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REINFORCED CONCRETE ELEMENTS SUBJECTED TO REVERSED SHEAR

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SUMMARY

Three reinforced concrete panels (1600 × 1600 × 285 mm) were subjected to reversed cyclic pure shear at the University of Toronto. An average stress versus average strain response was measured and the stresses in the concrete and steel components deduced.

The tests indicate that reinforced concrete subjected to repeated cycles of shear stress at any level above that which causes yielding of the weaker reinforcement will eventually fail by concrete crushing. This has direct implications for the seismic resistant design of regions such as beam-column joints.

A constitutive model for reinforced concrete subject to general membrane loading is proposed and compared with the test results.

INTRODUCTION

Many serious problems relating to the behaviour of reinforced concrete structures under severe seismic action can be traced to the poor characteristics exhibited by reinforced concrete when subjected to reversed cyclic shear. Understanding the behaviour of reinforced concrete under such conditions and being able to model it analytically are important aims in the quest for safe and economical structures.

The University of Toronto's Shell Element Tester (Ref. 1) was used to conduct three tests on reinforced concrete elements subjected to reversed cyclic pure shear so as to obtain a detailed understanding of the behaviour of reinforced concrete elements under such loading and to acquire the data necessary to formulate a constitutive model for reinforced concrete subject to general, reversed cyclic membrane loading. Of particular interest were: the deterioration of concrete compressive strength due to the transverse tensile strains and due to cycling; the post-cracking tensile stresses in the concrete after cycling; and the relationship between the direction of the principal stresses in the concrete and the direction of the principal strains.

TESTS ON REINFORCED CONCRETE ELEMENTS SUBJECTED TO REVERSED CYCLIC SHEAR

Three tests were conducted on specimens designated as SE8, SE9, and SE10. The geometry of a typical specimen is shown in Fig. 1. The principal variables were the reinforcing ratios and the loading type (see Table 1).

Table 2 shows the detailed material properties for each of the specimens.

Test Setup The tests were conducted in the University of Toronto's Shell Element Tester which is described in detail elsewhere (Ref. 1). Here, only a brief description of the load application and deformation measurement will be provided.

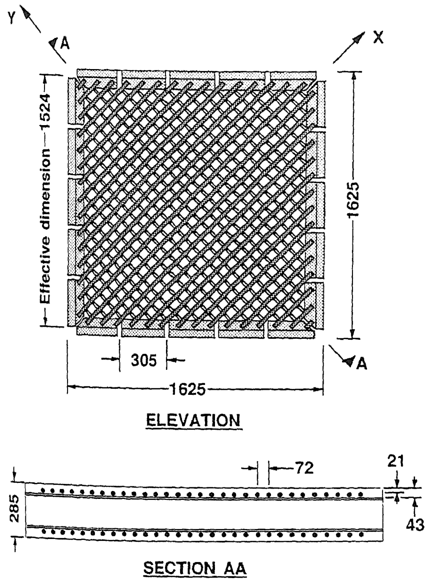

Fig. 1 Typical specimen dimensions.

SPECIMEN	REINF. RATIOS		LOADING
	ρ_{sx}	ρ_{sy}	
SE8	3%	1%	v_{xy} cycled, $f_x=f_y=0$
SE9	3%	3%	v_{xy} cycled, $f_x=f_y=0$
SE10	3%	1%	v_{xy} cycled, $f_x=f_y=-\lvert v_{xy}\rvert/3$

Table 1 Summary of reinforcement and applied loading.

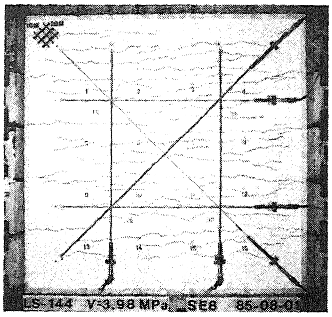

Fig. 2 LVDTs and Zurich targets on specimen SE8.

Material properties.			SPECIMEN		
			SE8	SE9	SE10
CONCRETE		f'_c [MPa] Peak Compressive Strength	37.0	44.2	34.0
		ϵ_{c0} [mε] Strain at Peak Compressive Strength	2.60	2.65	2.20
		f_{sp} [MPa] Tensile Strength (Split Cylinder)	3.4	4.3	4.0
		f_{ps} [MPa] Tensile Strength (Double Punch)	3.2	4.1	3.5
REINFORCING	x-direction	Bar Size	20M	20M	20M
		Bar Area [mm^2]	300	300	300
		ρ_{sx} Reinforcing Ratio	.0293	.0293	.0293
		f_{syx} Yield Stress [MPa]	492	422	422
	y-direction	Bar Size	10M	20M	10M
		Bar Area [mm^2]	100	300	100
		ρ_{sy} Reinforcing Ratio	.0098	.0293	.0098
		f_{syy} Yield Stress [MPa]	479	422	479

Table 2 Material properties.

A state of uniform stress is applied to the element by means of forty double acting 1000 kN hydraulic jacks - twenty acting vertically and twenty acting horizontally. With the reinforcing steel and the $x-y$ axes oriented at 45° to the horizontal, the jacks, in effect, apply the principal stresses to the specimens. If the vertical stress is equal in magnitude and opposite in sign to the horizontal stress, then the element is in a state of pure shear with repect to the $x-y$ axes. This was the case for specimens SE8 and SE9. Specimen SE10 was subjected to biaxial compression in addition to pure shear.

A continuous record of the stress and strain state in the specimen is achieved by applying and measuring the loads and measuring the resulting deformations. The average strain state in the element is calculated from the displacements by six Linear Variable Displacement Transducers (LVDTs) mounted on each face of the specimen as shown in Fig. 2. As well, electrical strain gauges were placed on some of the reinforcing bars to provide information on the local variation of the steel strains.

From the recorded data, the average stress and strain state for the reinforced concrete can be calculated directly. The stress state in the concrete alone can then be determined by subtracting the forces carried by the reinforcement. The stress in the reinforcement is calculated from the measured strain in the reinforcement.

TEST RESULTS

Complete and detailed information on the test program including full test results has been given elsewhere (Ref. 2). In this paper, only a few particular aspects of these results will be discussed.

Overall Behaviour The shear stress vs. strain responses for the three specimens are shown in Fig. 3. Comparing the response of SE9 with that of SE8 (both were subjected to pure shear) shows a large strength increase due to the larger percentage of reinforcement in SE9. Similarly a comparison of SE10 and SE8 shows a large strength increase and a delay of cracking due to the presence of biaxial compression.

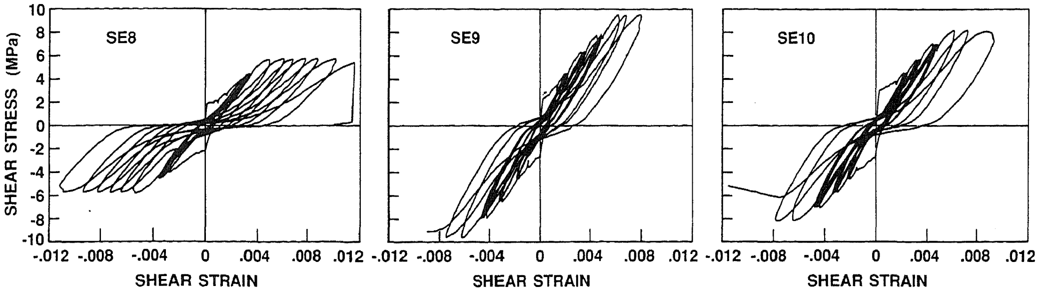

Fig. 3 Reversed cyclic shear response for specimens SE8, SE9, and SE10.

All three responses display the highly pinched hysteresis loops that are characteristic of shear dominated behaviour. For early cycles, before any yielding has occurred, the hysteresis loops are quite stable. However, when cycling between peak shear stresses that are above yield, large strain increments occurred on each successive cycle, until eventually the specimens failed by concrete crushing.

The peak concrete compressive stresses varied from $.31f'_c$ to $.48f'_c$. The deterioration of the concrete compressive strength as a function of the coexisting principal tensile strain has been quantified by Vecchio and Collins (Ref. 3). This deterioration is especially significant to the behaviour of elements under reversed cyclic shear because if cycling is performed at levels causing repeated yielding of the steel, then the principal tensile strain will continue to increase with each cycle. This means that the concrete compressive strength will decrease with each cycle until eventually failure will occur by concrete crushing. Thus, cycles of shear stress at any level above yield will cause failure of a reinforced concrete element. This has direct implications for the design of reinforced concrete components such as beam-column joints.

Table 3 gives the ratio of peak concrete compressive stress to cylinder strength for the three specimens. Also given is the ratio that would be predicted using the Vecchio and Collins formula for monotonic loading. As would be expected, the ratios measured for reversed cyclic loading are lower than those predicted for monotonic loading.

	SE8	SE9	SE10
$-\frac{f_{c2}}{f'_c}$ Experiment	0.31	0.42	0.48
$-\frac{f_{c2}}{f'_c} = \frac{1}{0.8 - 0.34\frac{\epsilon_1}{\epsilon_{c0}}}$	0.42	0.62	0.52

Table 3 Comparison of concrete crushing stresses.

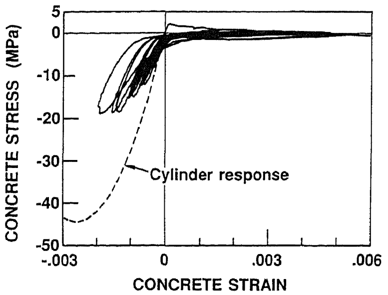

Fig. 4 SE9 concrete response showing full cylinder curve.

Concrete Stress vs. Strain Response for SE9 The concrete stress vs. strain response for SE9 is chosen for discussion because this specimen had equal reinforcing steel in the $x-$ and

y−directions and therefore the principal strain and stress directions coincided, and were at ±45° to the x−axis.

The concrete response for the horizontal direction is shown in Fig. 4. Also plotted is the cylinder response for concrete from that specimen. Several observations can be made:

- For a given compressive strain, there is a dramatic reduction of the concrete compressive stresses caused by the coexisting tensile strains in the perpendicular direction.
- For cycling at low compressive stresses the loops are quite stable.
- For tensile strains, there is significant average tension in the concrete and this tension is virtually completely reattainable on subsequent cycles.
- The response of the concrete when unloading from compression does not pass through the origin but levels off at a small compressive stress (in this case about -1 MPa) and then is almost linear, with very low stiffness, and passes through the tension envelope at the point of previous maximum tensile strain.
- On reloading from tension to compression a small amount of compression is quickly reattained. Then the response levels off, but as reloading continues the compressive stress starts to increase again well before the original compressive envelope is reached. Thus, compression is increasing even before the cracks are closed.

Concrete Principal Stress and Strain Directions One of the aims of this test series was to investigate the relationship between the concrete principal stress direction and the principal strain direction. The compression field approach for reinforced concrete under monotonic loading assumes that these directions are coincident, however it is clear that for an element subjected to reversed cyclic shear, the directions must diverge substantially as the shear stress and strains change sign. Fig. 5 shows the concrete principal stress and strain direction plotted against loadstage for specimen SE8. While the two lines track each other quite closely, there is a small region for each cycle when the principal directions differ by about 90°. As unloading proceeds and the shear stress changes sign, the principal tensile stress direction also quickly changes by 90°. However the principal tensile strain direction does not change by 90° until the point of zero shear strain is crossed.

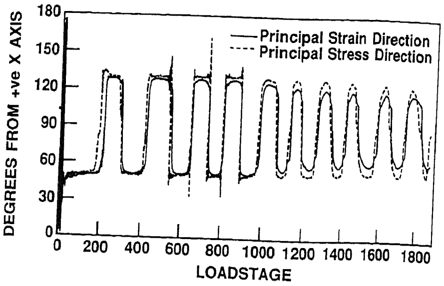

Fig. 5 Relationship between concrete principal directions for specimen SE8.

Before yielding has occurred, the principal directions are close to coincident at high values of shear stress and strain. However once yielding has occurred, subsequent cycles result in an increasing divergence of the principal directions. It can be seen that the principal strain direction progressively decays and becomes closer to the y−axis (90°). This is explained by repeated tensile yielding of the weaker y−reinforcement, each time resulting in larger permanent y−strains. The principal concrete stress direction returns to virtually the same orientation on each cycle.

A further observation can be made regarding crack directions. If it is assumed that the orientation of the cracks is coincident with the principal strain directions, then the observed divergence of the directions after a large number of cycles would imply that shear stresses on the cracks are actually increasing as the number of cycles, crack widths, and damage to the specimens all increase. A better explanation is that the crack orientations coincide approximately with the principal stress directions. The observed crack directions verified this.

CONSTITUTIVE MODEL FOR REINFORCED CONCRETE SUBJECT TO GENERALIZED LOADING

Analytical modelling of the seismic response of reinforced concrete elements is a necessary complement to large scale structural testing, both as an economical means of extending test results and as a tool for understanding complex behaviour.

The proposed model works strictly in terms of average stresses for both the steel and the concrete. Both the reinforcement and the cracks are treated as "smeared". The total stresses at a point are the sum of contributions from the steel and the concrete, calculated separately. Hence, constitutive models are required for both the concrete and the reinforcing steel, subject to generalized loading.

Constitutive Model for Reinforcing Steel The model for the reinforcing steel is one suitable for cyclic loading in the inelastic range that takes into account the Bauschinger effect. It assumes that the steel stiffness at any point is a function of the state of stress and strain, and of the particular strain history. Moreover, the stiffness function is such that:

- after any strain reversal, the stiffness equals the initial stiffness
- as the steel stress approaches f_y or the previous maximum stress (if greater than f_y), the stiffness approaches a limiting value which may be 0 or a strain hardening value greater than 0
- the stiffness varies between the initial and limiting values according to the strain history and the strain state at the last strain reversal.

The basic model, represented in Fig. 6, is modified slightly to account for the effect of embedment of the steel in concrete.

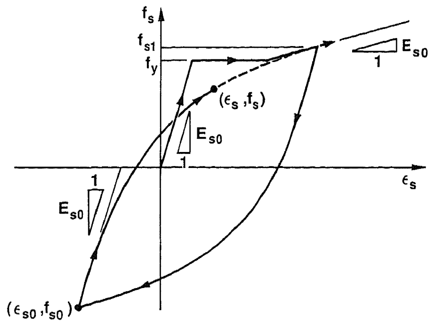

Fig. 6 Parameters in reinforcing steel model.

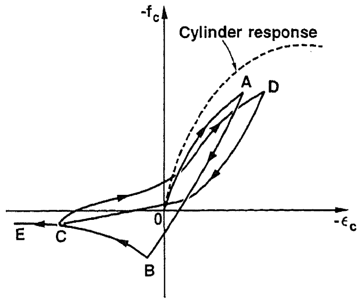

Fig. 7 Idealized concrete response.

Constitutive Model for Concrete For this non-linear problem, increments of stress and strain are considered and it is assumed that the principal direction of the concrete stress increment, $\theta_{\Delta c}$, coincides with the principal direction of the strain increment, θ_{Δ}.

Stated very briefly, the solution consists of determining the concrete stress state $(f_{cx}, f_{cy}, v_{cxy})$ for a given strain state $(\epsilon_x, \epsilon_y, \gamma_{xy})$ as a function of the stress and strain states at the end of the previous loadstage and of the strain history, in the following manner:

- calculate the strain increments relative to the previous loadstage
- calculate the principal strain increments and their direction, θ_{Δ}
- in the direction, θ_{Δ}, calculate the current strain state and the stress and strain states at the previous loadstage
- using the assumption that $\theta_{\Delta c} = \theta_{\Delta}$ and a stress-strain relationship for the concrete which takes account of the strain history and the total strain state, calculate the current stress state in the direction, θ_{Δ}
- calculate the concrete principal stresses and their direction
- check the concrete principal stress envelopes to account for shear stresses in the direction, θ_{Δ}
- calculate the concrete stresses in the $x - y$ direction.

Stress-Strain Relationship for Concrete A reversed cyclic stress-strain relationship for concrete is used to calculate the concrete stresses in the direction, θ_{Δ}. It is essentially a uniaxial model, based on experimental results, which incorporates the influence of the strain history, the effective reinforcing content and of the strain state in the normal direction. The model is represented in Fig. 7 and is described in detail, in Reference 2.

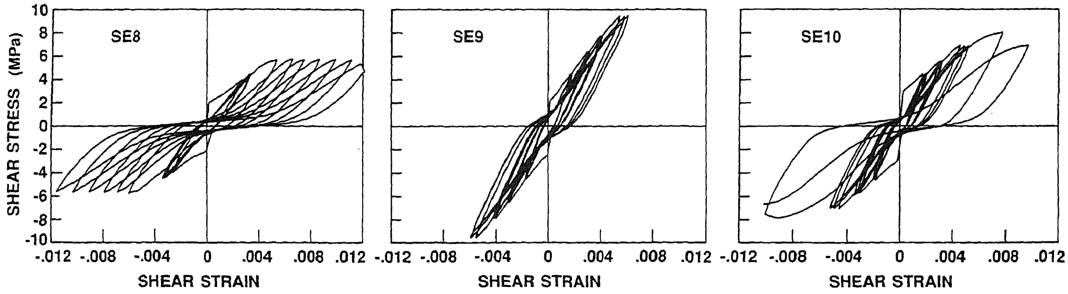

Fig. 8 Analytical shear stress vs. shear strain response for SE8, SE9, and SE10.

Comparison with Test Results Fig. 8 represents the load-deformation response of the three panels tested, based on the proposed model (see Fig. 3). The analytical predictions agree well with the experimental findings.

CONCLUSIONS

These test results provide detailed information on the behaviour of reinforced concrete subjected to reversed cyclic shear which is useful both to the engineer involved in the design of seismic resistant reinforced concrete structures and to the analyst who is interested in formulating and calibrating constitutive models for the analysis of such structures.

Some fundamental design related conclusions can be drawn.

1. Repeated cycles of shear stress at any level above that causing yield of the steel will eventually cause a reinforced concrete element to fail by crushing of the concrete.
2. It follows that if elements such as beam-column joints are to resist a large number of cycles, the reinforcement should be designed to remain elastic.
3. Shear reinforcement for beam-column joints and other regions subjected to high intensity reversed cyclic shear should be fabricated from steel that has no yield plateau but exhibits immediate strain hardening. This would prevent the shear reinforcement from reyielding on repeated cycles, effectively limiting the growth of the principal strains. An identical conclusion was reached by Uzumeri (Ref. 4), as a result of tests on exterior beam-column subassemblies.
4. Reversed cyclic loading under biaxial conditions results in a reduction of the peak concrete compressive stress in excess of that observed for monotonic loading.

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