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DYNAMIC RELIABILITY OF HYSTERETIC STRUCTURES UNDER COMBINED RANDOM LOADS

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SUMMARY

A method to estimate the dynamic reliability of hysteretic structures under combined time varying loads is investigated. The response statistical moments of structures, with hysteretic characteristics approximated by Wen's mathematical model, are obtained by using the Fokker-Planck equation method with a cumulant truncation technique. The uncertainties of power spectral densities pertinent to loads as stochastic processes, are also considered through the derivation of first passage probability. This paper describes the analysis of a single-degree-of-freedom system, and demonstrates that resulting variances and covariances of the response agree well with Monte Carlo simulation.

INTRODUCTION

In those structures which are subject to random dynamic loads such as earthquake, gust of wind, wave, etc., there are the possibilities of the structures receiving loads with very severe, great input level in future. The structures frequently show elasto-plastic behavior represented by hysteretic characteristics in the occurrence of such loads. In the evaluation of reliability, all loads which are likely to occur in future should be considered for the load level which is essentially uncertainty, and the loads should naturally include loads with the level at which the structures show nonelastic response. Therefore, probabilistic treatment based on the nonlinear random vibration theory is finally required in the dynamic reliability analysis.

Further, only any one of the above various dynamic loads does not always occur during the service life of the structure, but also several loads generally occur, and yet some loads may be applied at the same time. Though the effect of load combination is reflected in the guidelines for structural design in various countries including Japan, most of them are determined empirically and subjectively, and it seems that these have not completely studied theoretically and rationally.

Under these circumstances, this paper describes a method to estimate the dynamic reliability (first passage failure) of hysteretic structures taking into consideration the coincidence effect of dynamic loads.

FIRST PASSAGE PROBABILITY CONSIDERING LOAD COMBINATION

For the load combination problem of a structure, since the occurrence time t, duration time d and amplitude level h are naturally random variables in the dynamic load process, it is not easy to presume the occurrence possibility, maximum value, etc. of loads to be combined. For this problem a method considered to be effective for dynamic reliability analysis is Wen's method (1), the outline of which is given below.

Formulation of first passage failure of the system for random process is generally used in the following form:

$$P_f(T) = \nu^* T \tag{1}$$

where $P_f(T)$: Failure probability of the system during service life
ν^* : Mean rate of upcrossing
T : Service life

Since Wen's method to determine ν^* is simple in expression and can be easily utilized in the dynamic reliability theory, this paper adopts this method.

According to Wen, ν^* is expressed in the following way (2).

$$\nu^* = \sum_{i=1}^{M} \nu_i \widetilde{P}_i + \sum_{\substack{i=1 \\ (i \neq j)}}^{M} \sum_{j=1}^{M} \nu_{ij} \widetilde{P}_{ij} \tag{2}$$

where,
$\widetilde{P}_i$: Mean occurrence rate when only load F_i exists and occurs.
$\widetilde{P}_{ij}$: Mean occurrence rate when only loads F_i and F_j exist and occur.
ν_i : Conditional mean rate of upcrossing which the response value exceeds the limit state level when only load F_i exists and occurs.
ν_{ij} : Conditional mean rate of upcrossing which the response value exceeds the limit state level when only loads F_i and F_j exist and occur.
M : Total number of types of load

In equation (2), coincidence probability of three or more types loads is omitted since it is considered to be very small. For $\widetilde{P}_i$ and $\widetilde{P}_{ij}$, the following expression is approximately possible when stochastic Poisson process has been assumed.

$$\widetilde{P}_i = \lambda_i \tag{3}$$

$$\widetilde{P}_{ij} = \lambda_i \lambda_j (\mu_{di} + \mu_{dj}) \tag{4}$$

where $\lambda_k (k=i,j)$: Mean occurrence rate of load F_k
μ_{d_k} : Mean duration time of load F_k

Therefore failure probability $P_f(T)$ will be obtained by calculating ν_i and ν_{ij}.

A SINGLE-DEGREE-OF-FREEDOM NONLINEAR HYSTERETIC MODEL

This paper takes up a single-degree-of-freedom hysteretic model as shown in Fig.1, and analyzes a load combination problem when two types of dynamic loads - a load due to base excitation and general external force-operate. In Fig.1, x and x_g indicate the absolute displacement and ground displacement, and m and C indicate mass and damping coefficient respectively. Also, $F(t)$ and Q indicate the external force and nonlinear restoring force respectively.

A general equation of motion for Fig.1 is

$$m\ddot{x}_r + C\dot{x}_r + Q(x_r, t) = -m\ddot{x}_g + F(t) \tag{5}$$

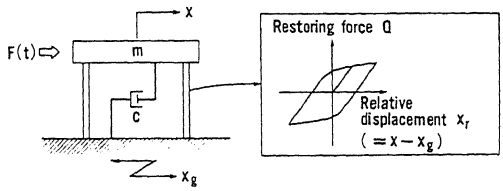

Fig. 1 Single-Degree-of-Freedom Hysteretic Model

Though various models have been proposed so far for the hysteretic model required for equation (5), effective models for theoretical analysis of dynamic reliability are less than expected. Out of these, Wen's model (3) has comparatively wide application, and can be easily utilized even in the theoretical analysis since it is given in mathematical expression. For this reason, this paper makes use of Wen's model shown in the following equation.

$$Q(x_r, t) = \alpha k x_r + (1-\alpha) k Z \tag{6}$$

$$\dot{Z} = A\dot{x}_r - \beta|\dot{x}_r||Z|^{n-1}Z - \gamma\dot{x}_r|Z|^n \tag{7}$$

MEAN RATE OF UPCROSSING WHEN TWO LOADS SIMULTANEOUSLY APPLIED

Estimation of the Response Covariance Using the Fokker-Planck Method Though there are the stochastic equivalent linearization technique, perturbation method, Fokker-Planck method, etc. as typical methods for nonlinear random vibration, the Fokker-Planck method is used here in accordance with the reference (4).

Assuming $x_r = X_1$, $\dot{x}_r = X_2$ and $Z = X_3$, indicating equations (5) to (7) using state variables,

$$\left.\begin{aligned} \dot{X}_1 &= X_2 \\ \dot{X}_2 &= -\frac{C}{m}X_2 - \frac{\alpha k}{m}X_1 - \frac{(1-\alpha)k}{m}X_3 - \ddot{x}_g + \frac{F(t)}{m} \\ \dot{X}_3 &= AX_2 - \beta|X_2||X_3|^{n-1}X_3 - \gamma X_2|X_3|^n \end{aligned}\right\} \tag{8}$$

Hence, if $\ddot{x}_g$ and $F(t)$ in equation (8) are considered to be random process, equation (8) becomes a probability differential equation with (X_1, X_2, X_3) as state random variables.

$\ddot{x}_g$ and $F(t)$ in equation (8) are regarded as nonstationary random processes which are independent to each other, and are assumed as follows.

$$\ddot{x}_g = e_g(t) n_g(t) \tag{9}$$

$$\frac{F(t)}{m} = e_f(t) n_f(t) \tag{10}$$

where $e_g(t)$ and $e_f(t)$ are assumed to be shape functions which slowly change with time, and $n_g(t)$ and $n_f(t)$ Gaussian white noises having the following characteristics.

$$
\left.\begin{aligned}
&E[n_I(t)] = 0 \\
&E[n_I(t)\, n_I(t+\tau)] = 2\pi\, S_{0I}\,\delta(\tau) \\
&\qquad (I = g, f)
\end{aligned}\right\} \tag{11}
$$

where $E[\quad]$: Ensemble average
S_{0I} : Power spectral intensity of $n_I(t)$
$\delta(\tau)$: Delta function

In equation (8), the dynamic load is composed of two: $\ddot{x}_g$ and $F(t)$, and each of them consists of white noises which are independent to each other. Therefore, state random variables $X = (X_1, X_2, X_3)^T$ are considered to be Markov vector component at this time, and its transient probability density $q_{(X)}(X^T, t\,|\,X_0^T, t_0)$ satisfies the Fokker-Planck equation.

When the initial condition is assumed to be known as probability 1, the equation which probability density function $P_{(X)}(X^T, t)$ should meet is determined as follows.

$$
\frac{\partial P}{\partial t} = -X_2\frac{\partial P}{\partial X_1} - AX_2\frac{\partial P}{\partial X_3} + \left(\frac{C}{m}\right)P + \left\{\left(\frac{\alpha k}{m}\right)X_1 + \left(\frac{C}{m}\right)X_2 + \frac{k(1-\alpha)}{m}X_3\right\}\frac{\partial P}{\partial X_2}
$$

$$
+\beta|X_2|\frac{\partial}{\partial X_3}\left\{|X_3|^{n-1}X_3P\right\} + \gamma X_2\frac{\partial}{\partial X_3}\left\{|X_3|^n P\right\} + \pi\left\{S_{0g}\,e_g^2(t) + S_{0f}\,e_f^2(t)\right\}\frac{\partial^2 P}{\partial X_2^2} \tag{12}
$$

Though various values are considered as the value for n in equation (12), $n=1$ seems to well represent a general elasto-plastic behavior according to Wen (5), and also it can be easily interpreted theoretically. Therefore, this paper limits to a case of $n=1$ to carry forward the formulation. Moreover, for the term for absolute value appeared in equation (12), quadratic curve approximation is performed as follows.

$$
|X_2| \approx a_1 X_2^2\ ,\ |X_3| \approx a_2 X_3^2 \tag{13}
$$

in which a_1 and a_2 are determined to minimize the statistical mean square error coming through approximation. Substituting equation (13) into equation (12), the equation without absolute values which probability density function $P_{(X)}(X^T, t)$ should satisfy is obtained. After repeating integration by parts against its equation, second order moment equations are determined as follows.

$$
\frac{\partial}{\partial t}E[X_1^2] = 2E[X_1 X_2] \tag{14}
$$

$$
\frac{\partial}{\partial t}E[X_2^2] = -4\xi\omega_0 E[X_2^2] - 2\alpha\omega_0^2 E[X_1 X_2] - 2(1-\alpha)\omega_0^2
$$

$$
\times E[X_2 X_3] + 2\pi\left\{S_{0g}\,e_g^2(t) + S_{0f}\,e_f^2(t)\right\} \tag{15}
$$

$$
\frac{\partial}{\partial t}E[X_3^2] = 2AE[X_2 X_3] - \left(\frac{8\beta}{3\sqrt{2\pi}}\right)\frac{1}{\sigma_{X2}}E[X_2^2 X_3^2] - \left(\frac{8\gamma}{3\sqrt{2\pi}}\right)\frac{1}{\sigma_{X3}}E[X_2 X_3^3] \tag{16}
$$

$$
\frac{\partial}{\partial t}E[X_1 X_2] = E[X_2^2] - \alpha\omega_0^2 E[X_1^2] - 2\xi\omega_0 E[X_1 X_2] - (1-\alpha)\omega_0^2 E[X_1 X_3] \tag{17}
$$

$$
\frac{\partial}{\partial t}E[X_2 X_3] = AE[X_2^2] - (1-\alpha)\omega_0^2 E[X_3^2] - \alpha\omega_0^2 E[X_1 X_3]
$$

$$
-2\xi\omega_0 E[X_2 X_3] - \left(\frac{4\beta}{3\sqrt{2\pi}}\right)\frac{1}{\sigma_{X2}}E[X_2^3 X_3] - \left(\frac{4\gamma}{3\sqrt{2\pi}}\right)\frac{1}{\sigma_{X3}}E[X_2^2 X_3^2] \tag{18}
$$

$$\frac{\partial}{\partial t}E[X_1X_3] = AE[X_1X_2]+E[X_2X_3]-\left(\frac{4\beta}{3\sqrt{2\pi}}\right)\frac{1}{\sigma_{X_2}}$$
$$\times E[X_1X_2^2X_3]-\left(\frac{4\gamma}{3\sqrt{2\pi}}\right)\frac{1}{\sigma_{X_3}}E[X_1X_2X_3^2] \tag{19}$$

in which replacement is performed as $(k/m) = \omega_0^2$ and $(C/m) = 2\xi\omega_0$.

Though equations (14) to (19) are not closed formed equations, closed forms can be easily obtained by using the cumulant truncation technique in the same manner as reference (4). In those derived closed equations, the parameters of S_{0g}, S_{0f}, $e_g(t)$ and $e_f(t)$ are random actually. However, this paper takes up only S_{0g} and S_{0f} as random variables, and limits $e_g(t)$ and $e_f(t)$ to definite constants for the simplicity. In order to facilitate the later reliability calculation, response moments may be obtained as a function of $(S_{0g}+S_{0f})$ from considering equation (15). The response moments take the following form at this time.

$$E[X_1^2] = H_1(S_{0g}+S_{0f}) \tag{20}$$

$$E[X_2^2] = H_2(S_{0g}+S_{0f}) \tag{21}$$

H_1 and H_2 in the above equations are functions obtained by the least square method.

<u>Calculation of the Conditional Mean Rate of Upcrossing</u> First determine joint probability density function $P(x_r, \dot{x}_r)$ for x_r $(=X_1)$ and $\dot{x}_r(=X_2)$. Since covariance for x_r and $\dot{x}_r$ is 0, $P(x_r, \dot{x}_r)$ becomes as below when the response can be approximately assumed to be small and Gaussian process.

$$P(x_r, \dot{x}_r) = \frac{1}{2\pi\sigma_{x_r}\sigma_{\dot{x}_r}}\exp\left\{-\frac{1}{2}\left(\frac{x_r^2}{\sigma_{x_r}^2}+\frac{\dot{x}_r^2}{\sigma_{\dot{x}_r}^2}\right)\right\} \tag{22}$$

By using equation (22), conditional mean rate of upcrossing ν_{gf}^+ that x_r exceeds a certain level $(x_r=d)$ at a positive gradient is determined as follows:

$$\nu_{gf}^+ = \int_0^\infty \dot{x}_r P(d, \dot{x}_r)\, d\dot{x}_r = \frac{1}{2\pi}\left(\frac{\sigma_{\dot{x}_r}}{\sigma_{x_r}}\right)\exp\left(-\frac{d^2}{2\sigma_{x_r}^2}\right) \tag{23}$$

If it is considered that the above almost applies in the case of a negative gradient, conditional mean rate of upcrossing ν_{gf} that x_r exceeds a region of $|x_r| \leqq d$ becomes

$$\nu_{gf} = \nu_{gf}^+ + \nu_{gf}^- \approx 2\nu_{gf}^+ = \frac{1}{\pi}\left(\frac{\sigma_{\dot{x}_r}}{\sigma_{x_r}}\right)\exp\left(-\frac{d^2}{2\sigma_{x_r}^2}\right) \tag{24}$$

Considering equations (20) and (21), ν_{gf} is expressed in the following form:

$$\nu_{gf} = \frac{1}{\pi}\sqrt{\left(\frac{H_2}{H_1}\right)}\exp\left(-\frac{d^2}{2H_1}\right) \tag{25}$$

If S_{0g} and S_{0f} are definite values in equation (25), ν_{gf} will be completely determined. Since, however, S_{0g} and S_{0f} are random variables, the following form considering it will be the final expression for ν_{gf}.

$$\nu_{gf} = \frac{1}{\pi}\int_0^\infty\int_0^\infty \sqrt{\left(\frac{H_2}{H_1}\right)}\exp\left(-\frac{d^2}{2H_1}\right) f_{S_{0g}}(S_{0g})\, f_{S_{0f}}(S_{0f})\, dS_{0g}\, dS_{0f} \tag{26}$$

in which $f_{S_{0g}}$ and $f_{S_{0f}}$ are probability density functions for S_{0g} and S_{0f} respectively. Since equation (26) is numerically integrated, $f_{S_{0g}}$ and $f_{S_{0f}}$ may have any form.

NUMERICAL CALCULATION

Various constants in hysteretic models adopted for numerical calculation are $A=1.0$, $\alpha=0.05$, $\beta=0.75$, $n=1$ and $\gamma=0.47$. Further as constants showing vibrational characteristics of the structure, $\omega_0=10.0$(rad/s) and $\xi=0.01$ are used.

The determined response moments are shown as a function of $(S_{0g}+S_{0f})$ in Fig.2, and the calculation results of the failure probability with parameter of service life are shown in Fig.3.

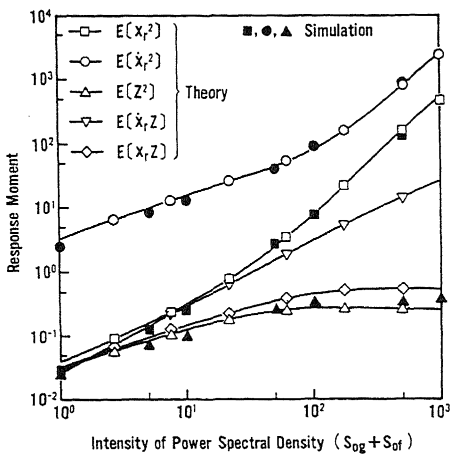

Fig. 2 Relationship between Power Spectral Density and Response Moment

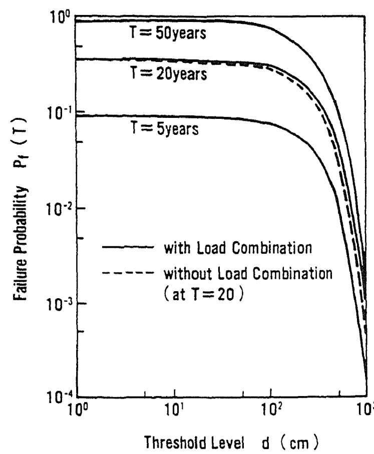

Fig. 3 Effects of Service Life on Failure Probability

CONCLUSION

(1) When a single-degree-of-freedom hysteretic structure is excited due to some random dynamic loads without reproducibility, the analysis method to estimate the first passage failure of the structure has been developed in consideration of the coincidence effect of the dynamic loads.

(2) For the response moments obtained by applying the cumulant truncation technique to the Fokker-Planck method, the Monte Carlo simulation is also performed. The comparison of both results shows good agreement and describes that the analytical theory is valid.

REFERENCES

1. Wen, Y. K., J. Am. Soc. Civil Eng., ST5, 103 (1977), 1079.
2. Wen, Y. K., Proc. 3rd ICOSSAR, (1981), 89.
3. Wen, Y. K., J. Am. Soc. Civil Eng., EM2, 102 (1976), 249.
4. Mochio, T., Trans. of JSME, 49-445, C (1983), 1455.
5. Wen, Y. K. et al., Civ. Eng. Stud., Struct. Res. Ser. No. 506, Univ. Illinois, (1983).