SG-14

MOMENT REDISTRIBUTION IN EARTHQUAKE RESISTANT DESIGN OF REINFORCED CONCRETE FRAMES

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SUMMARY

The effect of design moment redistribution on the earthquake response was studied for reinforced concrete frame structures. A permissible limit of the moment redistribution was discussed on the basis of the earthquake response analysis. The ductility demand tends to increase at the beam ends of reduced design capacity by the redistribution. The difference, however, in attained ductility with and without the redistribution under a strong intensity earthquake motion was observed small for a practical range of the redistribution.

INTRODUCTION

A statically indeterminate moment resisting frame, with a progress of damage under monotonically increasing loads, develops a moment distribution different from the initial elastic distribution, an excessive elastic moment at the formation of plastic hinges being redistributed to the other less critical sections. The reinforced concrete beam member, designed as a part of a ductile weak-beam strong-column framed structure, is generally accepted to yield during a strong earthquake. Hence, large amplitude bending moments at beam critical sections, obtained from a linearly elastic structural analysis, may be redistributed to other less critical beam sections in the course of design. Such technique is commonly used in the plastic design of the steel structure.

MOMENT REDISTRIBUTION

Objectives: The moment redistribution is desired in girders of framed building because the gravity load combined with earthquake load develops a large negative moment but a small positive moment at their ends, requiring the amount of reinforcement significantly different at the top and the bottom of the section. The Capacity Design Procedure, defined in New Zealand Reinforced Concrete Code for a ductile weak-beam strong-column frame (Ref.1), permits the redistribution of the elastic beam bending moment under combined gravity and earthquake loads, thus reducing the flexural capacity at beam critical sections, but providing these member ends with ductility. The reduced moment must be redistributed to less critical beam sections at the same floor level so that the story lateral resistance should not be affected by the redistribution. The moment redistribution, in this manner, enables a structural designer (a) to achieve a lateral load resisting capacity, barely exceeding the required design value, at the formation of the desired collapse mechanism,
(b) to equalize the top and bottom reinforcements at a beam critical section for a better flexural ductility, 
(c) to equalize the top or bottom reinforcements on the both sides of a beam-column connection for an easy placement of the beam longitudinal reinforcement through the connection, and 
(d) to achieve uniform distribution of the beam longitudinal reinforcement in each floor level (Ref.2).

**Procedure:** Before the moment redistribution, the frame is analyzed by a routine elastic analysis method using cracked stiffness under combined gravity and earthquake loads. The moment redistribution is carried out for each isolated sub-frame (Ref.2), removed from the original frame by cutting at the column inflection points immediately above and below the floor level. During the redistribution, the equilibrium of beam and column moments at a beam-column joint and the equilibrium of story shear and column shears must be maintained. Two types of redistribution may be conducted; 
(a) a moment at a beam top is moved to the adjacent beam bottom on the opposite side of the beam-column joint (beam redistribution),
(b) a moment at a beam top is moved to the beam bottom at a remote beam-column joint (column redistribution).

The beam redistribution is carried out over an interior beam-column joint, whereas the column redistribution is normally carried out between the two exterior beam-column joints. The beam redistribution does not alter the sum of the beam end moments at a joint, hence, maintaining column moments and shears at the joint. However, the change in a beam moment at an exterior beam-column joint inevitably alters the column moments connected at the joint, thus changing the column shears. Therefore, the column redistribution must be exercised with caution.

**Limitation:** It should be noted that the moment redistribution is likely to cause damages from medium intensity earthquakes at localities where the design moment was reduced from the elastic moment. Furthermore, plastic deformation tends to concentrate at such localities during a large intensity ground motion. Therefore, New Zealand Code (Ref.1) limits the amount of the beam and column moment redistributions as follows:

a) In any span, a beam redistribution ratio, defined as ratio of a redistributed moment to the absolute maximum moment of the span, should not exceed 30 percent.

b) A column redistribution ratio, defined as ratio of a redistributed moment to the maximum column moment of a joint, should not exceed 15 percent.

This paper examines the acceptability of the limits through earthquake response analyses of designed examples.

**DESIGN OF EXAMPLE FRAMES**

**Example Frames:** Two eight-story (Buildings A and B) and one twenty-story (Building C) moment resisting frames in the longitudinal direction were studied as example. The frames were of four spans in the longitudinal direction, and provided with two exterior structural walls in the transverse direction (Fig.1). The bay width in the longitudinal direction was the same in the four spans; i.e., 6.00 m and 8.00 m in Buildings A and B, respectively, and 6.00 m in Building C. The span length in the transverse direction was common in the three buildings and 6.00 m. The story height was uniform and 3.30 m. Column and girders dimensions are shown in Table 1. The slab was 120 mm thick. Floor beams, 250x400 mm, were deliberately placed in the transverse direction to increase bending moment demand in the longitudinal girders. Unit weight of reinforced concrete members was 2.4 ton/m³. Unit live load was 180 kg/m², but only 80 kg/m² was assumed to contribute to the earthquake inertia mass.

**Design Procedure:** The structures were first designed by the allowable stress
design principle under dead and live loads satisfying the minimum reinforcement requirements. The amount of longitudinal reinforcement was further proportioned by the ultimate strength principle for the combined design actions by gravity and earthquake loads allowing the moment redistribution. The elastic frame analysis under lateral loads considered member flexural, shear and column axial deformation, rigid beam-column connections, whereas the analysis under gravity loads ignored the column axial deformation and beam-column connections. The effective slab width for the girder stiffness was assumed to be 10 percent of the girder clear length. The structure was assumed to be fixed on the rigid foundation.

Table 1: Member Dimensions

<table>
<thead>
<tr>
<th>Building Story</th>
<th>Girders</th>
<th>Columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>A and B</td>
<td>300x500, 500x500</td>
<td>400x650, 650x650</td>
</tr>
<tr>
<td>2-1</td>
<td>300x600, 600x600</td>
<td></td>
</tr>
<tr>
<td>20-16</td>
<td>300x500, 500x500</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>400x600, 600x600</td>
<td>400x700, 700x700</td>
</tr>
</tbody>
</table>

(Unit in mm)

**Design Earthquake Loads:** In a Japanese building, the elastic moment associated with earthquake loads is normally significantly larger than the corresponding actions caused by gravity loads. Thus, even in a reinforced concrete building having span length as long as 8.0 m, the beam moment redistribution is not controlled by the 30 percent limit except at top few levels; the beam redistribution ratio rarely exceeded 10 percent at lower levels. Accordingly, the moment redistribution did not increase ductility demand at the critical beam sections during medium and large intensity earthquakes (Ref.3). In other words, the largest demand of moment redistribution and its worst effect on the earthquake response will be observed when earthquake actions are small compared with gravity induced moments in a frame. For this reason, the earthquake loads in this study were chosen to be approximately 30 percent of the Japanese requirements to enhance the moment redistribution; i.e., the base shear coefficients at the formation of the lateral collapse mechanism under earthquake loads were selected to be 0.09 and 0.066 for eight-story and twenty-story buildings, respectively. The maximum ground velocity of design earthquake motion was arbitrarily assumed to be 100 mm/sec.

**Moment Redistribution:** The frames were designed to develop ductile flexural yielding at the girder critical sections at the formation of lateral collapse mechanism under earthquake loads. The moment redistribution was carried out for a floor sub-frame, removed from the original frame by cutting at the column inflection points immediately above and below the floor concerned. The beam and column redistribution ratios were limited to 30 and 15 percent, respectively, in accordance with New Zealand Code (Ref.1). The girder moment at column face was expressed in terms of the number of reinforcing bars required by the ultimate strength design principle, and the fraction of the bars was moved elsewhere during the redistribution. The final reinforcement amount was kept greater than the amount required from the allowable stress design.

**RESULTS OF MOMENT REDISTRIBUTION**

The amount of moment redistribution is expressed by the beam or column redistribution ratio. The elastic actions, final design actions after the moment
redistribution, beam or column redistribution ratios are shown in Fig. 2 for typical floor girders of Frame Y1 in Building A. The elastic actions from gravity loads were significantly larger in upper floor levels than those from earthquake loads, resulting in larger redistribution ratios. Due to the limitation in allowable redistribution amount, however, the amount of negative beam longitudinal reinforcement at upper floor levels was controlled by the minimum reinforcement requirements. Hence, the lateral load resisting capacity increased in the upper floor levels. The amount of moment redistribution decreased at the lower floor levels with an increase in the earthquake induced actions relative to the gravity induced actions. The negative elastic moments at girder ends were, in general, reduced by the beam redistribution, and girder positive moments were increased. Building B with a large bay width exhibited larger redistribution to the limits of the redistribution.

Figure 3 shows the maximum, mean and minimum values of beam redistribution ratios and column redistribution ratios at each floor in Frame Y1 of the three structures. The redistribution ratio in Building A was generally larger at upper floor levels where the gravity actions were dominant, and was reduced at lower levels where the earthquake actions governed. The column redistribution ratio was generally smaller in interior frame Y1 than in exterior frames Y0 and Y2. The redistribution in Building B with large bay width was controlled by the redistribution limits. Building C exhibited a wide scatter of beam redistribution ratios. Even in a floor level, some beam ends hardly required redistribution, whereas, the other required the redistribution to the allowable limit. Note that large redistribution is demanded in localities where design actions are governed by the gravity loads.

**EARTHQUAKE RESPONSE**

**Analysis Method:** The earthquake response analysis was carried out using a plane frame analysis program (Ref.4). The members were idealized by an elastic element with two nonlinear rotational springs (one-component model) with finite rigid zones at the member ends. The hysteretic model for the rotational springs were taken as Takeda Model. The initial moment caused by the gravity loads was considered in the hysteretic model. Damping was assumed to be proportional to the instantaneous stiffness of the structure with the initial first mode damping factor of 0.05 and 0.03 for the eight-story and twenty-story buildings, respectively. The El Centro (NS) 1940 waveform was used in the earthquake response analysis by changing the amplitude. The maximum ground velocities were chosen for the medium, strong, and extraordinary intensity ground motions to be 50 mm/sec
(max. acc. of 430 mm/sec²), 100 mm/sec, and 200 mm/sec. For comparison, Buildings A', B' and C', designed without the moment redistribution, were also analyzed.

![Graphs showing moment redistribution ratio for Buildings A, B, and C](image)

Fig.3: Moment Redistribution Ratio (Frame Y₁)

**Earthquake Response:** The ductility demand of Building B, with largest moment redistribution of the three buildings, is shown in Fig.4; Buildings A and C showed a trend similar to Building B. Note that the moment redistribution changed the level of yield resistance as well as yield rotation at beam ends. In other words, the ductility demand increased for the same rotation when the design moment was reduced from the elastic moment.

Although a large moment redistribution was suspected to cause damage even from a medium intensity earthquake, the beams in Buildings A, B, and C did not yield; i.e., the ductility demand was less than 0.8 at beam top, and less than 0.5 at beam bottom. Therefore, the redistribution limits of 30 percent for the beam redistribution and 15 percent for the column redistribution were sufficient to protect beams from a medium intensity earthquake.

Buildings A, B, and C with redistribution developed ductility demand greater than corresponding Buildings A', B', and C' during a strong earthquake. However, the ductility demand was less than 2.0, and the differences were small. In Buildings B and B' in Fig.4, the ductility demand decreased above the third floor.
levels because the longitudinal reinforcement was controlled by the minimum reinforcement requirements. Building A, in which the gravity induced actions were relatively small, developed ductility less than 1.6. Buildings C and C' with a relatively long period, developed beam ductility less than 1.0 except for the top three floors in Building C.

The ductility demands were comparable in the buildings with and without redistribution during an extraordinary intensity earthquake motion.

CONCLUSIONS

The effect of moment redistribution was studied for three uniform frame structures, and the following three conclusions were obtained:
1) Medium intensity earthquake did not cause yielding in members designed with the moment redistribution.
2) The moment redistribution increased the ductility demand during an intense earthquake motions, the rate of which was not significant.
3) The earthquake resistance of a frame structure was not affected appreciably by allowing the moment redistribution in design if the beam and column redistribution ratios were limited to 0.3 and 0.15, respectively.

REFERENCES