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ANALYTICAL MODELS FOR THE BIAXIAL RESPONSE OF REINFORCED CONCRETE BUILDINGS

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SUMMARY

A refined element model is presented for the analysis of reinforced concrete beam-columns under generalized bi-directional flexural and axial load fluctuations. A section/fiber idealization is adopted. The formulation is capable of describing the internal damage distribution and the finite deformability and softening characteristics of the member. The element, which is implemented in a general purpose space frame analysis program, is used herein for correlations of column experiments under biaxial bending and the analysis of a test frame under oblique uniaxial earthquake base excitation. The analyses demonstrate the reliability of the formulation and the biaxial response characteristics of columns that influence global structural response and concentration of damage.

INTRODUCTION

Refined finite element discretizations are often computationally prohibitive for the analysis of entire building systems and they may be unsuitable for cyclic response analysis. For the analysis of entire structural systems different approaches usually have been followed for modelling the biaxial bending of individual beam-columns; these range from the lumped plasticity representations that assume all nonlinearity is confined to idealized end springs (3,6, and 13), to distributed damage models which account for the spread of damage along the length (2,5,7,9, and 12). A detailed review of alternative models and their restrictions is given in Refs. 8 and 15. The applicability of existing analytical models is often limited because they may oversimplify some key aspects of physical behavior, or may employ restrictive theoretical assumptions. Similarly, most formulations suffer from numerical problems when a drop in resistance is encountered with increasing deformation. A refined formulation which overcomes some of these deficiencies is presented herein.

ELEMENT FORMULATION

A series of nonlinear models has been developed for the static and dynamic analysis of individual biaxially loaded columns. The section discretization employed in the finite element models is similar to that adopted in the interactive program BICOLA, used for the analysis of arbitrary-shaped column sections under generalized biaxial bending-axial load (or deformation) control (16). The member models have been implemented in ANSR, a general purpose finite element analysis program for the nonlinear static and dynamic analysis of three-dimensional structural systems (10).

Section Idealization The section can be arbitrarily shaped and is defined as an assemblage of uniaxially stressed steel and concrete fibers in an orthonormal right-hand coordinate system. The coordinate set is normal to the reference axis of the member and the section origin is located on the member axis (Fig. 1). Following the usual fiber model assumptions, plane sections are assumed to remain plane and normal to the member axis while full compatibility is enforced between neighboring steel and concrete fibers. Therefore, the section state is completely defined by the reference strain at the origin and two orthogonal curvatures in the orthonormal system.

Different material constitutive models for the steel and concrete can be used within the section. For the steel three different models are implemented (Ref. 15). For the concrete a general model is adopted, capable of describing unconfined as well as confined behavior.

Member Idealization The element models prismatic members with a straight longitudinal axis. A column is monitored at sections located along its length (Fig. 1). At least two sections (at the ends of the member) must be defined. The member degrees of freedom are shown in Fig. 1.

Small displacement theory defines all interior kinematic transformations; however, the distribution of damage within the element is taken into account in the definition of these transformations. Global level second order geometric effects are considered assuming a simple truss mechanism. Shear deformations at the global level, as well as shear force-flexure interaction, are ignored. Under the plane sections assumption, effects due to bond slip are neglected within the element. However, fixed end rotations at the column ends can be included in the analysis of the entire structure through additional joint spring elements especially developed for this purpose (15).

Distributed transverse loads are not considered; this assumption is typically correct for columns. Variation of section properties is permitted along the longitudinal axis; however, interior sections must be at least as strong as the end sections to avoid convergence problems.

Member Stiffness For computational economy a linear flexibility variation is assumed between monitored sections. Therefore, the location of internal sections is dictated by the need for realism in establishing an adequate flexibility distribution. The member tangent stiffness matrix is obtained by inverting the local element tangent flexibility F_m. Following established virtual work procedures, this is evaluated during changes of fiber state through a weighted integration of the current section flexibilities $f_s(z)$,

$$F_m = \int_0^L b_s^m(z)^T f_s(z) b_s^m(z) dz \tag{1}$$

where $b_s^m(z)$ is the equilibrium transformation matrix of a simply supported member. Section flexibilities $f_s(z)$ are calculated by inverting the individual section stiffnesses (17).

State Determination Numerical problems with displacement advancement strategies due to negative definite stiffness following section disintegration, as well as due to nonproportional loading, have been reported in Ref. 5 and were examined in Ref. 16. In order to account for possible deformation softening of the critical regions, the process of determining the internal fiber stresses and section forces differs from conventional displacement formulations.

Changes in the end section fiber states within a time step are monitored through a modified event-to-event step advancement (15). During each event, section deformations and fiber strains are evaluated from end section deformations (dv_s) using displacement transformation matrices $a_s^m(z)$ that account for the current internal flexibility distribution, following

$$dv_s(z) = f_s(z) b_s^m(z) F_m^{-1} dr_m = a_s^m(z) dr_m \tag{2}$$

where F_m is the current tangent member flexibility. The use of the above variable transformation functions has been introduced in Ref. 7 for modelling steel tubular members.

After the section resistance vectors are obtained, remaining interior sections are updated iteratively to conform to the equilibrium diagram defined by the end section forces. As a result, numerical instability problems of the displacement formulation (illustrated in Refs. 8, 15, and 17) are resolved. For positive definite section behavior the adopted procedure is entirely equivalent to standard displacement formulations (15).

The equilibrium iteration procedure (15) consists of a three-level nested iteration loop establishing the section strain state (given the past history) that equilibrates the target section forces. At the upper level a bisection scheme is adopted in the two-dimensional moment space in order to establish the section deformation state that minimizes the difference between applied and resisting section moments. Using the lower level iterations, the evaluation of each trial point satisfies axial load equilibrium. A similar displacement controlled advancement of the critical end section is enforced in the event of instability under nonproportional loading associated with the onset of deformation softening.

VERIFICATION STUDIES

Several column tests have been used for element verification (11 and 13). A square, symmetrically reinforced column tested under a constant axial load of sixteen tons is considered here. The specimen is displaced in a square pattern in plan with cycles of gradually increasing amplitude. Reported material characteristics are used in modelling the member. The element is monitored at four unequally spaced sections, with approximately 200 fibers per section. Steel is modelled using an explicit exponential idealization.

The predicted base shear-tip displacement characteristics are compared with experimental results in Fig. 2. Analytical correlations of the above tests have also been reported, using other plasticity or lumped nonlinearity formulations in Refs. 3 and 13. The correlations between recorded and predicted responses demonstrate that the model is able to adequately simulate the biaxial bending phenomena observed experimentally in the variation of stiffness, strength, and hysteretic response.

Following the projected hysteretic shapes, several phenomena associated with biaxial response can be identified which are not encountered in strictly uniaxial response. As a consequence of the biaxial strength interaction, increase in the out-of-plane deformation (while keeping the in-plane displacement fixed) causes a significant reduction of the in-plane resistance due to the influence of a maximum capacity surface. Comparing subsequent cycles through the same displacement level with intervening out-of-plane excursions, a reduction is observed in the flexural stiffness of the member as a consequence of intermediate out-of-plane bending cycles.

ANALYSIS OF A SIMPLE FRAME

A two-story test frame was analyzed under the first seven seconds of the Taft earthquake record, scaled to 0.57g (Test W2, Ref. 4). Different inclinations (Θ) of the base input relative to the structural principal axes were considered, from 0° through 90° at increments of 15°, assuming full correlation of the base input orthogonal components. The recorded acceleration records acting simultaneously (the transverse motion acceleration being scaled with the same scale factor as the longitudinal one) and the longitudinal component acting simultaneously along both axes (cases UNC and 0+90, in Figs. 3 and 5, respectively). The detailing of the members conformed with standard ductile moment-resisting frame practices. Columns were rectangular in section, oriented such that strong bending occurs for longitudinal (denoted as x) motion. The diaphragm was assumed to be rigid in its own plane. Beams were modelled as simple two-component elements while columns were modelled by the fiber element, monitored at six unequally spaced sections. Rigid zones were specified to approximate the beam-column joints.

The predicted particle motions at the roof centroid are compared in Fig. 3 for the different Θ's. Overall, drifts are higher in the transverse direction due to the relatively higher flexibility of the structure in this direction, as well as due to a different pattern of member hinging. For this particular earthquake, centroidal drifts are comparable to those attained under uniaxial excitation in either direction. The maximum predicted drift (case 0+90) is 2.5 in (6.3cm), corresponding to an interstory drift of 1.8%. Projected column drifts exceed somewhat the corresponding centroidal values as a result of torsion.

The initial response loops are strongly affected by Θ, although this dependence breaks down as significant nonlinearity is induced in the system. Most nonlinear action is confined to the ground floor columns, with the first story gradually turning into a soft story. The maximum vectorial displacement ductility at this level, accounting for the reduction in yield deformation due to biaxial effects, is equal to 5.3, compared to the maximum uniaxial demand of 3 (obtained in y excitation). Despite the initial symmetry of the frame, torsional response is induced as a result of unsymmetric inelastic distributions of lateral stiffnesses between columns due to biaxial bending; this is further accentuated by axial load fluctuations.

Projected x motion bending moment-chord rotation characteristics for a typical ground corner column are compared in Fig. 4. Projected chord rotation demands are comparable for different Θ's. However, due to higher axial load fluctuation and biaxial bending, flexural strengths are reduced by up to 40% compared to uniaxial excitation.

Despite the near equality of global drifts, the presence of biaxial demands in the column critical regions produces a significant difference in the local damage. Comparing the energy dissipated by the reinforcement at the base critical section of this column normalized by the energy at yield in pure tension (Fig. 5), it can be seen that the induced damage is considerably higher than that induced under uniaxial excitation.

CONCLUSIONS

A fiber element model is proposed for the cyclic analysis of reinforced concrete beam-columns under biaxial bending and axial load fluctuations. The formulation adopted can reliably model members that exhibit deformation softening. Detailed information can be obtained on the internal curvature and strain demands, as well as the concentration of damage within the member.

The analyses of the test members indicate that biaxial bending of columns results in a reduction of strength and flexural stiffness relative to their uniaxial response counterparts. As a result of biaxial excitation, unusually high strains can be locally induced.

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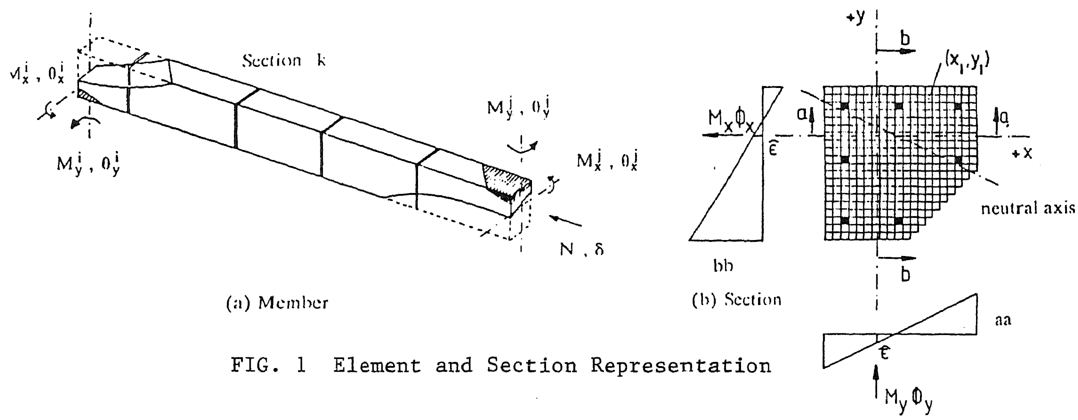

FIG. 1 Element and Section Representation

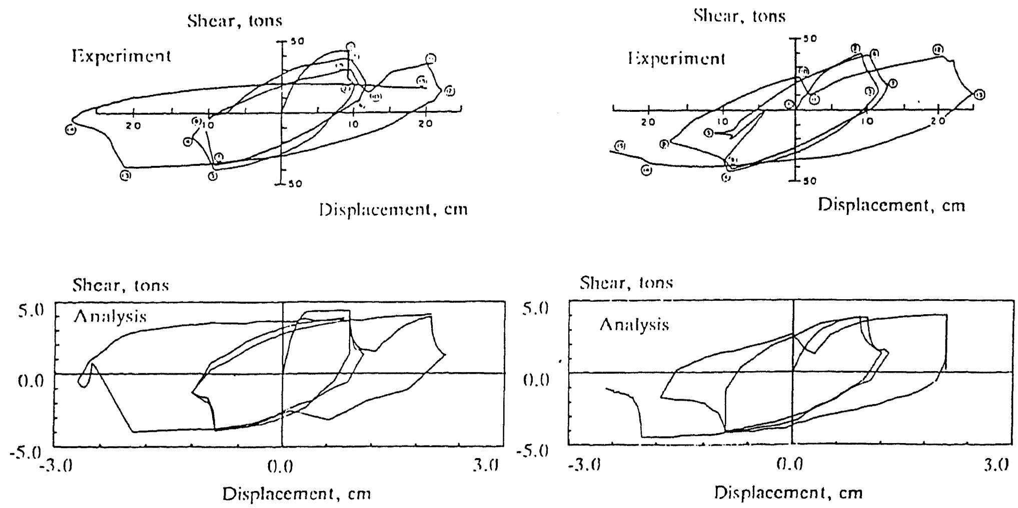

FIG. 2 Correlations With Experiments, Takizawa and Aoyama Column

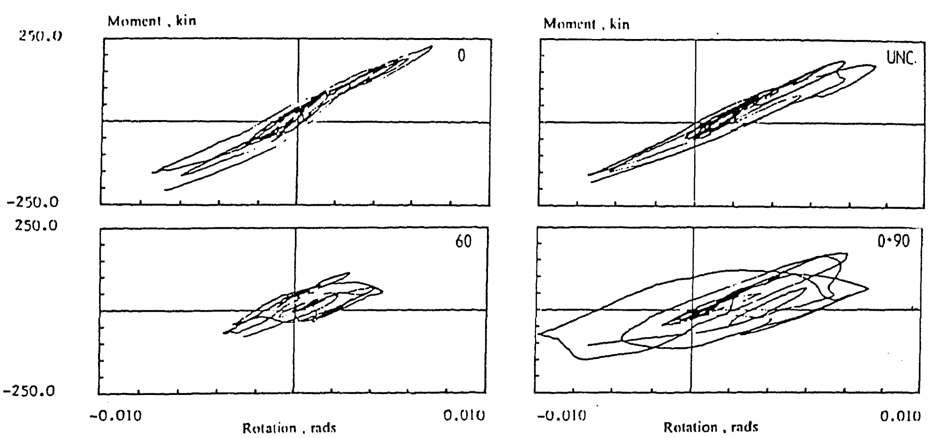

FIG. 3 Roof Centroidal Motions for Different Incident Angles

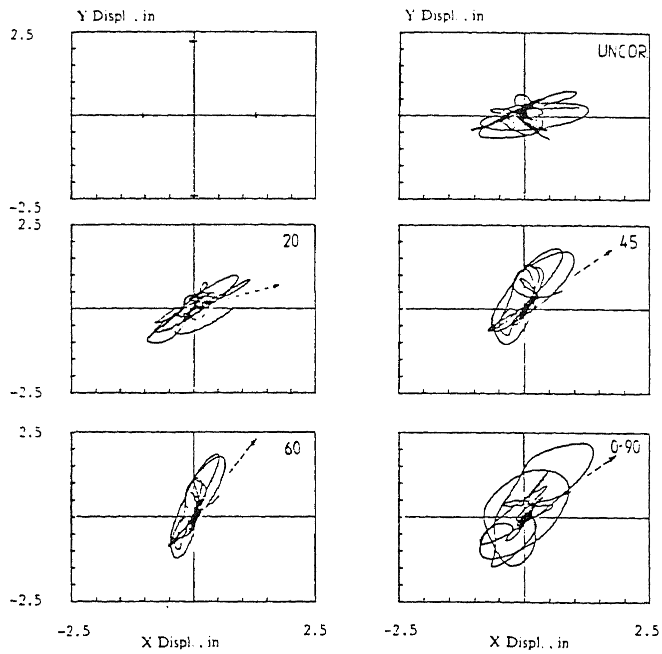

FIG. 4 Column Base Moment - Chord Rotation for Different Incident Angles

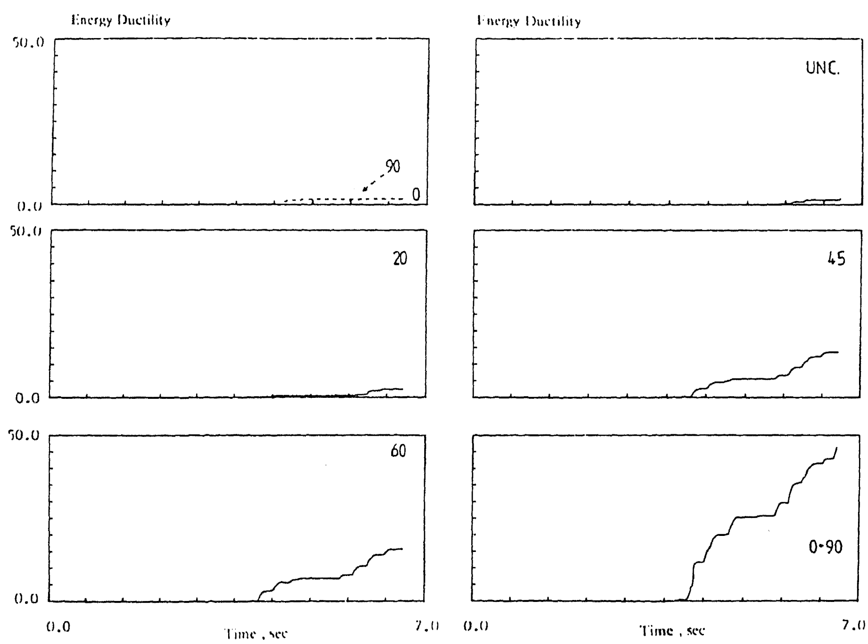

FIG. 5 Cumulative Inelastic Energy Ductility, Ground Story Column, For Different Incident Angles