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RESPONSE OF A SECONDARY SYSTEM TO SEISMIC EXCITATIONS

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SUMMARY

The statistical properties of the response of a linear secondary system are investigated. The supporting primary system, a multi-story building, can be linear or hysteretic. The effects of vertical ground motion and gravitation are shown to be important for the hysteretic case.

THE LINEAR TIME-INVARIANT COMBINED SYSTEM

When both primary and secondary systems behave linearly, the effect of vertical ground motion is generally unimportant. In such a case the combined system may be treated as being time-invariant, and the equations of motion can be written in the following matrix form:

$$\begin{bmatrix} M & 0 \\ 0 & m_u \end{bmatrix} \begin{bmatrix} \ddot{X} \\ \ddot{Y}_u \end{bmatrix} + \begin{bmatrix} C & C_{au} \\ C_{ua} & c_u \end{bmatrix} \begin{bmatrix} \dot{X} \\ \dot{Y}_u \end{bmatrix} + \begin{bmatrix} K & K_{au} \\ K_{ua} & k_u \end{bmatrix} \begin{bmatrix} X \\ Y_u \end{bmatrix}$$

$$= - \begin{bmatrix} M & 0 \\ 0 & m_u \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \ddot{G} \qquad (1)$$

where X is a vector of N displacements (relative to the ground) of the primary system, Y_u is a vector of unattached degrees of freedom of the secondary subsystems, and G is the input horizontal ground motion. However, a complete analysis of the combined p-s system is computationally not feasible in many cases and several approximate procedures of the secondary system have been proposed (e.g., Refs. 1,2).

For the special case of an N-d.o.f primary system and one-d.o.f secondary system, accurate response can be obtained from Eq. (1) by considering only the primary mode which is in tune with the secondary system, and those other primary modes lower than the tuned mode (Ref. 3). However, this approximate procedure is not efficient when the secondary system is tuned to a high primary mode, in which case a modified cascade procedure is more efficient; namely, the response near the natural frequency of the secondary system is calculated from Eq. (1) by including only the tuned primary mode and a few lowest primary modes, whereas the response for the remaining frequency region is calculated using the traditional cascade procedure.

The modified cascade approximation is used to calculate the r.m.s shear force acting on an equipment located on the fourth floor of a 20 story building. The equipment, with damping ratio of 0.03 and mass ratio $M_e/M=0.01$ where M is the mass of a typical story, is tuned to the ninth primary mode of

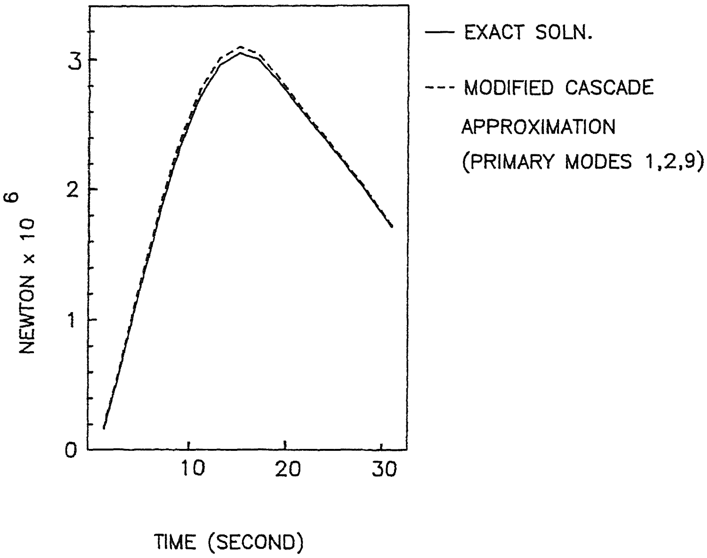

FIG. 1. COMPARISON OF EXACT AND APPROXIMATE SOLUTION FOR R.M.S. FORCE ACTING ON SECONDARY SYSTEM

the building which has story masses of 3.456×10^6 kg and interstory stiffness and damping coefficients of 3.404×10^9 N/m and 1.0×10^6 N/m/s, respectively. An evolutionary Kanai-Tajimi model for the ground acceleration (Ref. 4) was used in the calculations with ground resonant frequency of π rad./sec. and a damping ratio of 0.5. It has been shown that an evolutionary Kanai-Tajimi model can be simulated by a sequence of independently arrival pulses (Ref. 4). In the calculation, the pulse arrival rate is assumed to vary as $\mu(t)=t^2(1-t/\tau_1)^2$, with $\tau_1=20$. This model is capable of incorporating the variations of both intensity and frequency contents with time. The computed approximate result is shown in Fig. 1 along with the exact result for comparison. It can be seen that the accuracy of the approximate result is quite good.

HYSTERETIC PRIMARY SYSTEM

Under intense seismic excitation, a building can be forced into the inelastic regime. In this section we assume a shear-column construction for the building model and local hysteresis effects confined to the columns of the first floor. A hysteresis model proposed by Bouc (Ref. 5), and later generalized by Wen (Ref. 6), is used to model the restoring force in the first floor columns. Neglecting differential compressions between the columns in each floor, the modified equation of motion for the first story unit can be written as follows:

$$M_1 \ddot{X}_1 + C_1 \dot{X}_1 - C_2 (\dot{X}_2 - \dot{X}_1) - C_e(\dot{X}_{N+1} - \dot{X}_s)\delta_{1s} + a_1 K_1 X_1$$
$$-K_2 (X_2 - X_1) + (1-a_1)K_1 H - K_e(X_{N+1} - X_s)\delta_{1s} = -M_1 \ddot{G} \quad (2)$$

where

$$\dot{H} = -a_2 |\dot{X}_1| H - a_3 \dot{X}_1 |H| + a_4 \dot{X}_1 \quad (3)$$

where subscript e indicates the equipment and subscript s denotes the floor on which the equipment is located, and a_1 through a_4 are parameters. Parameters a_2 through a_4 can be varied to obtain different hysteresis behavior. The other equations in Eq. (1) are unchanged. The vertical component of ground motion V and gravitational acceleration g can be included by substituting $K_j (1-P_j/P_{cr,j})$ for K_j, where $P_{cr,j}$ is the pre-yielding buckling load (Ref. 7) of column j, and P_j is given by

$$P_j = \sum_{i=j}^{N} M_i(g + \ddot{V}) + M_e (g + \ddot{V}) U(s-j); \qquad j=1,..N \quad (4)$$

In Eq. (4), s denotes the supporting floor of the equipment, and

$$U(s-j) = 1 \quad \text{if} \quad s-j \geq 0$$
$$= 0 \quad \text{if} \quad s-j < 0 \quad (5)$$

The N+1 second order equations (building + equipment) can be converted to 2(N+1) first order equations which, together with Eq. (3), forms a system of 2N+3 equations. These equations can be written in matrix form as follows:

$$\{Z\}' = [F]\{Z\} + \Gamma''[G]\{Z\} - \Phi''\{H\} \quad (6)$$

in which each prime denotes one differentiation with respect to a non-dimensional time τ, and Φ and Γ are nondimensionalized horizontal and vertical ground excitations, respectively. We shall model these excitations as amplitude modulated Gaussian white noise processes, i.e.,

$$\Phi''(\tau) = e_1(\tau)\, S_1(\tau) \quad (7a)$$

$$\Gamma''(\tau) = e_2(\tau)\, S_2(\tau) \quad (7b)$$

in which $e_1(\tau)$ and $e_2(\tau)$ = deterministic envelope functions and $S_1(\tau)$ and $S_2(\tau)$ = stationary Gaussian processes with the following correlation functions:

$$E[\, S_1(\tau)\, S_1(\tau+s)\,] = 2\pi D_{11}\delta(s) \quad (8a)$$

$$E[\, S_2(\tau)\, S_2(\tau+s)\,] = 2\pi D_{22}\delta(s) \quad (8b)$$

$$E[\, S_1(\tau)\, S_2(\tau+s)\,] = 2\pi D_{12}\delta(s) = 2\pi D_{21}\delta(s) \quad (8c)$$

Eq. (6) can be converted to the Ito type stochastic differential equations (Ref. 8), which can in turn be used to construct moment equations of any order for the response. It can be shown (Ref. 9) that the equations for the first moments are:

$$\frac{d}{d\tau} E[Z_k] = E[m_k] = \begin{cases} f_{kr} E[Z_r], & k=1,\ldots 2(N+1) \\ E[f_{(2N+3)r} Z_r], & k=2N+3 \end{cases} \tag{9}$$

and the equations for the second moments are

$$\frac{d}{d\tau} E[Z_k Z_j] = f_{jr} E[Z_k Z_r] + f_{kr} E[Z_j Z_r] + 2\pi \{ D_{11} e_1^2(\tau)$$
$$- D_{12} e_1(\tau)e_2(\tau) (g_{kp}E[Z_p] + g_{jq} E[Z_q])$$
$$+ D_{22} e_2^2(\tau) g_{kp} g_{jq} E[Z_p Z_q] \},$$
$$\text{if } k = 1,\ldots N+1 \quad \text{and } j = 1,\ldots N+1 \tag{10a}$$

$$\frac{d}{d\tau} E[Z_k Z_j] = f_{jr} E[Z_k Z_r] + f_{kr} E[Z_j Z_r],$$
$$\text{if } N+1< k \le 2(N+1) \quad \text{or} \quad N+1< j \le 2(N+1) \quad \text{or both} \tag{10b}$$

$$\frac{d}{d\tau} E[Z_k Z_j] = E[f_{jr} Z_k Z_r] + E[f_{kr} Z_j Z_r],$$
$$\text{if } k = 2N+3 \quad \text{or} \quad j = 2N+3 \quad \text{or both} \tag{10c}$$

where f_{kr} and g_{kp} in the above expressions are elements of matrices [F] and [G] in Eq. (6). Although Eqs. (9) and (10) are written for the first and second order moments, higher order moments also appear in the expressions. This occurs when a f_{ij} term appears inside the square brackets for ensemble average as seen in Eq. (10c). Similarly, equations derived for the third order moments will involve still higher order moments. Therefore, the entire set of moment equations forms an infinite hierarchy which is a common property of nonlinear random vibration problems. To solve for lower order moments a suitable closure scheme must be used to truncate the infinite hierarchy of equations. In this study we choose the simplest truncation scheme, namely Gaussian closure, although more complicated closure schemes are possible.

Since Gaussian white noise is of a zero mean, it may be concluded on physical grounds that the first moments of the response variables are zero unless the initial state is nonzero or if the parametric and external random excitations are correlated, or both. Assuming a zero mean response and applying the Gaussian closure technique to truncate the third and higher order moment terms result in the following approximation to Eq. (10c):

$$\frac{d}{d\tau} E[Z_k Z_{2N+3}] = f_{kr}E[Z_{2N+3} Z_r] - B_1 E[Z_k Z_{2N+3}] - B_2E[Z_1Z_k]$$
$$k = 1,\ldots 2(N+1) \tag{11a}$$

$$\frac{d}{d\tau} E[Z_{2N+3}^2] = -2\{ B_1 E[Z_{2N+3}^2] + B_2 E[Z_{2N+3} Z_1]\} \tag{11b}$$

where

$$B_1 = \sqrt{(2/\pi)} \left[a_2\sqrt{(E[Z_1^2])} + a_3 \frac{E[Z_1 Z_{2N+3}]}{\sqrt{(E[Z_{2N+3}^2])}} \right] \tag{12}$$

$$B_2 = \sqrt{(2/\pi)} \left[a_2 \frac{E[Z_{2N+3} Z_1]}{\sqrt{(E[Z_1^2])}} + a_3\sqrt{(E[Z_{2N+3}^2])} \right] - a_4 \tag{13}$$

Numerical computations were carried out for the simplest case of a 1-story building-equipment combined system with a mass ratio M_e/M of 0.001 and assuming a damping ratio of 0.05 for both the structure and the equipment. The modulation functions were chosen to be $e_1=e_2=2.6\ (\exp(-.25\tau)-\exp(-.75\tau))$. For the spectral levels, we used $D_{11}=.1$, and $D_{22}/D_{11}=0.64$. For parameters of the

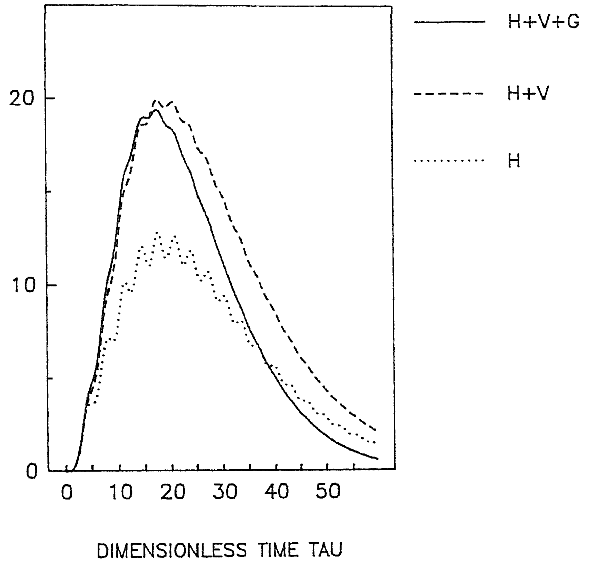

FIG. 2. NORMALIZED MEAN-SQUARE DISPLACEMENT RESPONSE

OF EQUIPMENT TO MODEL SEISMIC EXCITATION

hysteresis model, we selected the values $a_1=0.8$, $a_2=a_3=.5$, and $a_4=1$. These correspond to a softening nearly elastoplastic system with smooth transition (Ref. 6).

In Fig. 2 we show the normalized mean-square displacement response of the equipment when the equipment is tuned to the pre-yielding frequency of the structure. The labels H, V, and G, and their combinations, are used to denote which factors (horizontal, vertical ground motion, or gravity) are included in the computations. It is clear that the vertical ground motion increases the equipment response. The gravity effect is relatively unimportant in this example, but it can be more important in other cases (for different parameters a_2 through a_4. See Ref. 9.).

CONCLUDING REMARKS

We applied the modified cascade procedure in the first numerical example of linear p-s system under horizontal seismic excitation. Parametric studies have also been carried out for other cases with very accurate results (Ref. 3). The computed results shown in Fig. 2 for a softening hysteretic system indicated that the vertical ground motion had a greater effect on the secondary system response. Other computations for a hardening hysteretic system indicated that the effect of the gravitational force could be even greater (Ref. 9). These suggest that both the vertical ground motion and the gravitational force should be taken into account if the primary system is deformed into the hysteretic regime.

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REFERENCES

1. Asfura, A., and Der Kiureghian, A., "Floor Response Spectrum Method For Seismic Analysis of Multiply Supported Secondary Systems," Earthquake Engineering and Structural Dynamics, Vol. 14, 245-265, (1986).

2. Burdisso, R. A., and Singh, M. P., "Seismic Response Analysis of Multiply Connected Secondary Systems," Report VPI-E-86-10, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, (1986).

3. HoLung, J. A., Cai, J., and Lin, Y. K., "Frequency Response of Secondary Systems Under Seismic Excitation," Report NCEER-87-0013, National Center For Earthquake Engineering Research, State University of New York, NY 14261, (1987).

4. Lin, Y. K., and Yong, Y., "Evolutionary Kanai-Tajimi Earthquake Models," Journal of Engineering Mechanics, ASCE, 113, 1119-1137, (1987).

5. Bouc, R., "Forced Vibration of Mechanical System with Hysteresis," Proceedings, 4th Conference on Nonlinear Oscillations, Prague, Czechoslovakia, (1967).

6. Wen, Y. K., "Method for Random Vibration of Hysteretic Systems," Journal of Engineering Mechanics Division, ASCE, 102, No. EM2, 249-263, (1976).

7. Lin, Y. K., and Shih, T.Y., "Vertical Seismic Load Effect on Building Response," Journal of Engineering Mechanics, ASCE, 107, 331-334, (1982).

8. Ito, K., "On Stochastic Differential Equations," Memoirs of the American Mathematical Society, 4, (1951).

9. HoLung, J. A., "Response of Secondary Systems to Seismic Excitation," Dissertation presented to the Department of Mechanical Engineering at Florida Atlantic University, in 1988, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.