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FUZZY RANDOM VIBRATION OF ASEISMIC STRUCTURES

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SUMMARY

This paper is the improvement and development of our previous papers (Refs.1, 2, 3), in which a fuzzy random stationary model of future earthquake ground motion is established, and a basic analysis method of fuzzy random response of structures subjected to earthquake is presented. Furthermore, by taking account of the fuzziness of the damage criterion, the fuzzy random dynamical reliability of asismic structures is defined and the corresponding analysis method is proposed.

INTRODUCTION

The earthquake loads specified for the design of aseismic structures depend mainly on the predicted earthquake intensity and the site soil classification. The randomness of earthquake motion has been commonly acknowledged for a long time, but recently people have begun to recognize the fuzziness of earthquake descriptions resulting from the lack of distinct definitions and evaluation norms for the earthquake intensity and the site soil classification. In addition, the damage criterion of aseismic structures is often fuzzy. In this paper, on the basis of full consideration of the fuzziness and randomness abovementioned, a basic analysis method of fuzzy random response and fuzzy dynamical reliability of aseimic structures is presented.

FUZZY RANDOM STATIONARY MODEL OF EARTHQUAKE GROUND MOTION

Random Process with a Fuzzy Parameter Let $\underset{\sim}{A}$ be a fuzzy subset with membership function $\mu_{\underset{\sim}{A}}(u)$ on R^1, $R^1=(-\infty,\infty)$. Then, ξ is called a fuzzy variable on R^1, if ξ takes its value in R^1 and is restrained by the fuzzy restraint $R(\xi)=\underset{\sim}{A}$, which makes $\xi=u$ have possibility

$$\pi_{\xi}(u) = \mathrm{Poss}\{\xi=u\} = \mu_{\underset{\sim}{A}}(u) \tag{1}$$

π_{ξ} is called the possibility distribution function of ξ and $u\in R^1$ is called a fuzzy sample value (or F-sample value for simple) of ξ (Ref.4).

Similar to the definition of mathematical expectation of random variable, the expectation of fuzzy variable can be defined as

$$E_r[\xi] = \int_{-\infty}^{\infty} u\,\pi_{\xi}(u)du \Big/ \int_{-\infty}^{\infty} \pi_{\xi}(u)du \tag{2}$$

which is called F-expectation of fuzzy variable ξ. For distinguishing, the expectation of random variable is called R-expectation and denoted by $E_R[\xi]$.

Let ξ be a fuzzy variable on R^1, $X(\xi, t)$ is called a random process with the fuzzy parameter ξ, if when ξ takes any fixed value u in R^1, $X(u, t)$ is a random process. $X(u, t)$ $(u \in R^1)$ is called a F-sample process of $X(\xi, t)$. Evidently, any F-sample process of $X(\xi, t)$ is an ordinary random one. Therefore, the continuity, differential and integral of $X(\xi, t)$ can be defined by its F-sample processes. For example, let $T \subset R^1$, if for any $u \in G=\{u \mid \pi_\xi(u)>0\}$, $X(u, t)$ is differentiable on T in the mean sense (m.s), then we say that almost all F-sample processes of $X(\xi, t)$ are differentiable on T in m.s and say that $X(\xi, t)$ has a.f.m.s derivative. The a.f.m.s derivative of $X(\xi, t)$ is denoted by

$$Y(\xi, t) = dX(\xi, t)/dt \qquad (t \in T, \text{ a.f.m.s}) \tag{3}$$

It can be proved that a.f.m.s differential and integral for $X(\xi, t)$ has properties as same as m.s differential and integral for random processes.

<u>Fuzzy Random Stationary Model of Earthquake Ground Motion</u> Generally, earthquake ground motion can be simulated as a random process. But the parameters in the random model of ground acceleration employed in the design of aseismic structures are often determined according to the seismic intensity and site soil grade. As mentioned above, seismic intensity and site soil classification have strong fuzziness. In our previous papers (Refs.1, 2), by taking this fuzziness into account, a fuzzy random model of earthquake ground acceleration is established, which can be expressed as a random process with a fuzzy parameter and denoted by $\{a(\xi, t), \pi(u)\}$. For the stationary model, $a(\xi, t)$ has zero R-expectation and power spectral density $S_a(\xi, t)$ expressed by

$$S_a(\xi, \omega) = \frac{1+4\bar{\zeta}_g^2\dfrac{\omega^2}{\bar{\omega}_g^2}}{4\bar{\zeta}_g^2\dfrac{\omega^2}{\bar{\omega}_g^2}+\left(1-\dfrac{\omega^2}{\bar{\omega}_g^2}\right)^2}\frac{2\bar{\zeta}_g}{(1+4\bar{\zeta}_g^2)\pi\bar{\omega}_g}\,\xi \tag{4}$$

in which $\bar{\zeta}_g$ and $\bar{\omega}_g$ are respectively F-expectations of the damping ratio and predominant frequency determined according to the site soil fuzzy grade, the fuzzy parameter ξ is the stationary standard variance of $a(\xi, t)$ and has possibility distribution function as follows

$$\pi_\xi(u) = 0.5\sin(1.4427\ln u - I_J + 1.178)\pi + 0.5, \qquad u \in [f(I_J-1), f(I_J+1)\} = [u_1, u_2] \tag{5}$$

where $I_J=1, 2, \ldots, 12$, is the fuzzy intensity ordinal number and f is the relationship function between the intensity v and the standard variance u of ground acceleration, which can be expressed as

$$u = f(v) \approx 40 \times 2^{v-6}, \qquad v \in [0, 12] \tag{6}$$

The random process $a(\xi, t)$ with a fuzzy parameter ξ can be used as the fuzzy random model of earthquake acceleration and its power spectral density and possibility distribution function are respectively expressed by Eqs.(4) and (5).

FUZZY RANDOM EARTHQUAKE RESPONSE OF ASEISMIC STRUCTURES

<u>Model of SDOF Hysteretic Structures</u> If the earthquake ground acceleration is simulated by the fuzzy random model $a(\xi, t)$, the motion equation of the SDOF structure shown in Fig.1 can be expressed as

$$m\ddot{X} + c_o\dot{X} + f_s(X, \dot{X}) = -ma(\xi, t) \tag{7}$$

in which m and c_0 are respectively the mass and viscous damping coefficient of the structure, $f_s(X,\dot{X})$ is the hysteretic restoring force of the structure and may be taken as the slip-hysteresis (SH) model as shown in Fig.2.

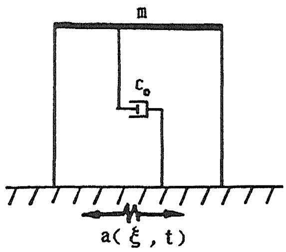
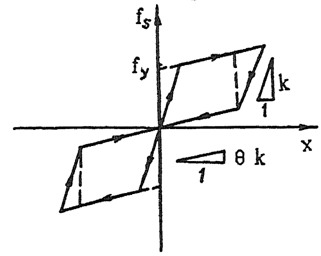


Fig.1 SDOF Structure under Fuzzy Random Ground Excitation



— SH Model --- CSU Model

Fig.2 Restoring Force Model of the Structure

A slip-hysteresis model can be replaced by a Coulomb-set-up (CSU) model with equal hysteretic loop areas (Ref.5). Thus, the restoring force $f_s(X,\dot{X})$ can be analytically expressed as

$$f_s(X,\dot{X}) = A_1 \mathrm{sgn}\dot{X} + A_2 X + A_3 \mathrm{sgn}X \qquad (8)$$

in which sgn[•] is the sign function, $A_1=A_3=(1-\theta)f_y$, $A_2=\theta k$, where k is the primary stiffness coefficient and f_y is the yield shear force, as shown in Fig.2.

<u>Fuzzy Random Earthquake Response of the SDOF Structure</u> Equation (7) is a stachastic differential equation with fuzzy parameter ξ. Noting that the fuzzy excitation $a(\xi,t)$ may be expressed by a series of F-sample processes such as $a(u,t)$, $u \in G=\{u|\pi_\xi(u)>0\}$, we define that $X(\xi,t)$ is the a.f.m.s solution of Eq.(7), if for any $u \in G$, $X(u,t)$ is the m.s solution of Eq.(7). According to this definition, by means of the stochastic equivalent linearization technique, the equivalent linearized equation corresponding to Eq.(7) can be expressed as

$$m\ddot{X} + c_e\dot{X} + k_e X = -ma(\xi,t) \qquad (9)$$

in which the equivalent linear damping and stiffness can be obtained as

$$c_e(\xi) = c_0 + \sqrt{2}A_1/[\sqrt{\pi}\,\sigma_{\dot{x}}(\xi)]; \quad k_e(\xi)=A_2+\sqrt{2}A_3/[\sqrt{\pi}\,\sigma_x(\xi)] \qquad (10)$$

Thus, the a.f.m.s solution of Eq.(9) can be easily obtained as

$$X(\xi,t) = \int_0^t h(t-\tau)a(\xi,\tau)d\tau \qquad (11)$$

in which $h(t-\tau)$ is the unit pulse response of the equivalent linearized system described by Eq.(9).

From a.f.m.s solution (11), by means of the random response analysis method and some proper simplifications, the stationary variances of earthquake responses $X(\xi,t)$ and $\dot{X}(\xi,t)$ of the structure can be obtained (Ref. 1);

$$\sigma_x^2(\xi)=\pi m^2 S_a(\xi,\omega_e)/[c_e(\xi)k_e(\xi)]; \quad \sigma_{\dot{x}}^2(\xi)=\pi m S_a(\xi,\omega_e)/[c_e(\xi)] \qquad (12)$$

where $\omega_e=\sqrt{k_e/m}$. It should be noted that they are all the functions of fuzzy variable ξ.

It can be seen from above analysis that the earthquake response of the structure clearly possesses fuzziness and randomness, owing to the fuzziness and randomness of earthquake excitation. This kind of fuzzy random response is also a random process with the same fuzzy parameter as in the earthquake excitation.

Fuzzy Safe Criterion The dynamical reliability analysis of aseismic structures is the problem to find the probability of the random response without exceeding the safe boundary in assigned time interval $[0,T]$. At present, this boundary is considered as a clear-cut line, which is very unreasonable for many practical problems.

In engineering practice, it is often very difficult to give a distinct definition for the damage of a structure. In general, there should be a gradually transient state from nondamage to damage. Especially in the earthquake engineering, the damage grades of a structure are often divided into the following five grades, slight, moderate, severe, destructive and collapsed damages. These damage grades possess strong fuzziness and we can not use clear-cut values of structural response as the boundaries between these grades. Therefore, if the response $X(t)$ of a structure, which may be the displacement, stress or cumulative energy response, is taken to define the damage grades of the structure and the damage grades are divided into

$$\{\underset{\sim}{B}_1, \underset{\sim}{B}_2, \underset{\sim}{B}_3, \underset{\sim}{B}_4, \underset{\sim}{B}_5\} = \{\text{slight, moderate, severe, destructive, collapsed}\} \tag{13}$$

then every damage grade $\underset{\sim}{B}_i$ $(i=1,2,\ldots 5)$ should be a fuzzy subset on the value region R^1 of the response $X(t)$. The membership functions of $\underset{\sim}{B}_i$ $(i=1,2,\ldots,5)$ have the character as shown in Fig.3.

Let $\underset{\sim}{B}_i^*$ represent the fuzzy safe region in which $\underset{\sim}{B}_i$ or more severe damage will not occur for the structure, its membership function curve should have the form as shown in Fig.4 and may be expressed by

$$\mu_{\underset{\sim}{B}_i^*}(x) = \begin{cases} 1 & (x \leqslant b_{i-1}) \\ 0.5\{1-\sin[(x-b_{i-1})/(b_i-b_{i-1})-0.5]\pi\} & (b_{i-1} < x \leqslant b_i) \\ 0 & (x > b_i) \end{cases} \tag{14}$$

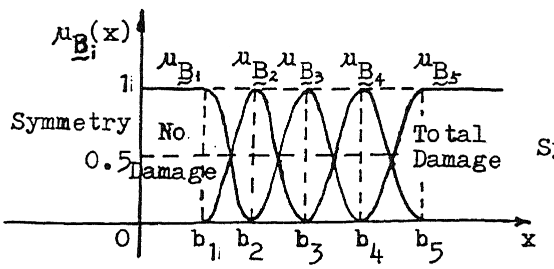


Fig.3 Membership Functions of $\underset{\sim}{B}_i$ $(i=1,2,\ldots 5)$

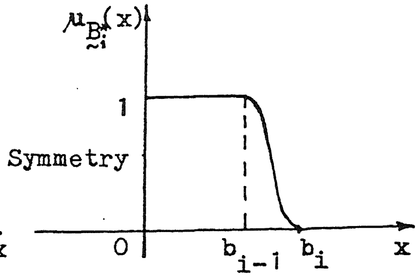


Fig.4 Membership Function of $\underset{\sim}{B}_i^*$

If an important displacement of a structure is taken as the control response of the structure safety, obviously when it is fallen into the fuzzy safe region $\underset{\sim}{B}_i^*$, the structure will not suffer $\underset{\sim}{B}_i$ or more severe damage to certain extent. Since the earthquake response $X(\xi, t)$ has fuzziness and randomness, this safe criterion can be expressed as

$$\underset{\sim}{\Theta} = \{X(\xi, t) \subseteq \underset{\sim}{B}_i^*,\ 0 \leqslant t \leqslant T\} \tag{15}$$

This is a fuzzy safe criterion. The probability that the response satisfies this fuzzy criterion is defined as the fuzzy random dynamical reliability of the structure.

Fuzzy Random Dynamical Reliability According to the fuzzy safe criterion (15), the fuzzy random dynamical reliability is defined as

$$P_s(\xi, \underset{\sim}{B}_i^*, T) = P(\underset{\sim}{\Theta}) = P\{X(\xi, t) \subseteq \underset{\sim}{B}_i^*,\ 0 \leq t \leq T\} \tag{16}$$

Since the fuzzy random earthquake response $X(\xi, t)$ of the structure consists of a series of F-sample processes $X(u, t)$ $(u \in R)$ with different possibilities $\pi_\xi(u)$, if all F-sample processes of $X(\xi, t)$ are fallen into the fuzzy safe region $\underset{\sim}{B}_i^*$, then $X(\xi, t)$ is fallen into the region $\underset{\sim}{B}_i^*$. Thus, firstly, we must find the probability $P_s(u, \underset{\sim}{B}_i^*, T)$ that the F-sample process $X(u, t)$ is fallen into the fuzzy safe region $\underset{\sim}{B}_i^*$ in the time interval $[0, T]$. It is obvious from Eq.(16) that

$$P_s(u, \underset{\sim}{B}_i^*, T) = P\{X(u, t) \subseteq \underset{\sim}{B}_i^*,\ 0 \leq t \leq T\} \tag{17}$$

By means of the random point process theory and based on proper assumptions, the concrete formula of $P_s(u, B_i, T)$ can be obtained as follows (Ref.3)

$$P_s(u, \underset{\sim}{B}_i^* T) = \exp\{-0.5\,\omega_e T H(u, \underset{\sim}{B}_i^*) / \pi\} \tag{18}$$

in which

$$H(u, \underset{\sim}{B}_i^*) = \exp(-0.5 b_{i-1}^2 / \sigma_x^2) + \exp(-0.5 b_i^2 / \sigma_x^2) + D_i$$

$$D_i = \int_{b_{i-1}}^{b_i} [z \exp(-0.5 z^2 / \sigma_x^2) / \sigma_x^2] \sin[(z - b_{i-1}) / (b_i - b_{i-1}) - 0.5)\pi]\, dz$$

where $\sigma_x = \sigma_x(u)$ is the stationary variance of the F-sample process $X(u, t)$ and may be obtained by Eq.(12).

Substituting ξ for u in Eq.(18), the fuzzy random dynamical reliability $P_s(\xi, \underset{\sim}{B}_i^*, T)$ can be obtained, which is the function of the fuzzy variable ξ. Taking F-expectation for $P_s(\xi, \underset{\sim}{B}_i^*, T)$, the corresponding dynamical reliability in nonfuzzy form can be obtained, i.e.

$$\bar{P}_s(\underset{\sim}{B}_i^*, T) = E_F[P_s(\xi, \underset{\sim}{B}_i^*, T)] \tag{19}$$

Since it is difficult to obtain the analytical expression of F-expectation $\bar{P}_s(\underset{\sim}{B}_i^*, T)$, by taking evenly N F-samples of ξ in $G = \{u | \pi_\xi(u) > 0\}$, the F-expectation $\bar{P}_s(\underset{\sim}{B}_i^*, T)$ can be approximately obtained as

$$\bar{P}_s(\underset{\sim}{B}_i^*, T) = \sum_{j=1}^{N} P_s(u_j, \underset{\sim}{B}_i^*, T)\, \pi_\xi(u_j) / \sum_{j=1}^{N} \pi_\xi(u_j) \tag{20}$$

$\bar{P}_s(\underset{\sim}{B}_i^*, T)$ is a nonfuzzy value, it can be used in the similar way to ordinary dynamical reliability. Of course, its physical meaning is different from that of the latter, because it has taken into account the fuzziness of the earthquake excitation and of the damage criteria.

NUMERICAL EXAMPLE

Consider a SDOF structure. It is assumed that the restoring force of the structure can be described by the slip-hysteresis model using the structural parameters listed in Table 1.

Table 1 Parameters of the Structure

m (ts²/m)	c_0 (ts/m)	k (t/m)	f_y (t)	θ
3.5	10	2020	6	0.15

It is assumed that after the fuzzy site soil grade is obtained by the comprehensive evaluation technique, the F-expectations of the damping ratio and predominant frequency are respectively $\bar{\zeta}_g = 0.63$ and $\bar{\omega}_g = 28.53$ rad/s. Suppose the earthquake intensity ordinal number $I_j = 8$, i.e. the support set of the fuzzy intensity is $\text{supp}\underset{\sim}{I}_j = [I_{j-1}, I_j] = [7, 9]$. Substituting $\bar{\zeta}_g$, $\bar{\omega}_g$ and I_j into Eqs.(4) and (5), the fuzzy random stationary model of the earthquake ground motion can be obtained, where the F-sample interval of the fuzzy parameter ξ with possibilities more than zero is $G=[f(I_{j-1}), f(I_j)]=(u_1, u_2)=(80, 320)$.

Suppose the duration of the earthquake is T=28 s and the fuzzy safe region is B_4^*, which means that the structure will not suffer the destructive or more severe damage, and b_3=3.0cm, b_4=4.5cm. Dividing suppI_j=[7,9] into M discrete equal intervals and taking the F-samples of ξ corresponding to the dividing points (i.e. M+1 points in all or N=M+1) into consideration, then let M=2,4,6,8 and according to Eqs.(10),(12),(18) and (20), we can obtain the F-expectations of the fuzzy random dynamical reliabilities as listed in Table 2.

It can be seen from Table 2 that when M=8, i.e. 9 F-sample responses are taken into consideration, the dynamical reliability has possessed high accuracy.

Table 2 Dynamical Reliabilities to Difference M

N	2	4	6	8
$P_s(B_4^*, T)$	1.000	0.909	0.916	0.917

CONCLUSIONS

The method presented above to consider the fuzziness of the earthquake excitation and structural damage extent in the analysis of structural responses and dynamical reliability are not only more reasonable but also convenient to use. For example, they have been successfully used in the design of a prestressed dual-layer cable 60m×90m roof system of Jilin new ice-rink (Ref.6).

It should be pointed out that the method presented in this paper can be extended to MDOF and continuous structures.

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