Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.VIII)

SH-5

STOCHASTIC RESPONSE OF INELASTIC, MULTIPLY-SUPPORTED SECONDARY SYSTEMS

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SUMMARY

The dynamic response of inelastic structural systems have characteristics considerably more complicated than those of corresponding linear elastic systems. In this paper, methods of nonlinear random vibrations are used with analysis techniques of linear primary-secondary systems to determine the dynamic characteristics of multiply-supported secondary systems with inelastic components. The problem formulation and analysis method provides valuable insight which is useful for both design and research applications.

INTRODUCTION

Primary-secondary systems, consisting of a relatively heavy primary subsystem supporting a secondary subsystem, have complicated dynamic characteristics not found in ordinary structures (Ref. 1). A common design criterion is to maintain linear elasticity in all components of the secondary subsystem. Recent studies have shown that secondary subsystems allowed to respond in the inelastic range have significant capacity, indicating that the linear design strategy is excessively over-conservative (Ref. 2). However, inelastic dynamic systems are considerably more complicated than the corresponding linear system, and appropriate analysis techniques are necessary to gain insight into their characteristics.

In this paper, attention is on secondary systems with added inelastic structural members. The basic design strategy is to introduce energy dissipation in the added members while maintaining a safe, linear elastic response in the main and critical portion of the secondary subsystem. The analysis is based on methods of nonlinear random vibrations which have proven to be accurate and mathematically based (Ref. 3). The essential new idea in this paper is to separate the inelastic analysis of the support members with the linear analysis of the remainder of the secondary subsystem. Some approximation is necessary; however, the analysis is simplified and can be interpreted in terms of linear secondary subsystem with varying member properties. This interpretation is useful since the properties of linear secondary systems have been mathematically characterized (Ref. 1). Efficiency is introduced by using analysis techniques of modified dynamic systems which have been developed in the aerospace industry (Ref. 4).

ANALYSIS OF NONLINEAR PRIMARY-SECONDARY SYSTEMS

Powerful methods of random vibrations are available to analyze nonlinear structural systems. One class of methods uses the equivalent linearization technique to obtain accurate estimates of the mean-square stochastic response (Ref. 3). This technique has been successfully applied to systems with hysteretic nonlinearities and extended to include stiffness and strength degradation (Ref. 5).

The original nonlinear equations of motion are of the form

$$\dot{\mathbf{z}} = \mathbf{g}(\mathbf{z}) + \mathbf{f} \tag{1}$$

where **g**, **f**, and **z** are N-vectors representing the nonlinear system, its response, and the forcing function, respectively. The equivalent linear system is

$$\dot{\mathbf{x}} = \mathrm{A}\mathbf{x} + \mathbf{f} \tag{2}$$

where A is an $N \times N$ matrix of constant, real coefficients and **x** is an N-vector approximation for the response. The matrix A, chosen so that the difference between **x** and **z** are minimized in a statistical sense (Refs. 3 and 5), is a function of a set of linearization constants $c_1, \ldots, c_m$. These constants are dependent on the covariance of the response vector, $\mathbf{R} = \mathrm{E}[\mathbf{x}\,\mathbf{x}^{\mathrm{T}}]$

$$c_j = c_j(\mathbf{R}) \qquad j = 1, \ldots, m \tag{3}$$

The relationship between c_j and **R** makes the equivalent linear system response-dependent, which is a key point examined in this paper.

Nonlinear primary-secondary systems can be analyzed using the general formulation of Eqs. 1-3; however, greater insight can be obtained by examining the special characteristics of such systems (Ref. 6). The approach is to perform a modal analysis of the equivalent linear system (Ref. 7) and to make use of the theory of linear primary-secondary systems (Ref. 1). The eigenvalue problem is given by

$$\mathrm{A}\,\Phi_i = \lambda_i\,\Phi_i \qquad i = 1, \ldots, N \tag{4}$$

where λ_i and Φ_i are the i-th eigenvalue and mode shape, respectively. For non-interacting primary-secondary systems, the system matrix A is of the form

$$\mathrm{A} = \begin{bmatrix} \mathrm{A_p} & \\ \mathrm{B} & \mathrm{A_s} \end{bmatrix} \tag{5}$$

where $\mathrm{A_p}$ and $\mathrm{A_s}$ are the subsystem matrices for the primary and secondary subsystems, respectively, and B is the matrix describing the subsystems' interface. The particular form of A can be exploited by matrix algebra to obtain closed-form solutions for the eigenvalue problem in terms of subsystem modal properties (Ref. 6). Once the modal properties are determined, the response covariance is obtained by modal combination

$$\mathbf{R} = \mathbf{R}\left(\mathrm{A}, \{\Phi_i, \lambda_i, i=1, \ldots, N\}\right) \tag{6}$$

Full details of the eigenvalue and modal combination analysis can be found in Ref. 6. In the next section, this analysis formulation is simplified by separating the nonlinear and linear aspects of the problem.

SEPARATION OF LINEAR AND NONLINEAR ANALYSIS

A study of a fundamental two-degree-of-freedom primary-secondary system (Ref. 8) has shown that a separation of linear and nonlinear analysis can be used to provide valuable insight into the response characteristics. The response is determined by using a previously established linear analysis of the system (Ref. 1) modified by the notion of response-dependent subsystem natural frequencies and damping ratios. The basis of the approach is a narrow-band condition for the response (Ref. 9) which is satisfied for tuned systems at moderate response levels (less than 50% of yielding).

In this section, these results are generalized to multiply-supported secondary systems. Consider a secondary subsystem comprised of a main structural component and supporting elements. The proposed design approach is to allow inelastic deformation in the supports while maintaining linear elasticity in the more critical main structural component during large (but rare) dynamic excitations. The energy dissipative mechanism of the supports would act as passive control devices which would reduce vibrations, and thereby preserve the integrity of the main component. To satisfy the narrow-band condition for the response, it is assumed that the response is dominated by a set of tuned modes at a primary natural frequency ω_p, and that the response levels are moderate.

The nonlinear support members are characterized by equivalent linear stiffnesses, k_i, and viscous damping constants, c_i,

$$k_i = k_i(R_i, \omega_p) \tag{7}$$

$$c_i = c_i(R_i, \omega_p) \tag{8}$$

for $i = 1, ..., M$, where R_i is the mean-square response at support i and M is the number of supports. These equivalent properties are evaluated by the method of slowly-varying parameters (Ref. 6, 10) where ω_p is the mean value of the response frequency. The equivalent linear properties define the nonlinear characteristics of each support, are related to the primary subsystem only through the tuning frequency ω_p, and is independent of the main component of the secondary subsystem. Thus, in this step, the nonlinear analysis of the supports has become separated from the analysis of the remainder of the primary-secondary system.

The linear part of the analysis is based on Eq. 2, where the system matrix A is given by Eq. 5 and the subsystem matrices are of the form

$$\mathbf{A}_p = \begin{bmatrix} & \mathbf{I} \\ -\mathbf{M}_p^{-1}\mathbf{K}_p & -\mathbf{M}_p^{-1}\mathbf{C}_p \end{bmatrix} \qquad \mathbf{A}_s = \begin{bmatrix} & \mathbf{I} \\ -\mathbf{M}_s^{-1}\mathbf{K}_s & -\mathbf{M}_s^{-1}\mathbf{C}_s \end{bmatrix} \tag{9}$$

where $\mathbf{M}_p, \mathbf{C}_p$, and $\mathbf{K}_p$ are the mass, damping, and stiffness matrices of the primary subsystem, and $\mathbf{M}_s, \mathbf{C}_s(c_1, ..., c_M)$, and $\mathbf{K}_s(k_1, ..., k_M)$ are the corresponding matrices of the secondary subsystem with dependence on the support properties as noted. The response covariance, $\mathbf{R}$, given by Eq. 6, is dependent on the support properties

$$\mathbf{R} = \mathbf{R}(\{k_j, c_j, j=1, ..., M\}) \tag{10}$$

Since the support properties are dependent on $\mathbf{R}$ (Eqs. 7,8) the response is determined by simultaneously solving the nonlinear equations given by Eqs. 7, 8, and 10. The computational solution requires repeated evaluation of the eigenvalue problem in Eq. 4. Two complementary analysis techniques can be used to insure efficiency in the solution process: the closed-form primary-secondary modal properties described in the previous section (Ref. 6) and re-analysis algorithms for modified dynamic systems that have been developed in the aerospace industry (Ref. 4).

EXAMPLE ANALYSIS

Consider the primary-secondary system in Fig. 1 in which the primary subsystem is a continuous cantilever shear beam and the secondary subsystem is a moment beam modeled with five lumped masses (magnitude m = 1 unit) with relatively stiff translational and rotational spring supports at the upper and lower ends and a nonlinear support mounted at the midpoint. The basic properties of the system are given in Fig. 1. The effect of the support nonlinearities on the mean-square displacement response at the beam mid-span, R_m, is investigated, where the support properties are chosen such that fundamental mode of the secondary subsystem is nearly tuned to the second mode of the primary subsystem (natural frequency = 12 Hz, or $\omega_p = 24\pi$ rad/sec). For simplicity, a wide-band excitation, modeled by white noise with power spectral density G_0, is used for the input. A range of values for G_0 is exam-

examined to show the changes in the response characteristics as the system becomes increasingly nonlinear.

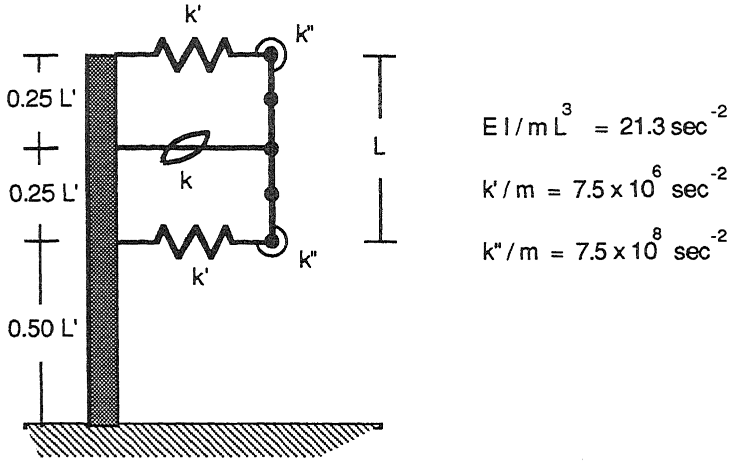

Fig. 1. Example Primary-Secondary System

The characteristics of the nonlinear support is investigated first. The relationship between the relative support velocity response, x, and the hysteretic displacement, z, is modeled by the nonlinear differential equation (Ref. 5)

$$\dot{z} = \dot{x} - |\dot{x}| z - \dot{x} z \tag{11}$$

in which yielding occurs at unit displacement. The inelastic force is $(1-\alpha) k_0 z + \alpha k_0 x$ where k_0 is the pre-yield stiffness, and $(1-\alpha) k_0$ is the post-yield stiffness. For this example study, α = 0.05 and three levels of k_0 representing three different designs for the support are considered: k_0/m = 10,000; 7,000; and 5,570 sec^{-2}.

The equivalent linear stiffnesses and viscous damping coefficients in Eqs. 7 and 8 are determined for varying mean-square response levels and ω_p = 24π rad/sec using the approximate analysis in Ref. 6. The results for k_0/m = 5,570 are shown in Figs. 2 and 3. As expected, the stiffness decreases and the damping increases with increasing response levels.

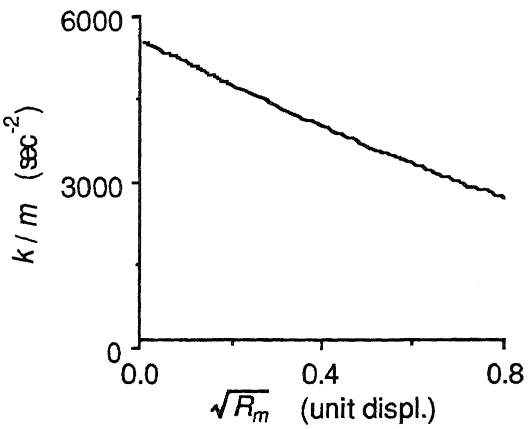

Fig. 2. Equivalent Support Stiffness

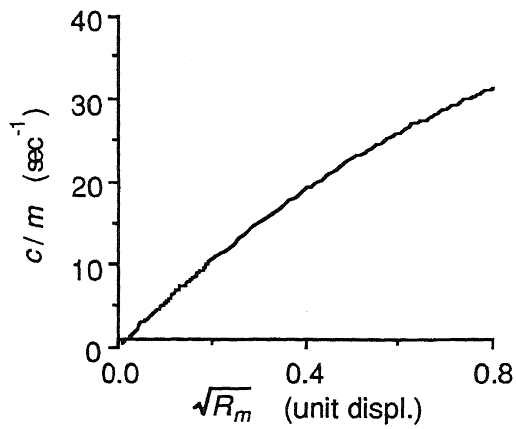

Fig. 3. Equivalent Support Damping

The linear analysis of the primary-secondary system with varying support properties is performed independent of the nonlinear analysis. The basic formulation is in Eqs. 4-6, and efficiency is introduced by using the methods described after Eq. 10. The results are shown in Fig. 4, where the response normalized by the input power spectral density is plotted for various linear stiffness and damping values for the support. The peaks in the curves which occur when k_0/m is near 5570 sec^{-2} are due to the tuning between the fundamental frequency of the secondary subsystem and the second mode of the primary subsystem. At the left of the peaks, the response becomes independent of the support damping. This can be mathematically explained in terms of the spatial coupling and tuning parameters (Ref. 1) and is commonly termed the "pseudo-static response."

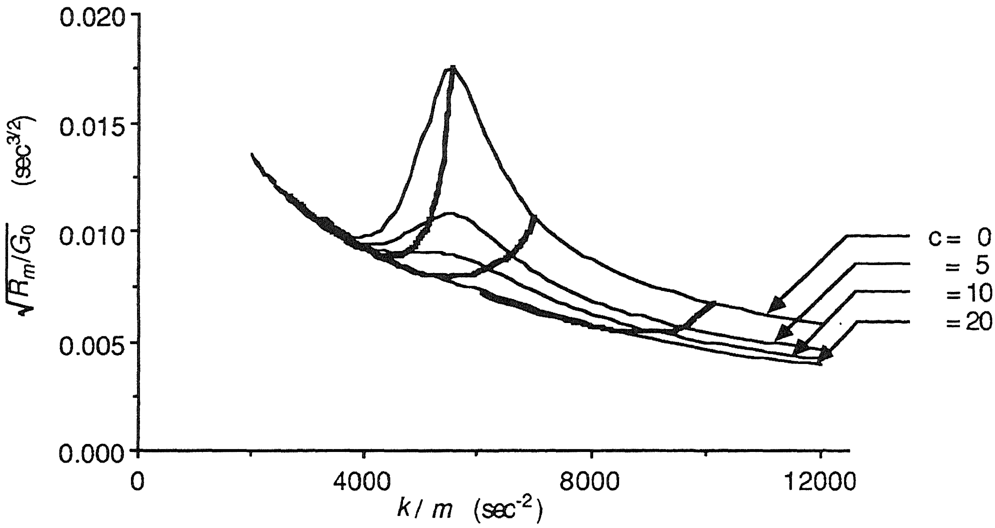

Fig. 4. Linear Response Curves with Nonlinear Trajectories

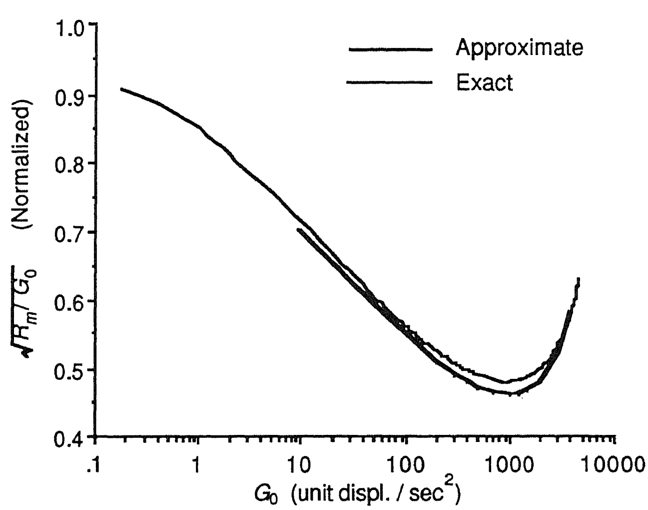

Fig. 5. Input/Response Relationships

The nonlinear and linear analysis is combined by mapping equivalent linear stiffness and damping pairs, defined by $(k;c)=(k(R_m,w_p);c(R_m,w_p))$ for various levels of R_m onto the set of curves in Fig. 4. The results are shown in three bold trajectories, corresponding to the three pre-yield stiffness values chosen for the support design. The trajectories begin at the

$c=0$ curve and move downward and leftward due to increased equivalent damping and decreased equivalent stiffness for increasing response level. Eventually the trajectories rise due to the softening of the support which results in an increased mid-span displacement response. An interesting aspect of the trajectories is that they appear to reach a lower limit as the equivalent damping becomes sufficiently high. Essentially, the high damping eliminates the tuning effect and the response is dominated by pseudo-static displacements. A rigorous mathematical analysis can be performed using the formulations developed in Ref. 1.

The relationship between the response and the input is obtained by comparing the response levels R_m in Eqs. 7 and 8 with the response ratio R_m/G_0 determined by the trajectories in Fig. 4. The result is normalized by the linear response result and is plotted in Fig. 5 for $k_0/m = 5570\ sec^{-2}$. In addition, the exact results, obtained by solving the full random vibration problem defined by Eqs. 1-3, is also shown. For a completely linear system, the response ratio R_m/G_0 is independent of G_0 and would appear as a straight horizontal line in Fig. 5. The results show that moderate nonlinearities in a support member can significantly reduce the response of a secondary system. In addition, good agreement is observed between approximate and the exact random vibration results.

SUMMARY AND CONCLUSIONS

Results from nonlinear random vibrations, linear primary-secondary systems, nonlinear two-degree-of-freedom systems, modal combination for nonlinear systems, and dynamic re-analysis techniques have been combined to formulate an approximate, simple method for examining inelastic secondary systems. The insight into the problem was briefly illustrated by an example illustration and can be developed further with more detailed examination of the relationships between the various theories. Further enhancements are possible in problems in optimal design (structural optimization theory), in the use of other aspects of random vibrations including nonstationary excitation and strength and stiffness degradation, and in a greater understanding of the relationships between linear and nonlinear response characteristics.

ACKNOWLEDGEMENTS

The research work in this paper was supported by the National Science Foundation under Grant No. CES-8707792, Dr. S.-C. Liu, Program Director. This support is gratefully acknowledged.

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