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PROBABILISTIC APPROACH TO IDEALIZATION OF EARTHQUAKE EXCITATION AND ASEISMIC SAFETY ANALYSIS OF STRUCTURAL SYSTEMS

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SUMMARY

An earthquake-like random excitation is idealized in the form of the root mean square value of the amplitude, the predominant angular frequency and the shaping factor of the spectral characteristics on the basis of the recorded earthquake accelerations. A fundamental equation of motion is derived for multistory building structures with strong non-linearity considering the frequency dependent soil-structure interaction. An ordinary differential equation is derived for the statistical moments of the response. Using these moments, the technique is developed for assessing the maximum displacement response and the reliability of the system. Numerical examples are presented to demonstrate the usefulness of this technique.

INTRODUCTION

The problem of seismic safety assessment of structural systems has been studied and discussed exclusively by a deterministic rather than a probabilistic approach in a dynamic sense. Considering the inherent randomness in earthquake excitations, however, the probabilistic approach could play more effective role on this problem if the idealization of the earthquake-like random excitation and the modeling of structural systems used for this assessment were made on the basis of their practical and essential characteristics. They include non-stationary amplitude and non-white spectral characteristics regarding the former, regarding the latter multistory, hysteretic characteristics with strong non-linearity and soil-structure interaction if necessary.

The refined and powerful approaches have been suggested for the random response analysis of non-linear structural systems. The author has also developed an efficient approach, particularly for the analysis of multistory structural systems with strong non-linearity (Ref. 1).

During the 1978 Miyagiken-Oki earthquake in Japan , the acceleration responses in the major high-rise building structures were recorded. They have provided researchers with interesting problems, such as explanation of damage distribution of structural systems. Based on these data available, this paper demonstrates the applicability of the proposed probabilistic approach to the analysis of the response and reliability distribution of structural systems which were exposed to the severe earthquake motion during this Miyagi earthquake.

Earthquake-like Random Excitation Model In order to introduce the non-stationary amplitude and the non-white spectral characteristics of earthquake excitations into the random response analysis of structural systems, it may be convenient and tractable to replace it with the well-known process defined as

$$f = 2h_g\omega_g\dot{z} + \omega_g^2 z \tag{1}$$

$$: \ddot{z} + 2h_g\omega_g\dot{z} + \omega_g^2 z = -\ddot{w} \tag{2}$$

in which $\dot{z}$ and z are respectively the velocity and the displacement of the surface layer excited by the white random base motion $\ddot{w}$. Two parameters ω_g and h_g are respectively the non-stationary predominant angular frequency and the spectral shaping factor governing the ensemble characteristics of earthquake excitations, whose occurrences are expected at the site of and during the life time of the structural system. However, it may be difficult not only to accumulate a large enough sample of earthquake accelerations but also to specify these characteristics even from the up-to-date knowledge of seismology. And a research objective here focuses on the examination of the applicability of a probabilistic approach to the seismic response or safety assessment of structural systems. Here is presented, therefore, a practical approach to the estimation of these parameters from the sample accelerations. The non-stationary Fourier spectrum of the recorded acceleration f can be estimated by the following formula:

$$S_f(\omega, t) = \frac{1}{T}\left|\int_{t-T/2}^{t+T/2} f(\tau)\cdot\exp(-i\omega\tau)d\tau\right|^2 \tag{3}$$

in which $i=\sqrt{-1}$, and T is a parameter associated with time properly chosen from the resolution and stability condition of the estimation. Using these spectra, the following spectral moments up to the second order can be estimated:

$$\lambda_j(t) = \int_0^\infty \omega^j S_f(\omega, t)d\omega \quad : j = 0,\ 1,\ 2 \tag{4}$$

The another analytical spectral moments corresponding to the Fourier spectra of the excitation in equation (1) can be estimated as

$$\overline{\lambda_j}(t) = \int_0^\infty \frac{\omega^j\{1+4h_g^2(\omega/\omega_g)^2\}s_0}{\{1-(\omega/\omega_g)^2\}^2+4h_g^2(\omega/\omega_g)^2}d\omega \tag{5}$$

in which s_0 defines the level of the spectra. The non-stationary parameters ω_g and h_g can be now obtained by equating the two spectral moments in equations (4) and (5). Using the zero-th order moment, the non-stationary root mean square value of the amplitude can be obtained by the following formula:

$$\sigma_f^2(t) = \frac{1}{\pi}\int_0^\infty S_f(\omega, t)d\omega \equiv \frac{1}{\pi}\lambda_0(t) \tag{6}$$

Note that the spectral parameters based on the recorded strong earthquake accelerations could include informations on the future earthquake excitation in that they are derived from the smooth spectral characteristics and that the strong or severe earthquake excitation often repeats itself with the similar spectral characteristics as was recognized in the recent 1978 Mexican earthquake.

Statistical Moments, Maximum Response and Reliability of Structural Systems A soil-lumped mass structure interaction model with a rigid rectangular foundation resting on the elastic half space ground is shown in Fig.1, in which the structural system having multi-degree-of-freedom undergoes the shear and moment force reactions through the foundation. The dimensionless equations of motion of this system may be written as

$$\ddot{x}_i = -l_i\left\{\omega_\theta^2\varphi_\theta - \sum_{j=1}^{n}\sum_{\nu=j}^{j+1}(-1)^{\nu-j}\mu_{\theta\nu}\psi_\nu h_j\right\} - \sum_{\nu=1}^{2}(-1)^\nu\sum_{k=i}^{i+1}(-\mu_\kappa)^{k-i}\psi_\kappa \tag{7}$$

$$: \psi_j = 2\xi_j\omega_j\dot{x}_j + \omega_j^2\varphi_j,\quad \kappa = k+\nu-2 \tag{8}$$

$$\ddot{x}_g = f - \omega_g^2\varphi_{x_g} + \mu_1\psi_1 \tag{9}$$

$$\ddot{\theta} = -\omega_\theta^2\varphi_\theta + \sum_{j=1}^{n}\sum_{\nu=j}^{j+1}(-1)^{\nu-j}\mu_{\theta\nu}\psi_\nu h_j \tag{10}$$

where f is the idealized earthquake excitation in equation (1); x_i, x_g and θ respectively are the relative displacement of the mass, the horizontal and rotational displacements of the rigid foundation; ω_i, ω_g and ω_θ are the angular frequency parameters; ξ_i is the viscous damping parameter; μ_i and μ_{θ_i} are the mass ratio parameters (Ref. 9).

The bi-linear hysteretic characteristics φ_i of the i-th floor with the elastic deformation δ_i and the second slope r_i can be expressed as a function of the displacement x_i and velocity $\dot{x}_i$ of the system and the relative displacement y_i of the spring element connected to the Coulomb slip element in the distributed element model (Ref. 2) as follows:

$$\varphi_i = r_i x_i + (1 - r_i)\, g_{1i} \tag{11}$$

$$\dot{y}_i = g_{2i} \tag{12}$$

$$g_{1i} = y_i\{\mathrm{u}(y_i + \delta_i) - \mathrm{u}(y_i - \delta_i)\} + \delta_i\{\mathrm{u}(\dot{x}_i)\,\mathrm{u}(y_i - \delta_i) - \mathrm{u}(-\dot{x}_i)\,\mathrm{u}(-y_i - \delta_i)\} \tag{13}$$

$$g_{2i} = \dot{x}_i\{\mathrm{u}(y_i + \delta_i) - \mathrm{u}(y_i - \delta_i) + \mathrm{u}(-\dot{x}_i)\,\mathrm{u}(y_i - \delta_i) + \mathrm{u}(\dot{x}_i)\,\mathrm{u}(-y_i - \delta_i)\} \tag{14}$$

The stochastic equivalent linearization (Ref. 3) reduces the two nonlinear functions in equations (13) and (14) to the following linear set:

$$g_{1i} = c_{1i}\dot{x}_i + c_{2i} y_i \tag{15}$$

$$g_{2i} = c_{3i}\dot{x}_i + c_{4i} y_i \tag{16}$$

in which the linearization coefficients can be given by the statistical moments of the response (Ref. 1).

Using the frequency dependent transfer function of a soil-foundation system in the form of an approximate rational function of the frequency, the rotational and horizontal reaction forces φ_{v} may be expressed (Ref. 4) as

$$\varphi_{\mathrm{v}} = (c_{2\mathrm{v}} q \dot{\mathrm{v}} + p_{2\mathrm{v}} \mathrm{v} + u_{\mathrm{v}}) / d_{1\mathrm{v}} \tag{17}$$

$$\dot{u}_{\mathrm{v}} = (p_{1\mathrm{v}} u_{\mathrm{v}} + p_{3\mathrm{v}} \mathrm{v}) / q \tag{18}$$

in which v denotes θ or x_g, u_{v} is a variable for the formulation of φ_{v}, and $p_{1\mathrm{v}} \sim p_{3\mathrm{v}}$ are the simulation parameters for the frequency dependent transfer function.

Using the linearized non-linear functions in equations (15) and (16) and the linear reaction forces of the soil ground in equations (17) and (18), the equations of motion (7)- (10) may be rewritten as a system of first order differential equations as follows:

$$\dot{x}_i = \sum_{l=1}^{N} a_{il} x_l + b_i \ddot{w} \quad : i = 1 \sim N,\ N = 3n + 8 \tag{19}$$

in which a_{il} are the coefficients associated with the system parameters and b_i is unity only if x_i denotes $\dot{z}$ and zero if otherwise.

Let the statistical moments of x_i and x_j be $m_{x_i x_j}$, and it can be shown that the moments satisfy the following differential equation:

$$\dot{m}_{x_i x_j} = \sum_{l=1}^{N} (a_{il} m_{x_j x_l} + a_{jl} m_{x_i x_l}) \tag{20}$$

$$m_{\dot{z}\dot{z}} = (\sigma_f{}^2 - \omega_g{}^4 m_{zz} - 4 h_g \omega_g{}^3 m_{z\dot{z}}) / 4 h_g{}^2 \omega_g{}^2 \tag{21}$$

where the moment $m_{\dot{z}\dot{z}}$ appearing in the right side of equation (20) demands replacement with the expression in equation (21) which was derived from equation (1) in order to take into account the non-stationarity of the amplitude and the non-white spectral characteristics of the excitation.

The maximum ductility factor response plays a central role in assessing the seismic safety of inelastic structural systems. Therefore, it is also important to estimate this response from a probabilistic viewpoint. The expected crossing number of a threshold level with positive slope per unit time may be expressed as

$$n_{a_i}^+(t) = n_0^+(t) \cdot \exp\left(-\frac{a_i{}^2}{2\sigma_1{}^2}\right)[\exp(-\gamma_1{}^2) + \gamma_1\sqrt{\pi}\{1 + \mathrm{erf}(\gamma_1)\}] \tag{22}$$

in which a_i is a maximum displacement response level, and

$$\mathrm{erf}(\gamma_1)=\frac{2}{\sqrt{\pi}}\int_0^{\gamma_1}\exp(-\nu^2)d\nu,\quad n_0^+(t)=\frac{\sigma_2}{2\pi\sigma_1}\sqrt{1-\rho^2},\quad \gamma_1=a_i\rho/\sigma_1\sqrt{2(1-\rho^2)},$$

$$\sigma_1{}^2=m_{x_ix_i}(t),\quad \sigma_2{}^2=m_{\dot{x}_i\dot{x}_i}(t),\quad \sigma_{12}{}^2=m_{x_i\dot{x}_i}(t),\quad \rho=\sigma_{12}{}^2/\sigma_1\sigma_2.$$

The maximum displacement response, therefore in an average sense, can be given by such that two times this expected number in an interval of duration t_d is equal to unity, that is

$$N_{a_i}(t_d)=2\int_0^{t_d}n_{a_i}^+(t)dt\equiv 1 \tag{23}$$

Some iterative procedures are necessary for finding the exact solution which satisfies this equation.

The reliability of the i-th story may be approximately expressed as

$$R_i(t)=1-P_i(t) \tag{24}$$

where $P_i(t)$ denotes the first excursion probability of the i-th floor displacement response process over the prescribed safety level x_{f_i}. Let the probability distribution function (p.d.f.) of a high level excursion of the displacement process be Poissonian, and the joint p.d.f. of the displacement and velocity response processes be Gaussian. Then, the reliability of the i-th floor may be approximately evaluated as

$$R_i(t)\cong\exp\left\{-2\int_0^t n_{x_{f_i}}(\tau)d\tau\right\} \tag{25}$$

where $n_{x_{f_i}}$ is given by equation (22) in which a_i is replaced by x_{f_i}.

Numerical Examples and Discussions As an example of the application of the proposed approach, three building structures (S-building, Sendai JR managing office building and architectural department building of Tohoku university (Refs. 5,6,7,)) were analyzed. They are all reinforced concrete buildings with shear walls. Here are presented the results only for the case of the S-building because of the space limitations.

Fig.2 shows the EW (upper) and NS (lower) components of the earthquake accelerations recorded at the basement of this building, to which non-stationary spectra were calculated by use of equation (3), and the corresponding non-stationary root mean square value, the predominant angular frequency and the spectral shaping factor were calculated by use of equations (4)-(6). These three parameters (EW-direction), by the dotted points, are plotted as a function of time in a dimensionless form in Fig.3 , where the solid lines correspond to the simulated smooth curves. Using these non-stationary parameters, the statistical moments were calculated by use of equations (20) and (21), and the maximum ductility factor response and the reliability of each floor were calculated by use of equations (23) and (25) respectively. All the necessary data for these calculations associated with the structure are in Ref. 5.

In Fig.4 the distributions of the maximum ductility factor response are plotted over the height against EW (left) and NS (right) directions of the structure, where the lines with triangular marks correspond to the case for the presented probabilistic approach, and the lines with asterisks the case for the deterministic approach by which time history response analyses were made against two directions of the structure subjected to the earthquake excitations in Fig.2.

From this figure, it is found that their mutual agreements are surprisingly good against both two directions, although this may not necessarily be true and actually as such was the case for the Tohoku university building. The results through the deterministic approach should be regarded as the sample ones through the probabilistic approach, and it was found that the agreement was strongly dependent upon the degree of realization of non-stationary spectral parameters from the actual earthquake process (Ref. 8).

In the left side of Fig.5 time histories of the reliability of the 5th, 10th and 20th floors are plotted as a function of time, while in the right side the space distribution of the reliability of each floor at the end of earthquake excitation duration are plotted over the height where three referential threshold levels 1.4, 1.7 and 2.0 in terms of ductility factor are selected throughout the floors. The examination of this figure indicates that the reliability of these three floors is lower than the other's and that the lower reliability with the lower threshold level is concentrated in these floors. No direct comparison of this analytical reliability can be made with the actual damage distribution because no damage had been reported on this building structure (Ref. 5), while the reliability distribution could explain fairly well the apparent damage distribution to the shear wall of the Tohoku university building (Ref. 8).

As to the non-stationary predominant angular frequency, it has a great effect on the response in general, but it can be replaced with the stationary one when tall building structures are built on the stiff soil ground where the higher predominant frequencies are generally observed (Ref. 9.).

Conclusions The earthquake-like random excitation model has been presented on the basis of the recorded earthquake accelerations. This model has been effectively used for assessing the statistical moments of the response of multistory bi-linear hysteretic building structures with soil-structure interaction. The technique has been presented for assessing the maximum displacement response and the reliability of the system, and it has been proven to be useful from the comparative observations and discussions on the maximum ductility factor response and reliability distribution of the existing building structures.

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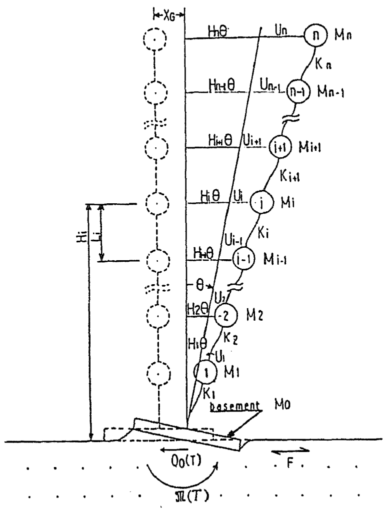

Fig.1 Soil-structure interaction model

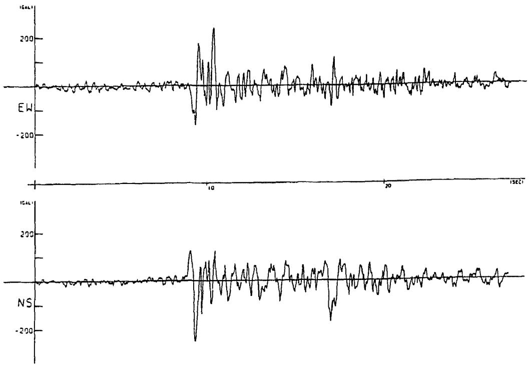

Fig.2 1978 Miyagiken-Oki earthquake accelerations recorded at the basement of S-building

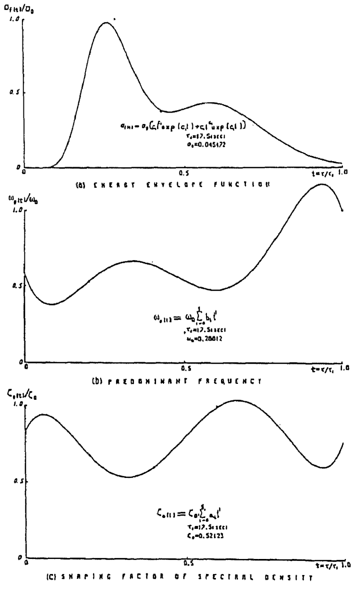

Fig.3 Non-stationary amplitude and spectral parameters as a function of time : EW-direction

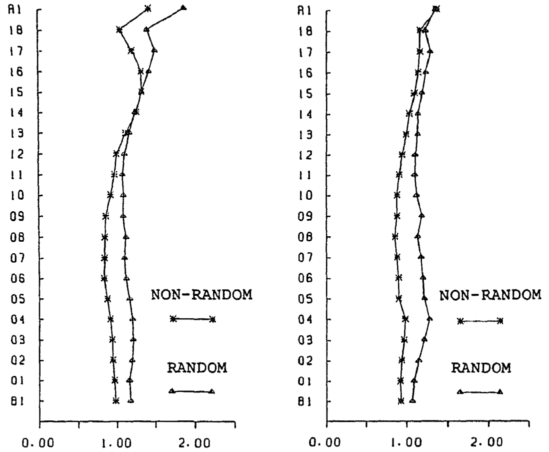

Fig.4 Comparison of distribution of the maximum ductility factor response over the height:EW(left) and NS(right)

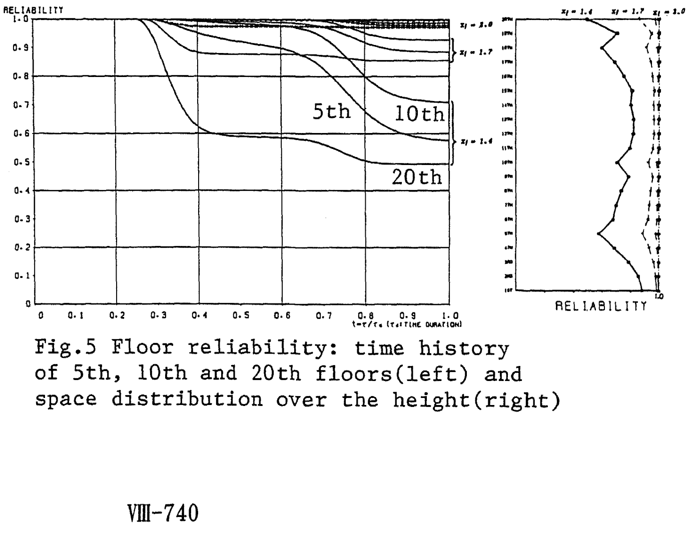

Fig.5 Floor reliability: time history of 5th, 10th and 20th floors(left) and space distribution over the height(right)