, as given in **Fig. 2.** Transverse deflections, u and v in the principal axes of the section at the mid-height of the column, and the other deformations shown in **Fig. 2** were measured by potentio-meters.

Method of Theoretical Analysis The load-deformation response of the column was solved by the numerical method of a second-order analysis taking into account the effects of deflections of the columns. The following basic assumptions have been made: (1) lateral deflections of the column are small compared to its length and the curvature is represented by the second derivatives of the deflections; (2) plane sections remain plane after deformation; (3) effects of shear on lateral deflection and rotation of the cross section are negligible; (4) the assumed stress-strain relationships for the concrete and the reinforcing bars are shown in **Fig. 3(a)** and **3(b)**, respectively. The concrete is unable to sustain any tensile stress. If the assumptions of perfect bonding and plane distribution of strains are made, the strain distribution in the section can be defined by the strain at the centroid of section and curvatures produced by bending moment components, M_x and M_y, about the x and y axes. Knowing the strain distribution, the axial force, N, and the bending moments, M_x and M_y, are calculated from the

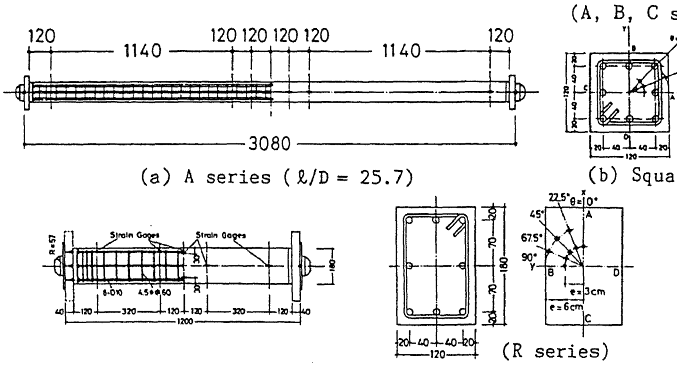
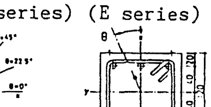



Fig. 1 Geometry of test specimen (units in mm)

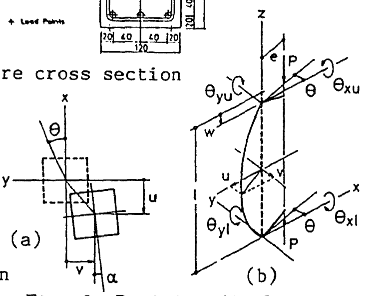


Fig. 2 Positive displacements

Table 1 Mix proportion of concrete and material properties

Test series ℓ/D or ℓ/b		C 5.7	B 15.7	A 25.7	R 10, 25	E 25
Mixing ratios by weight	Water	0.67	0.65	0.66	0.62	0.68
	Cement	1	1	1	1	1
	Sand	1.97	1.91	1.91	1.90	1.91
	Gravel	2.74	2.76	2.76	2.75	2.76
Concrete						
Compressive strength	σ_c (N/mm^2)	27.0	27.6	30.5	31.0	22.8
Strain at compressive strength	ε_p (%)	0.26	0.23	0.24	0.24	0.26
Tensile strength	(N/mm^2)	——	2.95	2.57	2.83	2.31
Reinforcement						
Upper yield strength	(N/mm^2)	365	365	356	370	383
Lower yield strength	(N/mm^2)	347	355	342	361	379
Ultimate strength	(N/mm^2)	517	507	539	508	567
Elongation	(%)	18.2	16.9	22.6	27.2	27.6

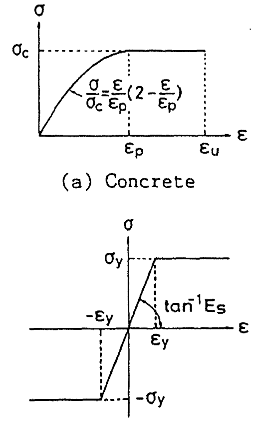
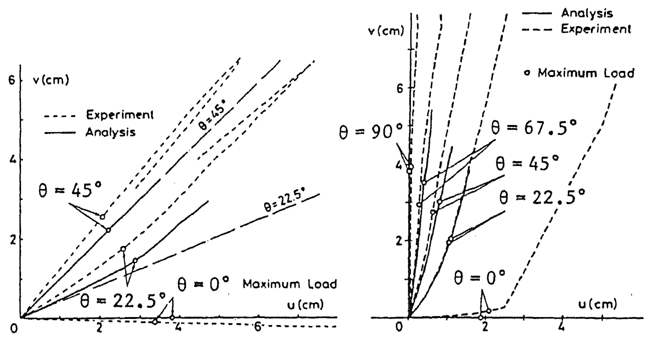


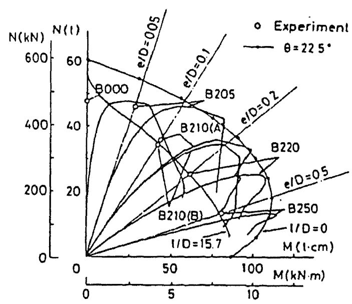
Fig. 3 Assumed stress-strain curves

assumed stress-strain relationships. The application of the Runge-Kutta-Gill numerical integration method yields the deflections u and v corresponding to the load, P. In the numerical analysis, the column length was divided into 10 segments of the same length, and the square and the rectangular cross sections were divided into 10 x 10 and 10 x 15 elements, respectively.

Deformation Behavior **Figure 4** shows the change of mid-height deflections with increasing load. In the case of square long columns under biaxial loading (**Fig. 4(a)**), the direction of deflection corresponds closely to the loading plane, that includes the loading points and the centroids of both end sections, before reaching the maximum load. However, after the attainment of the maximum load, the column has a tendency to deflect away from that plane. In the case of rectangular long columns (**Fig. 4(b)**), the lateral deflections significantly depend on bending about the weak axis of the section. When the columns are subjected to biaxially eccentric load to cause bending about the near-strong axis, they tend to deflect largely bending about the weak axis of the section. The test behavior is seen to be well predicted by the numerical analysis.



(a) Square long columns (b) Rectangular long columns
Fig. 4 Change of mid-height deflections with increasing load



ℓ/D = 15.7, θ = 22.5°

Fig. 5 Interaction diagrams

Ultimate Strength The behavior of the column under increasing load is illustrated by the axial load versus bending moment diagram for the critical section of the column, as given in **Fig. 5.** The lateral coordinate gives the magnitude of the biaxial bending moment, M. The points for the combination of axial load and nominal end moment obtained experimentally at the maximum strength are designated by small circles for each column. The curve denoted by "ℓ/D = 15.7" represent the corresponding analytical data. When the column is short, the additional eccentricity due to lateral deflection at the critical section is negligible and the maximum moment can be estimated by the end moment at all stages, so a material failure occurs. If the column is long, the additional eccentricity at the critical section increases more rapidly at high load levels, and maximum load of the column is eventually reduced by the amplified bending moment caused by additional eccentricity. There are two types of long column behavior for the case of ℓ/D = 15.7. When the eccentricity at the column ends is relatively large, a material failure occurs causing the N-M path to reach the interaction line of the section. However, when the load is applied with small eccentricity, the column becomes unstable before reaching the interaction line and even at the critical section the column does not give its full strength capacity. This is called an instability failure. For almost all columns of ℓ/D = 25.7, the instability failure occurred.

Figure 6 illustrates the family of square column interaction curves giving the combination of the axial load and the end moment which cause failure of the column. Solid lines are obtained analytically, indicating the reduction in ultimate load due to slenderness for various loading cases. All the test data are plotted by small circles. In the case of square long columns, it is remarkable that there is not much difference in ultimate loads of the columns in spite of variation of the angle of eccentricity of the applied load. It can be seen that the analytical results agree quite well with the test results. The columns loaded with equal eccentricity of the opposite sign show that their load carrying capacities suddenly decrease due to rapid changing of the column deflection shapes from double-curvature mode to single-curvature mode near the maximum load. Also in this case of loading condition, the maximum loads of square columns are rather insensitive to the angles of eccentric loading plane.

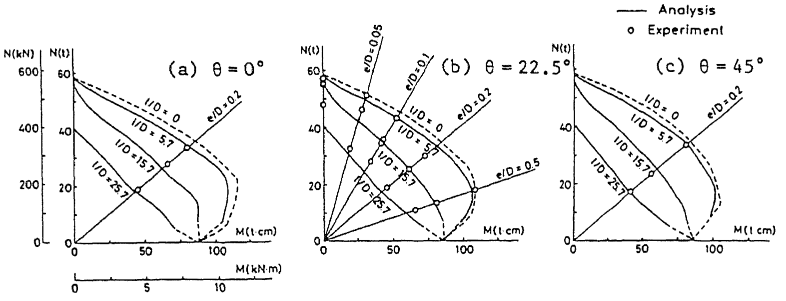


Fig. 6 Square long column interaction diagrams

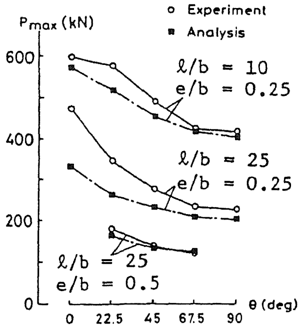


Fig. 7 Maximum load vs. loading angle relation (Rectangular columns)

Figure 7 shows how the maximum load P_{max} varies with changing the angle θ of eccentricity for rectangular columns subjected to uniaxially or biaxially eccentric loads. Because of the difference of load carrying capacity and bending stiffness about each of the strong axis and the weak axis, the maximum load of column decreases with increasing θ. For the long columns of ℓ = 300cm (ℓ/b = 25), the ultimate load was affected largely by increased deflections with increased load and the instability failure occurred. When the columns are subjected to a biaxially eccentric load causing bending about the near-weak axis, that is, the load is applied when angle θ ranges from about 45^o to 90^o, the maximum load and the deformation of the columns closely resemble the behavior under uniaxially eccentric loading conditions about the weak axis. Therefore, those columns can be approximated by the columns under only uniaxial bending about the weak axis. However, when the columns are subjected to biaxially eccentric load to cause bending about the near-strong axis, with θ ranging from 0^o to 45^o, the ultimate strengths seriously depend on the angle of eccentricity of the applied load, and consequently the column must be analyzed exactly by taking into account the biaxial loading effect.

DYNAMIC LOADING EFFECTS ON THE BEHAVIOR OF COLUMNS

As reported in the previous paper (Ref.2) concerning dynamic loading effects on the material properties, compressive strength of concrete and yield strength of steel increase due to strain rates expected during an earthquake. The elastic modulus of steel is not affected by a strain rate, but the initial elastic modulus of concrete increases with increasing strain rate. Therefore, in both elastic and plastic regions, it can be considered that buckling strengths of slender reinforced concrete columns are affected by loading rates.

Dynamic Loading Test In order to investigate the effects of loading rates on the elastic-plastic buckling behavior, four short columns (ℓ/D = 6) and eight long columns (ℓ/D = 16, 26), with a 6cm(b) x 6cm(D) cross section shown in **Fig. 8**, were tested under the prescribed longitudinal strain rate in quasi-static or dynamic loading. The test variables involved in the program were column length, eccentricity of the applied load and loading rate, as shown in **Table 2.** The specimens were tested by an electro-hydraulic servo-controlled testing system. Both ends of the specimen were pin-supported by knife-edges. Load was

Table 2 Test series and loading type

l/D	e/D	Loading	Specimen Name
6	0	Static Dynamic	S06C D06C
	0.25	Static Dynamic	S06E D06E
16	0	Static Dynamic	S16C D16C
	0.25	Static Dynamic	S16E D16E
26	0	Static Dynamic	S26C D26C
	0.25	Static Dynamic	S26E D26E

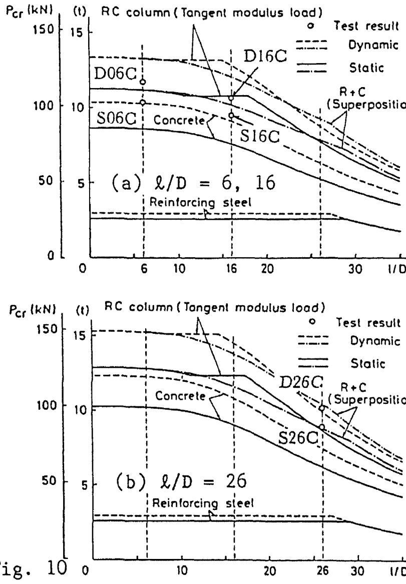
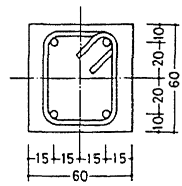
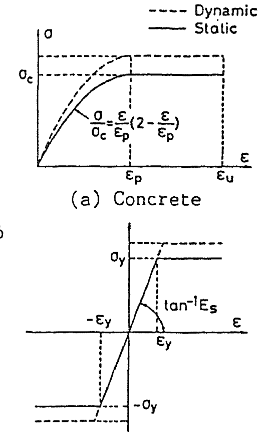


Fig. 10 Column curves; Centrally applied loading



Main reinforcement: 4-4.5mm φ

Fig. 8 Column cross section



ℓ/D	6, 16	26
σ_c (N/mm^2)	23.5	28.0
ε_p (%)	0.25	0.26
σ_y (N/mm^2)	400	
σ_u (N/mm^2)	427	

Fig. 9 Assumed stress-strain relations

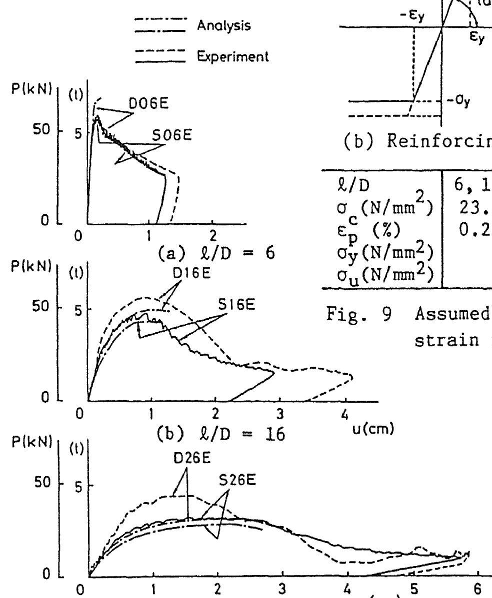


Fig. 11 Load-deflection relations; Eccentric loading

applied monotonically to impose uniform rate of actuator stroke, which was equivalent to a longitudinal strain rate in the specimens, through the whole process of loading. The specimens were tested at strain rates of 0.05sec^{-1} (dynamic) or 0.00005sec^{-1} (quasi-static). The measured strain rates at the testing part in dynamic loading were reduced about 1/2 of the prescribed rates. So the actual ratios of dynamic rate to static rate ranged 330-600 times. Increases of the buckling load with the increasing loading rate were precisely observed experimentally. In the case of dynamic centric loading, the maximum load of the column is 11-14% larger than in the quasi-static centric loading and, in the case of dynamic eccentric loading, 17-37% larger. Almost all the specimens failed at the mid-height section. It can be seen that the failure mode is not affected by the loading rate imposed during an earthquake.

Analytical Investigation The simple analytical method taking into account the strain rate effects (Ref.3) is applied, in which compressive strength of concrete and yield strength of steel are assumed to be uniformly 17-20% larger than the quasi-static values, as shown in **Fig. 9.** The buckling strength P_{cr} of the column subjected to centric load was calculated by the tangent modulus theory and the superposed strength method, which were used for composite columns by Bondale (Ref.4) and Wakabayashi (Ref.5), respectively, and which were modified for reinforced concrete columns. In the superposed strength method, the buckling load of a total reinforced concrete column was obtained as a simple sum of separately computed buckling load of reinforcing steel and tangent modulus load of concrete column. The results of the buckling analysis of columns are shown in **Fig. 10.** The values of loads are plotted against the arguments of the slenderness factor. Solid lines indicate the static solutions and dashed lines the dynamic solutions. In order to check the accuracy of the proposed method, the exact tangent modulus load of a reinforced concrete column was compared with the sum of the tangent modulus loads shown by dash-dotted lines. Small circles indicate the test results. It is found that the application of the present method on determining the buckling load of a centrally loaded column causes a small error in general, and that the analysis can estimate the increase of the load due to the loading rate effect. The dynamic load-deformation response of the eccentrically loaded column was solved by the same numerical method as the one in the static analysis, using the stress-strain relation in **Fig. 9.** The dynamic load carrying capacity of the eccentrically loaded column increases, as shown in **Fig. 11,** however is considerably higher than that predicted by the analysis. In the case of eccentric loading, this analysis predicts the increase qualitatively but is quantitatively not sufficient.

CONCLUSIONS

The following conclusions can be drawn for the ultimate load and the deformation behavior of slender reinforced concrete members subjected to statically and/or dynamically eccentric load. (1) On the basis of the results of static loading tests, it is recognized that the ultimate load of a slender column is reduced by the additional eccentricity due to lateral deflections, even in a column having a length to depth ratio of 15. (2) In the case of square long columns, there is not much difference on ultimate loads of the columns in spite of variation of the angle of eccentricity of the applied load. In the case of rectangular long columns, when the columns are subjected to biaxially eccentric load to cause bending about the near-strong axis, the ultimate loads and the deformation behavior seriously depend on the angle of eccentricity of the applied load. Consequently those columns must be analyzed exactly by taking into account the biaxial loading effect. However, the columns subjected to biaxial bending about the near-weak axis can be approximated by those under only uniaxial bending about the weak axis. (3) In the dynamic loading test, increases of the buckling load of the slender columns with the increasing loading rate were precisely observed. A simple analytical method taking into account the strain rate effects is applied, in which the material strength is assumed to increase up to about 20% larger than the quasi-static value. It predicts well the dynamic behavior of slender columns.

REFERENCES

1. Iwai, S.: Studies on the Elastic-Plastic Behavior of Reinforced Concrete Long Columns Subjected to Static and Dynamic Loads, Thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Engineering at Kyoto University, Kyoto, Japan, (1984; in Japanese).
2. Wakabayashi, M., Nakamura, T., Yoshida, N., Iwai, S., and Watanabe, Y.: Dynamic Loading Effects on the Structural Performance of Concrete and Steel Materials and Beams, Proc. 7th WCEE, Istanbul, Vol. 6, 271-278, (1980).
3. Wakabayashi, M., Nakamura, T., Iwai, S., and Hayashi, Y.: Effects of Strain Rate on the Behavior of Structural Members Subjected to Earthquake Force, Proc. 8th WCEE, San Francisco, Vol. 4, 491-498, (1984).
4. Bondale, D.S.: Column Theory with Special Reference to Composite Columns, The Consulting Engineer, Vol. 30, 43-48, (1966).
5. Wakabayashi, M.: A Proposal for Design Formulas of Composite Columns and Beam-Columns, Preliminary Report, Second International Colloquium on Stability, Tokyo, 65-87, (1976).