EVALUATION OF COLUMN AND WALL ACTIONS
IN THE ULTIMATE-STATE
DESIGN OF REINFORCED CONCRETE STRUCTURES

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SUMMARY

The paper presents a method of evaluating possible maximum column and wall actions in reinforced concrete structures during nonlinear earthquake responses. Using modal decomposition concept, dynamic effects on lateral forces are clarified in relation with input base accelerations. Maximum dynamic member forces can rationally be estimated on the basis of the general characteristics in dynamic forces, taking into account not only static but also dynamic shear distribution ratio of wall and column. Simple design formula is presented as dynamic magnification factor, which gives a fair upper bound of analytical responses and may be used in an ultimate-state design procedure.

INTRODUCTION

In the ultimate-state design guideline for reinforced concrete building structures which is to be proposed by the Architectural Institute of Japan, it is clearly prescribed that the overall beam-yielding mechanism of the structure should be selected, and the mechanism should be ensured using appropriate design factors for non-yielding members such as columns and walls in intermediate stories. The design factors should be based on the effects of (1) the possible upper bound of actual ultimate hinge strength, (2) the possible dynamic forces, and (3) the concurrence of bi-axial seismic forces, which are not deterministic in the calculation of the design procedure. The purpose of the paper is to present a generalized method of evaluating the possible maximum shear forces in columns and walls during an earthquake response, taking into account inelastic and dynamic behaviors of reinforced concrete structures, on the basis of which the dynamic magnification design factor in the guideline are prescribed.

DYNAMIC MAGNIFICATION OF COLUMN AND WALL ACTIONS

The full-scale seven-story reinforced concrete structure tested in U.S.-Japan cooperative research (Ref. 1) shown in Fig. 1 is analyzed. The structure is idealized as a plane frame model consisting of three different nonlinear member models; (1) a one-component model for beam and column, (2) there vertical line elements model for a wall, and (3) a vertical axial spring model for a transverse beam. Details of the analytical method, developed to simulate the full-scale pseudo-dynamic earthquake response tests, are described elsewhere (Ref. 2).
Three different earthquake records are used; El centro <NS>[1940], Hachinohe <EW>[1968], and Tohoku Univ. <NS>[1978] with maximum accelerations of 0.49G, 0.31G and 0.40G (G=earth gravity), respectively. The amplitude levels, which are 1.5 times of the original records, almost corresponds to those of the most severe design earthquakes currently used in the special design procedure of Japan for relatively high-rise buildings which requires special studies based on nonlinear and dynamic analyses.

Maximum dynamic and static story shear and wall shear computed for the test structure are shown in Fig. 2, normalized by the total weight of the structure. Static shear forces in solid lines are computed at the roof-level rotation angle of 1/100, which corresponds to the maximum dynamic responses, under an inverted triangular seismic coefficient distribution, that is, the distribution of inertia forces in a straight mode, which is dominant as averaged displacement mode in inelastic responses. The maximum dynamic shear forces in dashed lines are generally higher than the static due to the effect of dynamic magnification. The increment of the dynamic shear to the static shear distributes similarly along the stories in three cases and is larger in case of the higher base acceleration, and the wall carries most of it, which is important characteristics and is investigated in detail in the following section.

![Diagram](image)

(a) Plan (b) Elevation

Fig. 1 Full-Scale Test Structure

Fig. 2 Maximum Dynamic and Static Shear Forces

MODAL DECOMPOSITION OF DYNAMIC FORCES

The maximum shear forces are limited at the formation of mechanism, if the lateral force distribution pattern has not changed. However, dynamic force distribution changes due to higher modes of response. A modal decomposition method (Refs. 3, 4) is proposed and introduced to investigate characteristics of the dynamic restoring forces, summarized as follows.

By using assumed mode shapes which are orthogonal with respect to the mass matrix $M$, the restoring force vector $f$, obtained at every time step in dynamic analysis, can be decomposed into N components even in inelastic range. To make it applicable for a design procedure, the higher mode fluctuating force vector $f_p$ is defined as the residual to the decomposed fundamental force $f_1$ of the assumed mode $u_1$ as:

VIII-700
The amplitudes of the fluctuating forces become generally larger in the top and lower stories with contrary phases, which can generally be estimated from the input base acceleration $\ddot{x}_0$, based on the equations of motion (Ref. 3, 4). If constant amplification factor $D_j$ for the j-th modes is assumed for each response acceleration and $D_j$ is taken as unity for $j \geq 3$, then, the higher modes restoring forces are estimated in the form as:

$$f_F = f - f_1 = f - \left[ u_j^\top f/(u_j^\top M u_j) \right] M u_j \quad (1)$$

The second mode shape, $u_2$ is assumed in the estimation to be of a cubic curve orthogonal to the fundamental mode $u_1$ of an inverted triangular shape with respect to the mass matrix. The constant amplification factor for the second mode $D_2$ is taken as 1.7, which result in a good estimation of time history of fluctuating forces. Figure 3 compares the waveforms of the input base acceleration and the decomposed fluctuating base shear. The amplitudes of acceleration in dashed lines are normalized by the estimated relation from Eq. (2), which are in good agreement both in phase and amplitudes with those of the fluctuating base shear under three different earthquake records.

Analytical results of different types of structures with the same specific consideration are reported elsewhere (Refs. 3, 4, 5). The method is verified to give a fair estimation generally in other cases of wall-frame and open-frame structures with different number of stories or shear wall ratios. The suitable amplification factor for the second mode tends to become larger in higher structures, for example about 2.0 for 9-story structure, 2.4 for 12-story structures. However, approximated factor of about 2.0 is adequate for the estimation, because this part is relatively small in the formula.

**ESTIMATION OF MAXIMUM DYNAMIC SHEAR FORCES**

The maximum dynamic story shear does not exceed the sum of the modal maxima of the fundamental and the fluctuating shear forces. Maximum fundamental story shear $s_{1\text{max}}$ can be evaluated as static shear under the fundamental distribution of lateral forces. Maximum fluctuating shear $s_{f\text{max}}$ can also be estimated from the maximum ground acceleration based on Eq. (2). Then, the maximum dynamic magnification ratio may be estimated as the ratio of $(s_{1\text{max}} + s_{f\text{max}})$ to $s_{1\text{max}}$ as plotted with solid lines in Fig. 4 for El Centro and Hachinohe records. The estimated magnification ratio almost agree with the response ratio with dashed line and shade, which is computed as the ratio of the dynamic story shear to the static in Fig. 2.
The maximum dynamic wall shear can be estimated in the same way as above, if the distribution ratios of the fluctuating story shear into the wall and columns are specified. The ratios can also be investigated using the modal decomposition method. Under the fundamental lateral forces, constant relationships between the wall shear $v_1$ and the story shear $s_i$ are assumed using a matrix $K_1$ with static wall shear ratios in diagonal elements, then, the fluctuating wall shear due to the higher modes $v_F$ can be decomposed from computed dynamic wall shear $v$ as:

$$v_F = v - v_1 = v - K_1 s_1$$  \hspace{1cm} (3)

Figure 5 shows waveforms of base shear, wall and column shear forces in the first story under El Centro motion, decomposed into the fundamental mode and the fluctuating mode. The wall shear ratio for the fluctuating shear can approximately be determined from the amplitudes ratios of the waveforms. The wall ratios thus evaluated for all the stories are plotted in Fig. 6. The wall shear ratio under the fundamental mode is about 0.60 in the first story and become smaller in the upper story, whereas the fluctuating wall shear ratio is about 0.80 in all the stories, which is estimated approximately from elastic stiffness ratios of the wall and the columns under antisymmetrical bending moments as shown in the figure with dashed line. The estimation for the fluctuating shear ratio is appropriate for other structures with different shear wall ratios and may be used in a practical design procedure.
Using the shear ratios under the fundamental and the fluctuating mode, the maximum dynamic wall shear $V_{D\text{max}}$ can be estimated as:

$$V_{D\text{max}} = V_{\text{max}} + V_{F\text{max}} = R_1 s_{\text{max}} + R_F s_{F\text{max}} \quad (4)$$

Figure 7 shows the estimated dynamic magnification ratio of wall shear defined as the ratio of $V_{D\text{max}}$ to $V_{\text{max}}$, with the response ratio computed in dynamic analyses. It should be noted that if the input base acceleration levels are made higher, the amplitudes of the fluctuating shear forces are to become higher accordingly. Figure 8 show the maximum dynamic wall shear in the first story with relation to the maximum input base acceleration, which are varied by 1.0, 1.5, and 2.0 times from the original levels. The estimated dynamic increment with dashed line in proportion to the maximum base acceleration give a upperbound for all the cases.

(a) El Centro  (b) Hachinohe

Fig. 7 Computed and Estimated Dynamic Magnification Ratios of Wall Shear

**Fig. 8 Maximum Dynamic Wall Shear in Relation with Maximum Base Acceleration**

**DYNAMIC MAGNIFICATION FACTOR IN DESIGN FORMULA**

Based on the characteristics clarified as above, dynamic magnification design factor $D_{w1}$, which is to be used for the static maximum wall shear forces in 1-story under the fundamental mode, may be in the form:

$$D_{w1} = 1 + \left(\frac{C_i}{C_0}\right)\left(R_{F1}/R_{I1}\right) \quad (5)$$

where $C_0$ is the over-design factor of the structure, that is, the ratio of the ultimate lateral load carrying capacity to the specified standard capacity. Note that if the ultimate capacity is higher, the magnification is to be less accordingly. $C_i$ is the fluctuating mode factor of 1-story, which can be determined based on the estimation method with relation to the design earthquake level. $R_{F1}$ and $R_{I1}$ are the fluctuating and the fundamental wall shear ratio.

In case of relatively high-rise structures, the constant for the second mode amplification is a little higher, while the probability of concurrence of the maxima in the fundamental mode and in the fluctuating mode is less. Referring to the estimated upper bounds derived from ground acceleration of 0.30G and standard
base shear coefficient of 0.25, and also referring to the analytical results of higher structures, simple design formula is proposed for the fluctuating mode factor of i-story in N-story structure as:

\[
C_i = \begin{cases} 
0.25 & \text{if } i=1 \\
0.20 & \text{if } 2 \leq i \leq N/2 \\
0.20 + 0.10 (i - N/2) & \text{if } i > N/2 
\end{cases}
\]

(6)

The design magnification factors derived from this formula for the test structure are plotted in Fig. 9 with the dynamic magnification ratios computed from the nonlinear analyses.

![Fig. 9 Dynamic Magnification Factor in Design Formula](image)

(a) Story Shear  (b) Wall Shear

CONCLUSIONS

The following characteristics of nonlinear dynamic forces, which is available for a rational formula of dynamic magnification factor in an ultimate-state design provision, is clarified through the modal decomposition method:

1. The amplitude of the higher mode fluctuating force, which is defined as the residual to the specified fundamental mode, is in proportion to that of the base acceleration and can generally be estimated from Eq. (2).

2. The maximum story shear force during an earthquake response can rationally be evaluated as the modal maxima of the fundamental shear from static analysis and the the fluctuating shear estimated from the maximum base acceleration.

3. The distribution ratio of the fluctuating story shear into the wall is different from that under the fundamental mode, which should be considered in the ultimate shear design of a wall.

REFERENCES


