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SEISMIC DAMAGE AND RELIABILITY ANALYSIS OF HYSTERETIC MULTI-DEGREE-OF-FREEDOM STRUCTURES

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SUMMARY

An analytical method of stochastic damage analysis and reliability analysis of hysteretic multi-degree-of-freedom structures subjected to nonstationary seismic excitations is described. The aims of this paper are in systematically unifying the conventional stochastic response analysis including the damage analysis and the reliability analysis, and in directly evaluating the reliability function of the whole structural system under consideration of statistical dependence among the safety margins of the structural components.

INTRODUCTION

In this study, the stochastic damage and the reliability analyses of multi-degree-of-freedom hysteretic structural systems subjected to severe seismic excitations are dealt with. The measures of structural damages are, in general, defined as continuous and nondecreasing functions of time. For the failure criterion that the ith structural component fails when the structural damage η_i of the ith structural component exceeds the corresponding structural capacity c_{F_i}, the reliability function of the ith structural component is given by

$$R_i(t) = \mathrm{Prob}[\ \eta_i(t) < c_{F_i}\ ] = \int_0^{c_{F_i}} p(\eta_i)d\eta_i \tag{1}$$

If system failure occurs when at least one of the structural components fails, the reliability function of the whole structural system is expressed as

$$\begin{aligned} R(t;S_M) &= \mathrm{Prob}[\ \eta_1(t) < c_{F_1} \cap \eta_2(t) < c_{F_2} \cap \cdots \cap \eta_N(t) < c_{F_N}\ ] \\ &= \iint_{S_M}\cdots\int p(\eta_1,\eta_2,\cdots,\eta_N)d\eta_1 d\eta_2\cdots d\eta_N \end{aligned} \tag{2}$$

where N is the number of structural components, S_M is the safe domain of the whole structural system, and $\cap$ is the intersection operator of events. In order to evaluate the system reliability from Eq. (2), the joint probability density function $p(\eta_1,\eta_2,\cdots,\eta_N)$ of the damages of structural components is required.

The objects of this study are to obtain the joint probability density function of damages in the analytical form, and to evaluate explicitly the system reliability by taking account of the statistical dependence of the structural damages. The analytical method is based on the theory of continuous Markov processes. By using the differential forms of the measures of structural damages as well as the constitutive laws of hysteretic structures, it is possible to formulate the present problem in the form of stochastic differential equations. The method proposed for the reliability analysis of hysteretic systems (Refs. 3,4) is applied to multi-degree-of-freedom hysteretic systems.

FORMULATION IN THE FORM OF STOCHASTIC DIFFERENTIAL EQUATIONS

Consider multi-degree-of-freedom hysteretic structural systems. The nondimensional hysteretic characteristic ϕ_i of the ith story is expressed as

$$\phi_i = r_i x_i + (1-r_i) z_i \tag{3}$$

where the nondimensional relative displacement of the ith story, x_i, is normalized with reference to yield deformation, z_i is the nondimensional hysteretic component, and r_i is ratio. Both ϕ_i and z_i are normalized so that each initial rigidity is unity. The differential forms of hysteretic characteristics including the degrading or stiffening characteristics have been presented for a class of piecewise-linear hystereses (Ref. 1) and for the curved hystereses (Ref. 2). For the bilinear hysteretic model, the differential form is given by

$$\dot{z}_i = \dot{x}_i[1 - U(\dot{x}_i)U(z_i-\delta_i) - U(-\dot{x}_i)U(-z_i-\delta_i)] \equiv g_{z_i} \tag{4}$$

where $U(\cdot)$ denotes the unit step function. For the curved hysteretic model (Wen model) the differential form is given by

$$\dot{z}_i = \dot{x}_i[1 - \{\bar{\gamma}_i \mathrm{sgn}(\dot{x}_i z_i) + \bar{\beta}_i\}|z_i|^{\bar{n}_i}] \equiv g_{z_i} \tag{5}$$

It is necessary to define an appropriate measure of structural damage as the output response of each part of structural systems. The elementary measures of structural damages have been described by the differential forms in terms of single-valued nonlinear functions of the relevant state variables (Ref. 1). The cumulative plastic deformation ratio is expressed as in the differential form

$$\dot{\eta}_{pi} = (1-r_i)\mathrm{sgn}(z_i)(\dot{x}_i - g_{zi}) \equiv g_{\eta_{pi}} \tag{6}$$

where $g_{zi} \equiv \dot{z}_i$. The dissipated hysteretic energy ratio is in the form of

$$\dot{\eta}_{hi} = (1-r_i) z_i (\dot{x}_i - g_{z_i}) \equiv g_{\eta_{hi}} \tag{7}$$

The low-cycle fatigue damage factors associated with total and plastic deformations are, respectively, given by

$$\dot{\eta}_{ft_i} = a_i c_{F_i}^{-a_i} |x_i|^{a_i-1} |\dot{x}_i| \equiv g_{\eta_{ft_i}} \tag{8}$$

$$\dot{\eta}_{fpi} = a_i (1-r_i)^{a_i} c_{P_i}^{-a_i} |x_i - z_i|^{a_i-1} |\dot{x}_i - g_{z_i}| \equiv g_{\eta_{fpi}} \tag{9}$$

where a_i, c_{F_i} and c_{P_i} are parameters. Here the fatigue damage factors are so normalized as to take the value 1 when failure occurs. The composite measures of structural damages can be defined by making use of the elementary measures.

By using the differential forms of the hysteretic constitutive laws and the measures of damages, the state space equation of the whole dynamic system consisting of the hysteretic structure and the measures of damages may be given by

$$\dot{Z} = F(t,Z) + N , \quad F(t,Z) = AZ + G(Z) , \quad Z_{t=0} = 0 \tag{10}$$

where

$$Z = \begin{bmatrix} x \\ y \\ z \\ \eta \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ 0 \\ g_z \\ g_\eta \end{bmatrix}, \quad \mathbf{N} = \begin{bmatrix} 0 \\ V(t)W(t) \\ 0 \\ 0 \end{bmatrix}$$

$\mathbf{y} = \dot{\mathbf{x}}$, A is a coefficient matrix of the linear part of the structural system, $\mathbf{V}$ is a matrix-valued function of t, and $\mathbf{W}$ is a Gaussian white noise vector. Eq. (11) is equivalent to a special class of the Itô stochastic differential equation in the form

$$dZ = F(t,Z)dt + V(t)dB(t), \quad Z_{t=0} = 0 \tag{11}$$

where B is a vector-valued Brownian motion with zero-mean and diffusion intensity matrix **Q**. By defining the moments $M(k_1,k_2,\cdots,k_n)$ of the state vector **Z**

$$M(k_1,k_2,\cdots,k_n) = E[\psi(Z)], \quad \psi(Z) = \prod_{i=1}^{n} Z_i^{k_i} \tag{12}$$

the moment equations are derived from Itô's formula as

$$\dot{M}(k_1,k_2,\cdots,k_n) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \Gamma_{ij}k_ik_j\, M(k_1,\cdot\cdot,k_i-1,\cdot\cdot,k_j-1,\cdot\cdot,k_n)$$
$$+ \frac{1}{2}\sum_{i=1}^{n} \Gamma_{ii}k_i(k_i-1)\, M(k_1,\cdot\cdot,k_i-2,\cdot\cdot,k_n) + \sum_{i=1}^{n} k_iE[F_iZ_i^{-1}\prod_{j=1}^{n} Z_j^{k_j}] \tag{13}$$

where n is the number of state variables, F_i and Z_i are the ith components of **F** and Z, respectively, and Γ_{ij} is the ij element of $\Gamma = VQV^T$.

FINITE SERIES EXPANSION OF PROBABILITY DENSITY FUNCTION

In order to concretely determine the expectation operator E appearing in Eq. (13) and to truncate the moment equations to a finite set of simultaneous first order differential equations, it is necessary to introduce an analytical form of the joint probability density function p(**Z**). Here, the joint probability density function is approximated by a finite series expansion in terms of different orthogonal polynomials depending on the properties of state variables, especially on the regions of the state variables. For the case that the structural system has the bilinear hysteretic characteristics, the approximate probability density function is expressed as

$$p(Z) = \prod_{i=1}^{N} L_{z_i}\, w_N(x_i)\, w_N(y_i)\, w_N(z_i)\, w_G(\eta_i) \sum_{\sum_{j=1}^{N}\ell_j+m_j+n_j=0}^{L}\ \sum_{\sum_{j=1}^{N}q_j=0}^{M}$$
$$C_{\ell_1\cdot\cdot\ell_Nm_1\cdot\cdot m_Nn_1\cdot\cdot n_Nq_1\cdot\cdot q_N} \prod_{i=1}^{N} H_{\ell_i}(\hat{x}_i)\, H_{m_i}(\hat{y}_i)\, H_{n_i}(\hat{z}_i)\, L_{q_i}^{(\beta_i-1)}(\nu_i\eta_i) \tag{14}$$

where N is the number of degree-of-freedom of structural system, w_N and w_G are, respectively, the univariate normal and gamma density functions. H_{ℓ_i}, H_{m_i} and H_{n_i} are Hermite polynomials, $L_{q_i}^{(\beta_i-1)}$ is the generalized Laguerre polynomial, $\hat{x}_i$, $\hat{y}_i$ and $\hat{z}_i$ are standardized random variables, $\beta_i = E^2[\eta_i]/\sigma^2_{\eta_i}$, $\nu_i = E[\eta_i]/\sigma^2_{\eta_i}$. The folding operator L_{z_i} is defined as

$$L_{z_i}(\cdot) = [U(z_i+\delta_i) - U(z_i-\delta_i) + \delta(z_i+\delta_i)\int_{-\infty}^{-\delta_i}dz_i + \delta(z_i-\delta_i)\int_{\delta_i}^{\infty}dz_i]\ (\cdot) \tag{15}$$

The coefficients $C_{\ell_1\cdot\cdot\ell_Nm_1\cdot\cdot m_Nn_1\cdot\cdot n_Nq_1\cdot\cdot q_N}$ are determined as the functions of the moments from the following relations:

$$\sum_{\Sigma n_j=0}^{L-\Sigma\ell_j+m_j} C_{\ell_1\cdot\cdot\ell_Nm_1\cdot\cdot m_Nn_1\cdot\cdot n_Nq_1\cdot\cdot q_N} \prod_{i=1}^{N} J(k_i,n_i;\delta_i,\bar{\sigma}_{z_i})$$
$$= \frac{E[\prod_{i=1}^{N} H_{\ell_i}(\hat{x}_i)\, H_{m_i}(\hat{y}_i)\, \hat{z}_i^{k_i}\, L_{q_i}^{(\beta_i-1)}(\nu_i\eta_i)]}{\prod_{i=1}^{N} \ell_i!\, m_i!\, (\beta_i)_{q_i}/q_i!}. \tag{16}$$

where $(\beta)_q = \Gamma(\beta+q)/\Gamma(\beta)$

$$J(k,n;\delta,\sigma) = [1+(-1)^{k+n}]\{\int_0^{\delta} \hat{z}^k H_n(\hat{z})w_N(z)dz + (\frac{\delta}{\sigma})^k \int_{\delta}^{\infty} H_n(\hat{z})w_N(z)dz\}$$

For the Wen hysteretic model, $p(Z)$ is assumed as

$$p(Z) = \prod_{i=1}^{N} w_N(x_i)\, w_N(y_i)\, w_B(z_i)\, w_G(\eta_i) \sum_{\Sigma\ell_j+m_j+n_j=0}^{L} \sum_{\Sigma q_j=0}^{M}$$

$$C_{\ell_1 \cdot\cdot m_1 \cdot\cdot n_1 \cdot\cdot q_1 \cdot\cdot q_N} \prod_{i=1}^{N} H_{\ell_i}(\hat{x}_i)H_{m_i}(\hat{y}_i)P_{n_i}^{(\alpha_i,\alpha_i)}(z_i)L_{q_i}^{(\beta_i-1)}(\nu_i\eta_i) \qquad (17)$$

where $\alpha_i = (1/\sigma_{z_i}^2 - 3)/2$, w_B is the probability density function of Beta distribution, and P_{n_i} is the Jacobi polynomial. The coefficients are given by

$$C_{\ell_1 \cdot\cdot m_1 \cdot\cdot n_1 \cdot\cdot q_1 \cdot\cdot q_N} = \frac{E[\prod_{i=1}^{N} H_{\ell_i}(\hat{x}_i)H_{m_i}(\hat{y}_i)P_{n_i}^{(\alpha_i,\alpha_i)}(z_i)L_{q_i}^{(\beta_i-1)}(\nu_i\eta_i)]}{\prod_{i=1}^{N} \ell_i!\; m_i!\; B_{n_i}(n_i)\; (\beta_i)_{q_i}/q_i!} \qquad (18)$$

where
$$B_n(\alpha) = \frac{\Gamma(2\alpha+2)\;\Gamma^2(n+\alpha+1)}{n!\,(2n+2\alpha+1)\;\Gamma^2(\alpha+1)\;\Gamma(n+2\alpha+1)}$$

The nonlinear terms in Eq. (13) can be evaluated by using the approximate probability density function, and the resultant moment equations yield a finite set of first order nonlinear ordinary differential equations which are numerically solved under nonstationary state. The time-dependent joint probability density function and statistics of the whole state variables are obtained.

RELIABILITY ANALYSIS

The probability density function of the measures of structural damages is obtained as the marginal probability density function from Eq. (15) or (17) as follows:

$$p(\eta) = \prod_{i=1}^{N} w_G(\eta_i) \sum_{q_1+\cdot\cdot\cdot+q_N=0}^{M} D_{q_1q_2\cdot\cdot q_N} \prod_{j=1}^{N} L_{q_j}^{(\beta_j-1)}(\nu_j\eta_j) \qquad (19)$$

where $D_{q_1q_2\cdot\cdot q_N} = C_{00\cdot\cdot\cdot 0q_1q_2\cdot\cdot q_N}$

The reliability function of the ith structural component and the system reliability function of the whole structural system defined by Eqs. (1) and (2), respectively, are directly obtained by integrating the probability density function $p(\eta)$ over the safe domains as follows:

$$R_i(t;c_{F_i}) = \sum_{q=0}^{M} \frac{\Gamma(\beta_i+q)}{q!} \sum_{p=0}^{q} \sum_{s=0}^{q} (-1)^{p+s} \binom{q}{p}\binom{q}{s} \frac{\nu_i^s}{\Gamma(\beta_i+s)} E[\eta_i^s]\; I_G(\beta_i+p,\nu_i c_{F_i}) \qquad (20)$$

$$R(t;S_M) = \sum_{q_1+\cdot\cdot+q_N=0}^{M} \prod_{i=1}^{N} \frac{\Gamma(\beta_i+q_i)}{q_i!} \sum_{s_1=0}^{q_1} \cdot\cdot \sum_{s_N=0}^{q_N} E[\eta_1^{s_1}\eta_2^{s_2}\cdot\cdot\eta_N^{s_N}] \prod_{j=1}^{N} (-1)^{s_j}$$

$$\binom{q_j}{s_j} \frac{\nu_j^{s_j}}{\Gamma(\beta_j+s_j)} \sum_{\ell_1=0}^{q_1} \cdot\cdot \sum_{\ell_N=0}^{q_N} \prod_{k=1}^{N} (-1)^{\ell_k} \binom{q_k}{\ell_k} I_G(\beta_k+\ell_k,\nu_k c_{F_k}) \qquad (21)$$

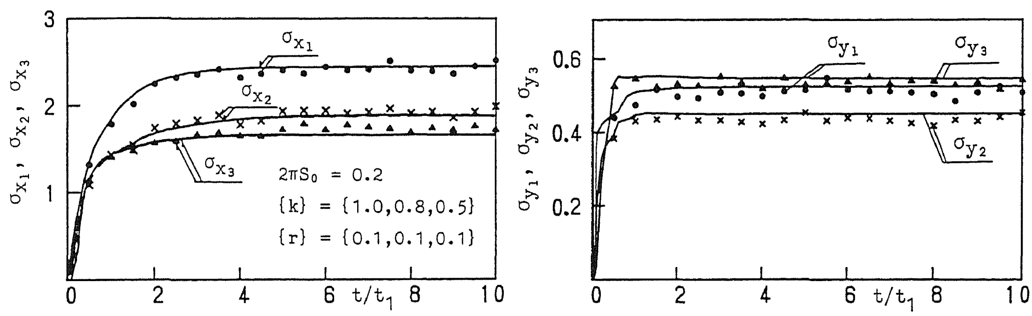
where $I_G(\beta,c) = \gamma(\beta,c)/\Gamma(\beta)$, $\Gamma(\cdot)$ is the gamma function, $\gamma(\cdot,\cdot)$ is the incomplete gamma function, and $\binom{\cdot}{\cdot}$ is the binomial coefficient.

NUMERICAL ANALYSIS

As a numerical example, a three-degree-of-freedom shear structure subjected to stationary white noise with the spectral level s_0 is investigated. Here the hysteretic model is the Wen type where $\bar{\beta}_i = \bar{\gamma}_i = 0.5$, $\bar{n}_i = 2$. The distribution of the nondimensional mass, initial stiffness and rigidity ratio are given as $\{m\} = \{1, 1, 1\}$, $\{k\} = \{1, 0.8, 0.5\}$ and $\{r\} = \{0.1, 0.1, 0.1\}$. The cumulative plastic deformation ratio is adopted as the measure of structural damage. In the series expansion of Eq. (17), L = 4 and M = 1 are used, and the number of the moments to be solved is 2169. To verify the accuracy and validity of the present method, a digital simulation of sample size 1000 has been carried out.

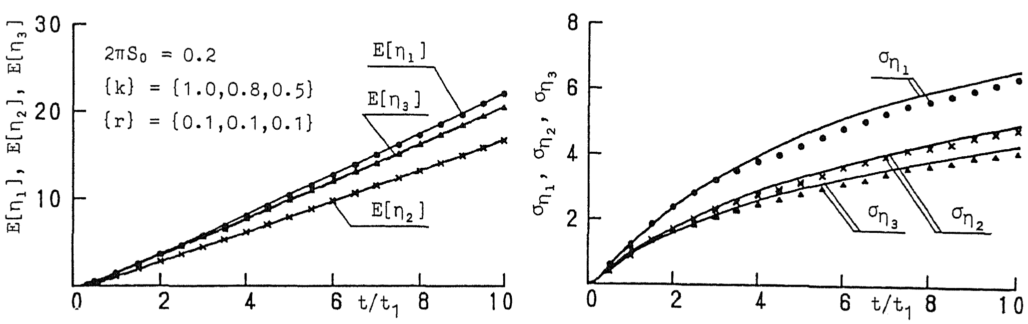
The time-dependent standard deviations of displacement and velocity responses are shown in Figs. 1(a) and 1(b), respectively. The abscissas are normalized with reference to the natural period. Figs. 2(a) and 2(b) show, respectively, the mean values and the standard deviations of the cumulative plastic deformation ratios. Fig. 3 shows the reliability functions of the structural components and the system reliability for the case that $c_{F_i} = c_F = 15$, i = 1, 2, 3. In these figure, the simulation results are also plotted. It is indicated that the analytical results are in good agreement with the simulation estimates.

The assumption of the statistical independence among the structural components is often used to obtain approximately the lower bound of the system reliability. On the other hand, unity correlations give the upper bound of the



(a) Displacement responses. (b) Velocity responses.

Fig. 1 Standard deviations of responses of 3DOF hysteretic system under stationary white noise. —— : Theory. ●, ×, ▲ : Simulation.



(a) Mean values. (b) Standard deviations.

Fig. 2 Statistics of cumulative plastic deformations of 3DOF system under stationary white noise. —— : Theory. ●, ×, ▲ : Simulation.

system reliability. From the definition of Eq. (2), the analytical expressions of two reliability functions are given as follows:

$$R(t) = \prod_{i=1}^{N} R_i(t) \quad \text{for } \rho_{\eta_i \eta_j} = 0 \quad (22)$$

$$R(t) = \min_{1 \leq i \leq N} R_i(t) \quad \text{for } \rho_{\eta_i \eta_j} = 1 \quad (23)$$

Fig. 4 shows the system reliability functions for various values of the structural capacity. In this figure, the approximate reliability functions obtained from Eqs. (22) and (23) are compared with the reliability function obtained by the present method.

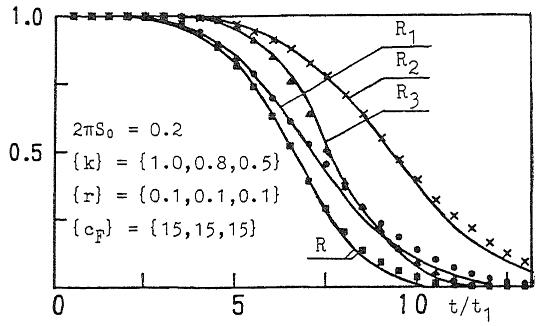


Fig. 3 Reliability functions of structural components and system reliability function of 3DOF system under stationary white noise.
—— : Theory. ●,×,▲,■: Simulation.

CONCLUDING REMARKS

An analytical method has been presented for seismic damage and reliability analysis of multi-degree-of-freedom hysteretic structures based on the stochastic differential equations. The proposed method has some significant features. First of all, by introducing the finite series expansion of the joint probability function of the whole state variables in terms of different orthogonal polynomials depending on the properties of state variables, the conventional stochastic response and reliability analyses are systematically unified into the analysis only including one time parameter. The second feature is that the system reliability can be obtained by simple integrating procedure under consideration of the statistical dependence among the structural components. The proposed method has the advantage of versatile applicability. However, since the number of differential equations to be solved considerably increases as the number of state variables as well as the order of the analysis increases, some effective and simplified schemes are required in order to apply the proposed method to large scale structural systems.

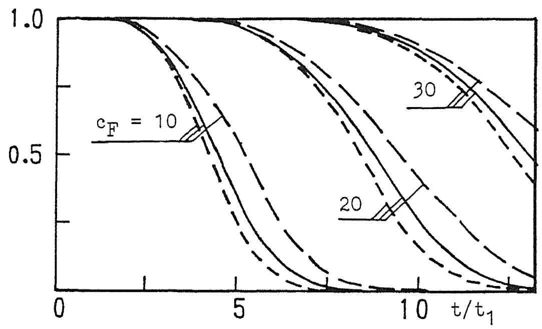


Fig. 4 System reliability functions of 3DOF system.
—— : Eq. (21), present method.
- - - : Eq. (22), $\prod_{i=1}^{3} R_i(t)$.
— — : Eq. (23), $\min_{1 \leq i \leq 3} R_i(t)$.

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