THEORETICAL MODELING OF EARTHQUAKE GROUND MOTION AND SEISMIC RESPONSE SENSITIVITY ANALYSIS OF STRUCTURAL SYSTEMS

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SUMMARY

The seismic response sensitivity of a structural system is investigated for the development of a reasonable seismic design methodology. The earthquake ground motion is modeled theoretically for the analyses of the seismic response sensitivity. The source function for the rupture process on the fault surface is found by the solution of Klein-Gordon equation. The two layered half-space consisting of surface soil layer overlying a semi-infinite random medium is supposed as a physical model of wave propagation path. The physical quantities characterizing real earthquakes and seismic response spectra are investigated on the variable parameters describing the ground motion model.

INTRODUCTION

The response sensitivity analysis of structural systems to uncertain soil and geological properties, and source characteristics of an earthquake is very important for the estimation of damage potential and development of a reasonable seismic design methodology.

In this paper, the earthquake ground motion is modeled theoretically for response sensitivity analysis. The source function is found by the dynamic behaviors of mass-spring system to stress-drop process on the rough surface, which can be described by Klein-Gordon equation. The two layered half-space consisting of surface soil layer overlying a semi-infinite random medium is supposed as a physical model of wave propagation path from source to site. The earthquake ground motion is presented by the convolution of the Green's function of the two layered half-space model and the source function, and the formation of the characteristics of earthquake ground motions is discussed on the basis of the variable parameters describing this model. The seismic response time histories and their spectra for the Green’s functions and the earthquake ground motion models presented here are analyzed to show how the physical laws, and the soil and geological properties, and the source rupture process give effect on seismic safety of a structural system.

MODELING OF EARTHQUAKE GROUND MOTION

Source Function for Rupture Process on Fault Surface The rupture process on the fault surface would be physically simulated by the dynamic behavior of mass-spring system to stress-drop process on the rough surface[3], which is expressed by Klein-Gordon equation. When the non-uniform stress-drop propagates in only x
direction with average constant velocity \( V_s \), as shown in Fig. 1, the particle displacement \( D(x,t) \) on the fault surface is governed by the equations

\[
\begin{align*}
V_s^2 \frac{\partial^2 D}{\partial t^2} + 2h_o \frac{\partial D}{\partial t} - \frac{\partial^2 D}{\partial x^2} + V_s^2 \left( \gamma_o^2 - b_o^2 \right) &= -Q(x,t) \\
Q(x,t) &= q(x) \left( \frac{x - x_j}{V_r} - H \left( t - \frac{V_r}{V_s} \right) \right), x_{j+1} = x_j + \Delta x
\end{align*}
\]

\[ [1] \]

where \( V_s \) and \( h_o \) are velocity of transverse wave and radiation damping parameter, \( b_o \) and \( \gamma_o \) are the parameters related to the stiffness per unit mass and the exponential dependence of the wave motion on \( z \) direction. The fault surface with length \( L_f \) and width \( W_f \) is divided into \( M \) local areas. We assume that the rupture front propagates along the definite line from \( A' \) to \( B' \) at focal depth \( Z_{0s} \) in Fig. 2, which shows the geometric relation between the causative fault and the observation stations. The rupture on the fault surface are estimated on the center point of each local area located at \( x_1, x_2, \ldots, x_j, \ldots, x_{12} \) along with the line \( AB' \). Fourier transformation of eq. (1) leads to the equations

\[
\begin{align*}
\hat{D}(x_1, \omega) &= -\frac{2q(x)T_1}{K(\omega)} \frac{\sin(\omega T_1)}{\omega T_1} \exp(-i\omega(\frac{x - x_1}{V_r})) \\
T_1 &= \frac{\Delta x}{2V_r}, K(\omega) &= (1 - \frac{V_r^2}{V_s^2})\omega^2 + V_s^2(\gamma_o^2 - b_o^2) \\
- \frac{D(x_1, \omega)}{V_r} &= \frac{M}{2} \sum_{i=1}^{12} \hat{D}(x_i, \omega)
\end{align*}
\]

\[ [2] \]

\( D(x_1, \omega) \) is particle displacement at \( i \)th local area. The function \( q(x) \) is proportional to the stress-drop on the fault surface. \( \omega \) and \( t \) are frequency and time.

The Fourier spectra of nondimensional particle displacements over local fault area and entire fault surface are shown in Fig. 3. In these figures, there seem clear oscillations which are the result of the cumulative contribution of discontinuous propagation of stress-drop on the fault surface. It is found that spectra in high frequency range would depend on nondimensional stiffness parameter \( b_{00} \) in eq. (1). From Fig. 3, the solution of Klein-Gordon equation seems to realize basically the numerical solution of the dynamical problem of expansion of crack on the fault surface.

**Dynamic Green’s Function for Layered Half-Space.** If the propagation spectra of seismic wave motion is governed with the amplitude characteristics of surface soil layer and the energy loss of multiple scattering in the heterogeneous earth structure beneath surface soil layer, it would be reasonable to express the wave propagation path from source-to-site by the two layered half-space model which consists of surface soil layer overlying a semi-infinite random medium [1]. When seismic wave motions radiated from a point source in the 2nd layer located at focal depth \( Z_{0s} \) propagates as shown in Fig. 2, their nondimensional surface displacements at epicentral distance \( R \) are evaluated as follows:

\[
\begin{align*}
\{ \hat{G}_D(\tau : r) \} &= \frac{1}{F} \{ C \}[1], \quad m_{12} = \frac{V_{S2}}{V_{S1}} \\
\left( \frac{k}{k_c} \right)^2 &= g_1(\varepsilon, \lambda_2, \mu_2, \rho_2, \delta, a_0), \quad \left( \frac{k}{k_s} \right)^2 = g_2(\varepsilon, \lambda_2, \mu_2, \rho_2, \delta, a_0)
\end{align*}
\]

\[ [3] \]
In eq. (3), the nondimensional surface displacements indicate the three components of Green's function vector of the two layered half-space model \(\{G_0(t,r)\}\), which includes the effective wave numbers \(g_1\) and \(g_2\), and nondimensional viscoelastic coefficients \(y(1)\) and \(y(2)\) for transverse waves of 1st and 2nd layers. \(k_c\) and \(k_s\) are wave numbers of longitudinal and transverse waves. \([C]\) consists of transmission and reflection coefficients concerning the above two waves. \(F\) is Rayleigh function. \([1]\) means unit vector concerning internal stresses and displacements which act at the focal depth \(Z_{0s}^i\). \(\nu_{0i}\) and \(\rho_{0i}\) are Lamé's constants and density of \(i\)th layer. \(\delta\) is inhomogeneity correlation length of a random medium[1]. The nondimensional and dimensional variables in eqs. (2) and (3) are related to the following parameters:

\[
a_0 = \frac{b u_0}{v_s}, \quad R = b r, \quad \gamma = \frac{\gamma_b}{\omega 0}, \quad b = b b, \quad n_1 = \frac{1 - 2\nu_1}{2(1-\nu_1)}
\]

\[
\frac{Z_{0s}}{Z_{0i}} = \frac{b \gamma_0}{b \gamma_0}, \quad Z_{01} = b \gamma_{01}, \quad \rho \gamma = \frac{\lambda_{01} + 2\mu_{01}}{\rho_{01}}, \quad \gamma \gamma = \frac{\mu_{01}}{\rho_{01}}, \quad i = 1,2
\]

\[\text{[4]}\]

\(u_1\) and \(u_2\) are velocity of longitudinal wave and reference length. \(\nu_1\) and \(\nu_2\) are Poisson's ratios of 1st and 2nd layers, respectively.

Fig. 4 shows the spectra and wave form functions of Green's functions of radial component for the ratios of epicentral distance to focal depth \(R/Z_{0s}=1\) and 5. From Figs. 4(a) and (b), in the near-field \((R/Z_{0s}=1)\), \(P, SV\) waves and their reflected waves characterize seismic wave motions. In the far-field \((R/Z_{0s}=5)\), \(P, SV,\) and Rayleigh waves and their reflected wave characterize them. Seismic wave motion is especially amplified with the increase of transverse wave ratio of 1st and 2nd layers \(m_{12}\). Seismic wave motions are not so much amplified by the thickness of surface soil layer. Surface wave is also especially amplified as \(m_{12}\) increases. However, the duration time increases with the increase of \(R/Z_{0s}\), \(m_{12}\), and the thickness of surface soil layer \(Z_{01}\). In Fig. 4(b), it is shown that the spectra is attenuated in the frequency range more than \(a_0=50\) by the inhomogeneity correlation length. Then, the ratio of epicentral distance to focal depth and transverse wave velocity ratio of 1st and 2nd layers are found to be very important physical quantity, and soil and geological parameters which describe essentially the ground motions. The two layered half-space model presented here provides some basic characteristics associated with real earthquake ground motions.

Earthquake Ground Motion Model The earthquake ground motion is obtained by superposition of seismic wave motion radiated at each time when the rupture front propagates at \(x_1, x_2, ..., x_{12}\) on the line \(A'B'\) shown in Fig. 1, as follows:

\[
Y_D(t) = \sum_{i=1}^{M} \int_{0}^{L_f} C_D(r - x_i; t - \tau)n_i(r_i, \tau)d\tau, \quad D = x, y, z
\]

\[
n_i(r, t) = \mu_2 L_f W_f D(r, t)
\]

\[\text{[5]}\]

In eq. (5), \(r_i\) and \(\mu_2\) are position of rupture front at the \(i\)-th local area and shear modulus of 2nd layer. Fig. 5 shows Fourier spectra of earthquake ground motion model. In the analyses of this model, the high frequency components increase with the increase of Mach number and decrease with the increase of stiffness parameter \(b_{0f}\). It is also found that the corner frequency \(f_c\) is almost described by the fault length \(L_f\), and that the spectra in high frequency range depend on the stiffness parameter \(b_{0f}\) and the transverse wave velocity ratio of 1st and 2nd layers \(m_{12}\), the inhomogeneity correlation length \(d\), and the ratio of epicentral distance to focal depth \(R/Z_{0s}\). It follows that the
ground motion model presented here is considered to be effective to simulate the characteristics of real earthquake ground motions.

SEISMIC RESPONSE SPECTRA CHARACTERISTICS

In order to investigate how soil and geological properties affect response characteristics of a structural system, the seismic responses to the Green's functions presented here are shown along natural frequencies and duration time in Fig. 6. The inhomogeneity correlation length \( \delta \), the perturbation parameter measuring inhomogeneity \( E \), nondimensional viscoelastic damping coefficients \( \nu(1) = \nu(2) \), and Poisson's ratios \( v_1 = v_2 \) are taken to be 0.0125, 0.02, and 0.25, respectively. In these figures, the responses are sensitive to P, SV waves and their reflected waves in short period range, and to Rayleigh wave and their reflected waves in long period range. Fig. 7 shows the response spectra of radial and cross-radial components of Green's functions for the cases \( R/20s = 1.0 \) and 5.0. In these response analyses, \( Z_{01}/Z_{02} \) and \( m_{12} \) are 0.05 and 5.0, respectively. As \( m_{12} \) increases, the natural frequencies of shear vibration of surface soil layer would be of large influence on peak seismic response of a structural system in the far-field. The appearance of P, SV, SH, and surface waves are described by the ratio of epicentral distance to focal depth, and surface waves are especially characterized by the transverse wave velocity ratio of 1st and 2nd layers. The response spectra are sensitive to such body waves as P, SV, and SH waves in the near-field and to such surface waves as Rayleigh and Love waves in the far-field. From the above analytical results, the responses for the fluctuations of the stiffness and thickness of surface soil layer, and the ratio of epicentral distance to focal depth would describe the upper and lower limits of seismic safety of a structural system for future earthquakes.

CONCLUDING REMARKS

The seismic response sensitivity is investigated how the physical laws and quantities describing earthquake ground motion give the effect on the seismic safety of a structural system. The theoretical modeling of the earthquake ground motion suggests that the ratio of epicentral distance to focal depth, the transverse wave velocity ratio of 1st and 2nd layers are basic physical quantities to describe seismic ground motions. It is pointed out that the seismic response sensitivity to variation of such important physical quantities describing the earthquake ground motion model would present the upper and lower limits of seismic safety of structural systems for future earthquakes.

REFERENCES


Fig. 1 Rupture Process Model

Fig. 2 Geometric Relation between Causative Fault and Observation Stations

Fig. 3 Source Function

(a) R/2O=1, m12=3

(b) R/2O=5, m12=3

Fig. 3 Fourier Spectrum and Wave Form Function of Green's Function
Fig. 5 Fourier Spectrum of Earthquake Ground Motion Model
(Vertical Component)

Fig. 6 Seismic Response to Green's Function

Fig. 7 Response Spectra of Green's Function