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STOCHASTIC EVALUATION OF INELASTIC SEISMIC RESPONSE FOR MULTI-DEGREE-OF-FREEDOM LUMPED MASS MODELS

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SUMMARY

Inelastic seismic responses were computed for six-degree-of-freedom lumped mass models with the normal bi-linear and the origin oriented hysteretic characteristics and various fundamental frequencies. The spectral characteristics of input ground motions were also varied considering those of recorded motions. Results for both hysteretic models are reduced into a general formula with the ratio of the dominant frequency of ground motion to the fundamental frequency of structure as a key parameter. Then a practical method is presented for estimating the second moment seismic safety index based on the log-normal distribution postulation for both the seismic load effects and the ultimate resistance.

INTRODUCTION

The probabilistic seismic safety evaluation of structures requires reliable estimations for uncertainties and variabilities of both the structural resistance and the seismic load effects. Some of difficulties may arise from the inelastic behaviour of structures which must be considered in the seismic safety and the large variability of seismicity in terms of the coefficient of variation. Some interesting measures for the inelastic response variability at various acceleration levels have been available based on an equivalent linearisation technique and sensitivity studies with quite a few model parameters (Ref. 1).

On the other hand relatively minor uncertainties and variabilities in comparison with the seismicity variability may be disregarded in order to provide an approximate and overall measure for the seismic safety, which could be more useful for practical applications. The present study examines the variability of inelastic responses in terms of the equivalent elastic response for various levels of simulated earthquake ground motions, then to apply the reduced results to determining a probabilistic seismic safety index proposed by one of the authors (Ref. 2). This is an extended version of the previous study for a reactor building model (Ref. 3).

SEISMIC INPUT AND STRUCTURAL MODEL

Ground Motion Model Commonly used four major parameters are considered to characterise the earthquake ground motion. They are the intensity, the duration, the dominant frequency and the spectral distribution form. The intensity is specified in terms of the peak acceleration and scalar factors, such as 2, 3 and

5, are multiplied to vary its level. The duration can be considered to represent the earthquake magnitude, M , and the envelope shapes proposed by Jennings et al (Ref. 4) are used to simulate the time history for $M=5.5, 7$ and 8.5 . Durations of the strong stationary phase for three magnitudes are 3.2sec, 9.4sec and 27.4sec respectively.

The Kanai-Tajimi spectrum (Ref. 5) is used to specify the spectral characteristics of ground motions, i.e.,

$$S(f) = S_0 \frac{1 + (2\eta f/f_g)^2}{[1 - (f/f_g)^2]^2 + [2\eta f/f_g]^2} \quad (1)$$

where S_0 determines the intensity, f_g represents the dominant frequency and η governs the spectral distribution form. Best fit values for f_g and η to the power spectra of many existing recorded motions have been examined (Ref. 6), then $f_g=1.25\text{Hz}, 2.5\text{Hz}$ and 5Hz and $\eta=0.3, 0.4$ and 0.5 are used herein as typical examples. Ten simulated time histories are generated to have a set of Fourier amplitude, as shown in Fig.1, specified by the power of Eq.(1) with different sets of random phases. 27 kinds of these ten ground motion time histories, i.e. with three variations for each of three parameters M, f_g and η , are used.

Structural Model Six-degree-of-freedom lumped mass shear-beam model is adopted to represent a typical multi-storey building model. Each mass has a uniform weight and each member has a spring constant proportional to its supporting weight. Their values are determined so that the fundamental frequency, $f_0=1\text{Hz}, 2\text{Hz}$ and 5Hz . Damping ratio is 5% for the elastic stiffness. The yielding strength distribution is due to the conventional practice in Japan as specified in the building regulation, i.e. so-called A_i distribution, where the period for the ground is given by the inverse of f_0 . The preliminary elastic response analysis has confirmed that the response distribution approximately corresponds to the A_i distribution.

The inelastic characteristics of members are specified by a bi-linear load-deflection skeleton with the second stiffness equal to 1/8 of the initial. Two typical hysteresis rules, i.e. the normal bi-linear type (N.B.) and the origin oriented type (O.O.), are adopted to model the hysteretic characteristics with the highest energy absorption capacity and the lowest respectively.

STOCHASTIC NATURE OF INELASTIC RESPONSES

Equivalent Elastic Response — Peak Acceleration Relationship The inelastic response (γ, Q) is examined in terms of the energy equivalent elastic response, Q^* , as defined in Fig.2, where the area ABCO is equal to the area ODE. The ultimate strength, R , in terms of the equivalent elastic response is also shown, corresponding to the ultimate deformation γ_u . Examples of computed results for

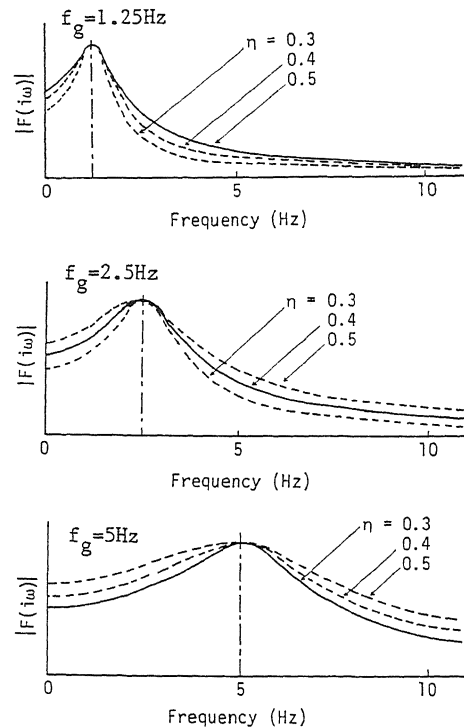


Fig.1 Fourier Amplitude Spectrum Used in Present Study According to Kanai-Tajimi Spectrum.

Q^* of member 1 (corresponding to the first storey) v.s. the peak acceleration, a , are shown in Fig.3 for two combinations of ground motions and fundamental frequencies of the structure, i.e., $M=8.5$, $f_g=1.25\text{Hz}$, $\eta=0.5$ and $f_o=5\text{Hz}$, and $M=5.5$, $f_g=5\text{Hz}$, $\eta=0.3$ and $f_o=1\text{Hz}$ and for two cases of hysteretic characteristics of the structure. Q^* and a are normalised by the yielding resistance, Q_y , and the corresponding peak acceleration, a_y , respectively. When $f_g < f_o$ in Fig.3 (a),(b), the slope becomes very steep in comparison with the case of $f_o < f_g$ in Fig.3 (c),(d). Results also show that Q^* for N.B. in Fig.3 (a),(c) are slightly less than that for 0.0. in Fig.3 (b),(d). Straight lines were obtained by the least squares fitting for individual cases.

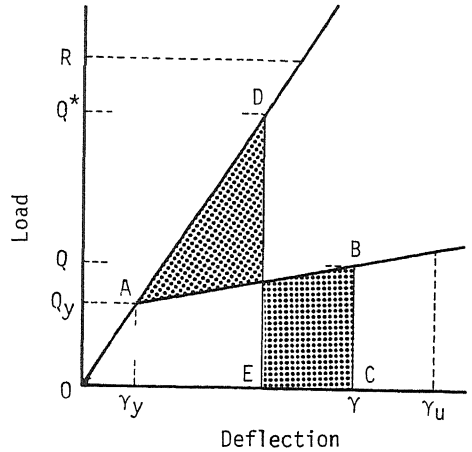


Fig.2 Definition of Equivalent Elastic Response Q^* with Bi-linear Skelton Curve.

Variation of Inelastic Responses In order to exemplify the variability of inelastic responses, frequency distributions of computed Q^* normalised by the rectilinear approximation value are shown in Fig.4 corresponding to Fig.3. The form of distribution could be regarded as log-normal. The c.o.v. values are approximately between 10 % and 20 % as shown in the figure, and might be neglected to estimate the c.o.v. of seismic load effects with a high value for the variability of seismicity such as 60% or more (Ref.7). This result also suggests the appropriateness of the rectilinear approximation of Q^* - a relationship.

Reduction of Inelastic Response Results Extensive examinations were carried out on the Q^* - a relationship for 6 members with 3 times 27 combinations of various natural frequencies and ground motions. It was found that the duration of motion or the earthquake magnitude, M , and the spectral form, η , have less significant effects on the inelastic response within the range of variation examined for M and η . Once the yield strength distribution is properly designed, the inelastic deflection seems to be also fairly uniformly developed in all members with the largest normalised equivalent elastic response generally occurring at the bottom member 1 (Refs. 1 and 6).

Therefore the slope in Fig.3 for various M and η , which characterise the inelastic response, was plotted against the ratio of the dominant frequency of input motion, f_g , to the fundamental frequency of structure, f_o . The slope in this study is expressed in terms of the normalised response acceleration ratio defined as $(Q^*/Q_y)/(a/a_y)$ at $Q^*=4Q_y$. Results are shown in Fig.5 (a) for N.B. and (b) for 0.0. The scatter due to the various M and η exists but this might be also neglected when a high variability of seismicity is taken into account. Following formula may be used to represent the tendency of plots.

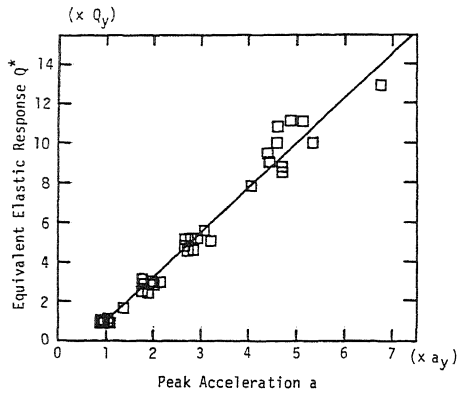
$$\left(\frac{Q^*/Q_y}{a/a_y}\right)_{a: Q^*=4Q_y} = \left\{ A + \left(\frac{B}{\frac{f_g}{f_o} + 1.0}\right)^C \right\}^{0.5} \quad (2)$$

Constants A, B, and C are obtained by the least squares method as,

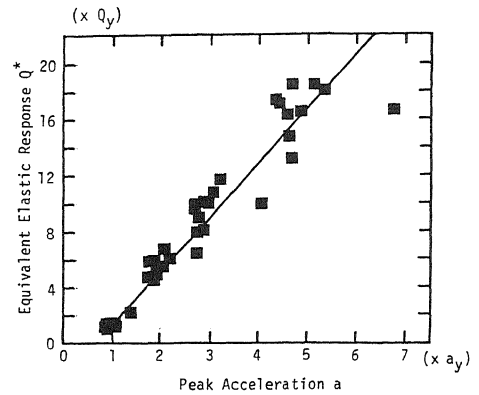
$$A = 0.567, B = 1.53, C = 5.70 \text{ for N.B. and}$$

$$A = 0.839, B = 1.65, C = 5.13 \text{ for 0.0.}$$

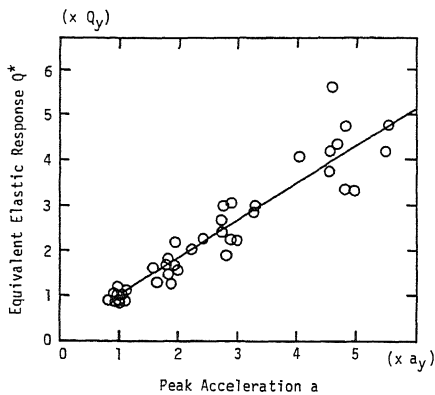
Although the total variance of response should consist of the contribution due to the uncertainties of f_g , η , M , f_o , Q_y , the damping ratio, the hysteresis rule



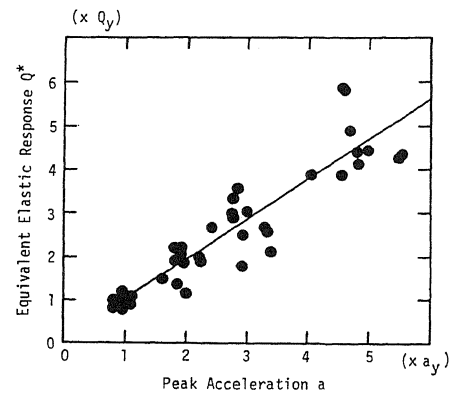
(a) : $M=8.5$, $f_g=1.25\text{Hz}$, $\eta=0.5$, $f_o=5\text{Hz}$ for Normal Bi-linear Model.



(b) : $M=8.5$, $f_g=1.25\text{Hz}$, $\eta=0.5$, $f_o=5\text{Hz}$ for Origin Oriented Model.



(c) : $M=5.5$, $f_g=5\text{Hz}$, $\eta=0.3$, $f_o=1\text{Hz}$ for Normal Bi-linear Model.



(d) : $M=5.5$, $f_g=5\text{Hz}$, $\eta=0.3$, $f_o=1\text{Hz}$ for Origin Oriented Model.

Fig.3 Examples of Computed Results for Q^* v.s. peak acceleration, a , for Member 1 with Rectilinear Approximation.

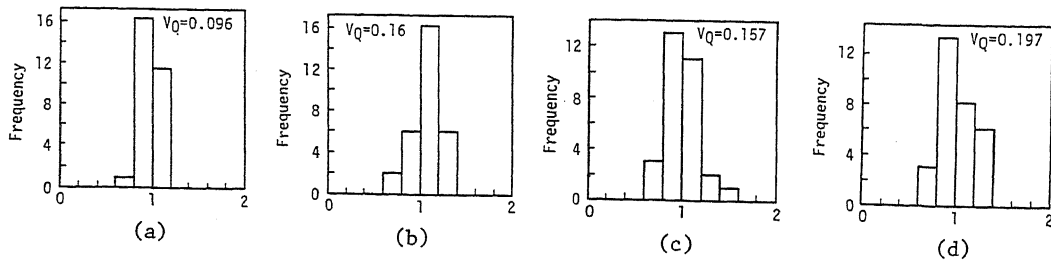
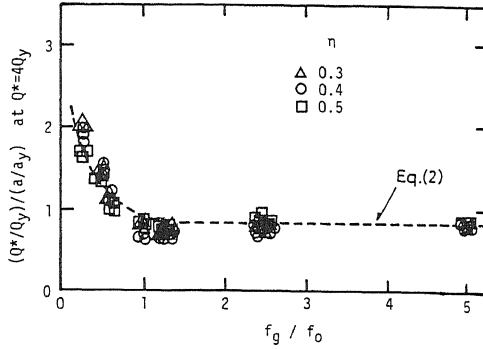
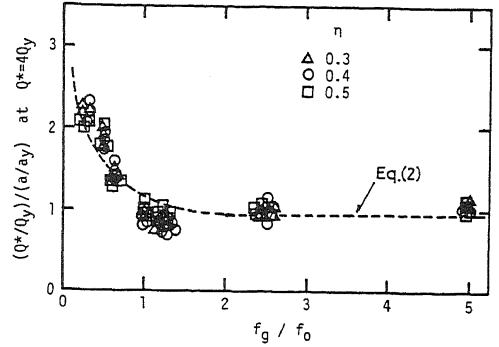


Fig.4 Examples of Frequency Distributions of Computed Results around Rectilinear Approximation ((a),(b),(c) & (d) respectively correspond to those in Fig.3).



(a) N.B. model



(b) O.O. model

Fig.5 Relationship between $(Q^*/Q_y)/(a/a_y)$ at $Q^*=4Q_y$ v.s. f_g/f_o .

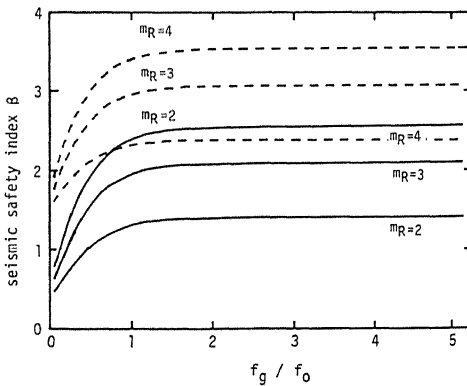
and so on (Ref. 1), results shown above suggest that the variability of seismicity is only predominant in estimating the c.o.v. of seismic load effects, as far as reasonably accurate values are given for these parameters.

PROBABILISTIC SEISMIC SAFETY MEASURE

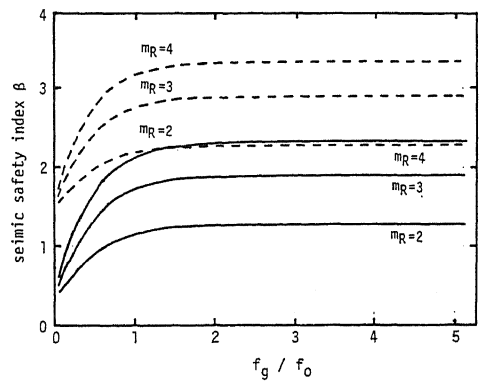
A probabilistic seismic safety index, β , has been proposed by the authors (Refs. 2,3) in the following form based on the second moment method by postulating the logarithmic distribution for both the resistance, R , and the peak acceleration, a ,

$$\beta = \frac{\ln m_R - \frac{\zeta_R^2}{2} + \frac{\ln m_R}{\ln m_a} \left(\frac{\zeta_a^2}{2} + \ln \frac{a_y}{\mu_a} \right)}{\sqrt{\zeta_R^2 + \left[\frac{\ln m_R}{\ln m_a} \zeta_a \right]^2}} \quad (3)$$

where m_R is the resistance margin μ_R/Q_y , μ_R is the mean ultimate resistance in terms of Q^* , m_a is the earthquake intensity margin a_u/a_y , a_u is the peak



(a) N.B. model



(b) O.O. model

Fig.6 Second Moment Safety Index, β , with f_g/f_o for $V_a=0.6$ and $V_R=0.3$, (----: $a_y/\mu_a = 2$ and —: $a_y/\mu_a = 1$).

acceleration corresponding to μ_R, μ_a is the mean maximum acceleration expected in the lifetime of a structure, $\zeta^2 = \ln(1+V^2)$. V_a and V_R are c.o.v. for a and R respectively. V_a value could be increased by a few percent from the c.o.v. value of maximum peak acceleration in lifetime, if the contribution of parameter uncertainties to the total variance of inelastic response is significant. The consistency of β with the probability of failure has been examined elsewhere (Ref. 3).

Once Q^* - a relationship is estimated by Eq.(2), β in Eq.(3) can be calculated for m_R and a_y/μ_a by substituting μ_R for Q^* at $a=a_u$. Results are shown for typical cases of $m_R=2$ to 4, $a_y/\mu_a=1$ and 2 in Fig.6 with $V_a=0.6$ and $V_R=0.3$ for (a) N.B. and (b) 0.0. As the inelastic response becomes greater for $f_g \ll f_0$, β becomes less with the same m_R and a_y/μ_a . In other words the level of β will be maintained when a greater m_R or a_y/μ_a is adopted in the case of $f_g \ll f_0$, where the latter is generally quite feasible since the elastic response for $f_g \ll f_0$ is considerably less than that for $f_g=f_0$.

CONCLUSIONS

Based on numerous parametric studies on the inelastic responses of six-degree-of freedom lumped mass models, following conclusions may be drawn;

- (1) Equivalent elastic response — peak acceleration relationship could be approximated to be rectilinear and the scatter was around 10 % to 20 % in terms of c.o.v., when input motion characteristics are specified.
- (2) An empirical formula was proposed to estimate the inelastic response against the ratio of the input motion dominant frequency to the structural fundamental frequency.
- (3) A proposed seismic safety index evaluation was demonstrated by applying the reduced results of inelastic responses.

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