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INELASTIC BEHAVIOR AND MODELING OF REINFORCED CONCRETE COLUMNS UNDER MULTIDIRECTIONAL SEISMIC EXCITATIONS

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SUMMARY

During the past two decades considerable research has been performed to assess the performance of reinforced concrete columns subjected to the biaxial flexure and/or varying axial forces associated with multidirectional seismic excitations. These studies are reviewed in this state-of-the-art report. Emphasis is initially placed on experimental studies on column elements and on observed behavioral characteristics. Techniques for modeling this behavior for seismic analysis are then presented. Some examples are shown to illustrate design implications. Recommendations for future experimental and analytical research are offered.

INTRODUCTION

The partial and complete collapse of reinforced concrete buildings during recent earthquakes has demonstrated the need to design columns to be able to withstand the multidirectional nature of seismic excitations and response (1- 3). However, the inelastic flexural behavior of such columns is complicated by the wide variety of physical phenomenon involved. At the section level, such phenomenon include yielding, Baushinger effects and buckling of the reinforcement, cracking and constitutive nonlinearity in the concrete, shear, bond deterioration, and spalling of the concrete cover. Moreover, in the case of beam-columns, the presence of axial load influences the closing of cracks, the location of the neutral axis along and across the member and the plastic hinge length (4, 5). These factors and their interaction complicate structural response and its prediction.

To mitigate such problems, model building codes often stipulate a strong column - weak girder design philosophy. However, it may not be possible to achieve this ideal situation in practice due to the special conditions existing at the base of a structure, the contribution of the slab to the strength of the beams, variations in axial loads (associated with overturning moments developed in the structure as well as with the vertical component of ground motions), the structural system used, the presence of nonstructural elements, and of course, biaxial bending effects. Thus, careful study is required to understand the behavior of individual columns, to assess the effects of this behavior on structural response and to devise appropriate design methods.

The intent of this report is to review current experimental and analytical research related to reinforced concrete columns subjected to bidirectional flexure and/or varying axial loads. Emphasis is placed on the behavior and modeling of individual elements. Implications for design will be highlighted as will future research needs. Additional sources of information may be found in the companion papers and state-of-the-art reports (6-13) as well as in Refs. 2 and 14.

EXPERIMENTAL STUDIES

A variety of experimental research programs have been carried out to assess the flexural response of columns under bidirectional excitations (14- 24). Most of these studies have been performed on simple cantilever specimens. While a variety of tests have examined the effects of monotonically increasing bilateral loads (e.g. Ref. 25), fewer studies have considered cyclic loading conditions representative of seismic excitations. In these latter tests, square or circular cross sections are generally considered, with relatively few tests of rectangular or more complex shaped sections (24). Predetermined displacement histories (resembling stylized diagonal, elliptical, diamond, square or clover leaf patterns) are generally employed in the tests. Because of the need for loading simplicity, many tests have been performed without axial load (19, 22). Other cases have considered constant axial forces (14, 15, 17, 18). Recently, members with varying axial loads (proportional to lateral forces) have been tested for uniaxial (4, 11, 26) and biaxial bending (12, 24). Only a few investigations have considered axial load variations that are not proportional to lateral displacement, and these have been generally restricted to uniaxial bending (4, 11, 26, 27). To have a better understanding of column behavior under earthquake like excitations, a few on-line control (pseudodynamic) tests have been performed (11, 17). Actual dynamic tests of column elements are few. However, several shaking table tests have been performed on framed structures with bidirectional base excitations (e.g., Refs. 28 and 29).

Experimental results indicate that well confined columns with adequate shear reinforcement subjected to biaxial flexure sustain more damage, suffer more deterioration of stiffness and strength, and exhibit substantially more complex and irregular hysteretic characteristics than comparable columns or beams loaded in uniaxial flexure (Figs. 1 and 2). The responses are very sensitive to the history and pattern of loading applied. In comparison with uniaxial response, stiffness under loading in one direction decreases significantly as a result of previous or concurrent loading in the transverse direction. This is primarily due to the effects of cracking associated with the additional load or of accumulated damage from the prior transverse loading (cracking and spalling of concrete, Baushinger effects in the reinforcement and bond deterioration). Stiffness reductions of 50% or more have been observed within the working load regime. While principles of mechanics indicate that reductions in projected strength would be expected under bilateral loading, the effect of cyclic loading in the transverse direction is generally much greater than would be anticipated on this basis alone (due in part to increased damage). In some cases, strength reductions in one direction have exceeded 80% when loads are applied in the orthogonal direction even though displacements are held constant in the first direction. Damages (concrete cracking and spalling as well as steel strains) are consistently larger than those for uniaxial loading to similar displacement levels. The larger the axial load level the more pronounced are these effects (12).

Some of these characteristics can be observed in the envelopes of cyclic response hysteretic loops (24) shown in Fig. 3. In this figure Curve 1 corresponds to a uniaxially loaded specimen. Curve 2 corresponds to an identical specimen loaded at a 45 degree inclination. As would be expected on the basis of flexural theory, this envelop is lower. Curve 3a corresponds to the point on a clover leaf displacement pattern which is in uniaxial deformation. Comparison of this curve to Curve 1 indicates a significant reduction. Curve 3b corresponds to the point on the clover leaf which is aligned with the points plotted on Curve 2. Thus, prior cracking, straining and damage due to transverse loads does adversely influence inelastic behavior.

Concurrent variation of axial load leads to even more complex hysteretic behavior as a result of the dependence of flexural strength and neutral axis position on axial load. Axial loads that vary in proportion with lateral deformation produce systematically unsymmetrical hysteretic loops for either uniaxial (4, 24, 26) or biaxial (12, 24) flexure. For low compressive mean loads, apparent stiffness and strength increase with increasing axial load and vice versa. For loads near the balanced point hysteretic loops are more symmetrical, but still show complex triaxial interaction effects (12). Results show that columns often suffer a preferred orientation for damage (toward the compression-most load direction) which can reduce the capacity in this direction (24, 26). For axial loading which varies non-proportionately with lateral deformations (4, 26) very irregular hysteretic loops can be obtained which do not match commonly used analytical representations or design approximations.

Results also indicate that a significant proportion of the specimen deformations are associated with slippage of the reinforcement at the end of the specimen (22, 23, 24). Few tests (e.g., Ref. 23) have been instrumented sufficiently to identify quantitatively the contribution of the resulting fixed end rotations to the total lateral displacement and to study the mechanism of bond deterioration.

Most of the specimens considered in the above studies were proportioned so that flexure dominated the behavior. A few test have, however, been made of short columns (effective length to depth < 2.5) in which shear would be expected to control response (22, 23, 30 - 32). These tests indicate that cycling and increased axial loading accelerates deterioration of the section once the lateral displacement at which maximum resistance occurs is exceeded. Figure 4 compares the cyclic response of a bilaterally loaded short column to an identical unidirectionally loaded one (marked M).

In view of the limited data on bilateral behavior of columns, additional tests would be desired to extend the range of sections considered, axial loads imposed, amounts and detailing of transverse and longitudinal reinforcement used, and loading histories and boundary conditions employed. On-line and shaking table tests are required to assess response under realistic earthquake excitations and to identify any rate effects. Loading histories (proportional and nonproportional variation of axial loads) need to be established to better evaluate analytical models. Tests to assess slenderness and lateral stability issues need to be performed. Specimens need to be instrumented to measure local deformations so that analytical models can be better assessed. In particular, information regarding bond slip and the distribution of flexural, axial and shearing deformations along the length of the member are needed. To facilitate analytical and design related studies data should be put in a form that can be easily exchanged between researchers.

ANALYTICAL MODELS

A variety of analytical techniques have been developed to model the inelastic cyclic behavior of reinforced concrete column sections and elements. The increasing availability of low cost, high performance engineering workstations has lead to the development of interactive, graphically oriented computer programs for the design (33) and analysis (34, 35) of biaxially loaded sections. For example, in the case of Ref. 35, arbitrarily shaped sections can be analyzed considering the section to be discretized into an arbitrary grid of concrete and steel fibers. The program is capable of considering a series of sequentially applied load and/or deformation histories. As such, information can be obtained on three dimensional interaction curves for various limit states (first cracking, yielding, ultimate strength, etc.), biaxial moment- curvature hysteretic loops, axial load-elongation relations, stress state and neutral axial position. The interactive nature of such programs permits rapid assessment of the effects of variations in design parameters and loading conditions.

The techniques used to model complete members depend on the type of information needed, the quality of information available regarding the column to be analyses and the nature of the applied loads. For example, at the preliminary stages of analysis or to assess the effects of overall design changes on global response quantities, relatively simple macroscopic models may be most appropriate (3, 36). If more detailed information is required regarding the damage state in individual elements, more traditional finite element approaches are appropriate. However, due to the level of computational effort required a variety of simplifying assumptions are generally introduced. To achieve computational economy phenomenological idealizations are often utilized which mimic the observed behavior of columns. The applicability of such models is limited to the loading and boundary conditions considered in their formulation. More refined physical and analytical representations are able to account for the spatial and temporal variation of inelasticity. However, they require more precise information regarding the constitutive modeling of materials, distribution of damage, bond slip relations, and so on. In many cases this information must be estimated.

Comprehensive bibliographies of the literature may be found in Refs. 2, 14 and 15. A brief summary is presented here to indicate the approaches used in modeling damage due to material inelasticity. Studies for considering geometric nonlinearities and long term effects are treated elsewhere (e.g., see Ref. 37). Two basic approaches have been used to model the damage at the element's critical regions: one in which inelasticity is averaged (lumped) at the ends, or another in which damage is monitored at various sections along the length. These will be referred to as lumped and distributed plasticity models, respectively. In nearly all of the available models, shear and torsional deformations are disregarded, and plane sections are assumed to remain plane.

Lumped Plasticity Models Emerging from series and parallel element representations for beams (38-40), various unilateral beam-column models have attempted to included axial loading-bending moment interaction by idealizing the critical regions by inelastic rotational springs governed by a series of phenomenological rules (41, 42). The portions of the member between these idealized plastic regions are assumed to remain elastic. Typically, the rules included do not account for the effects of axial load on member stiffness in the elastic range. Only a few of these phenomenological models account for the variations in axial stiffness that result from yielding or axial load fluctuations (42).

It is possible to extend such phenomenological rules into two or three dimensions using plasticity theory. While the basic approach used for elasto- plastic materials may be suitable for steel structures and certain ranges of behavior for reinforced concrete structures (36), this tends to ignore the stiffness degradation and pinching typically associated with reinforced concrete columns. Takizawa and Aoyama developed a modified formulation (14) utilizing a trilinear envelop curve and a set of degradation rules for cycling. Constant axial load is assumed. The plasticity model consists of two similar ellipses, representing "cracking" and "yielding" surfaces in a biaxial moment plane (Fig. 5). Since the "cracking" surface is used to control the stiffness changes on initial flexural cracking as well as on unloading from a yielded state, its shape must be empirically defined. Rules for movement, relative sliding and expansion of these two surfaces are based on plasticity theory. As shown in Fig. 1, results obtained correlated very well with overall nature of experimental data (14, 21). However, discrepancies are noted for local details in the hysteretic loops. This is apparently a consequence of the underlying phenomenological model and the assumptions inherent with the plasticity approach. This approach has been extended to more complex situations including varying axial load (13, 21, 43). In this case, phenomenological rules are established for axial force versus elongation and three dimensional plasticity models are employed. Good correlations (Fig. 1) have been reported in recent studies (13, 43).

Another approach to concentrated plasticity models is to employ a simplified physical representation of the critical cross section(s). Although a detailed fiber representation could be used, in most cases a simpler idealization is employed. The model proposed by Lai, Will and Otani (44) uses a five spring idealization for the critical end region (Fig. 6). The four corner springs represent the effective combined stiffness and strength of the steel rebars and concrete in compression and of the steel reinforcement in tension. The fifth center spring is used to represent the effective compressive properties of the concrete in this region. The effective properties of these springs are based in part on the deformations in a bar over its development length into the beam-column joint core. A bilinear, degrading stiffness, phenomenological model is used to represent the cyclic characteristics of these springs. The concrete contribution is modeled by an estimate of the indentation of the joint core concrete in compression and the strength of the concrete. The areas and locations of the effective concrete springs are assigned such that the moments and axial loads predicted match those for the balanced point based on flexural theory. The element has demonstrated good analytical correlation for members with constant axial loads (44) and incorporates many features not possible with other models, such as variation of elastic stiffness with axial loading.

Refinements have been made to these multi-spring elements based on the use of different methods for estimating the effective properties and locations of the springs (10, 12, 45, 46). Recent studies have considered cases of varying axial loads (12) and unsymmetric hysteretic loops of the type observed experimentally were predicted. The model is, however, unable at this stage to account for deterioration of the capacity and spalling of the concrete which can result in overall softening of the member. Nonetheless, the model provides an economical and realistic method for predicting the response of members which having loading and boundary conditions consistent with the development of the controlling analysis parameters and assumptions.

Distributed Plasticity Models To obtain solutions for more general loading conditions or more severe deformation ranges, a more complex approach is needed which monitors response at the stress or section level. A complete finite element approach would be possible, but is often disregarded as being computationally prohibitive. However, in a recent study of uniaxial column behavior under nonproportional unilateral loading, Saadeghaviri and Foutch (9) have employed isoparametric plane stress elements for the concrete and bar elements of the reinforcement. Post-crushing and cracking are modeled to account for the confinement and tension stiffening of the concrete which are not directly accounted for in previously mentioned models. However, extension to bidirectional excitations and inclusion of bond slip and shear cracking effects would substantially increase computational requirements so that this approach might be impracticable for studies of complete structures.

Because of these computational demands, most finite element studies have idealized behavior around the section. In some cases, this has been on the basis of classical plasticity (43), or as shown in Fig. 7, by explicitly refining the section into fibers (Refs. 2, 15, 37, 47-50). A high degree of refinement is generally necessary to approximate the plastification in the critical regions, making such models computationally intensive, especially if fiber section representations are used. Considerable computational economy can be achieve by approximating the distribution of section flexibilities, as introduced in Refs. 47 and 52.

Several recent studies have utilized a flexibility approach to formulate the element tangent stiffness matrix (2, 11, 47, 52). Nonetheless, most formulations follow the standard displacement approach to determine the distribution of deformations along the member (53). These are typically approximated by means of cubic Hermitian polynomials. Very good correlations with experimental results have been obtained for unilateral cases with large variations in axial load (11, 47) and for bilateral excitations (Fig. 8) under constant axial loads (2). For improved representations of internal damage and improved computational efficiency, recent models have introduced variable interpolation functions (2, 47, 51). The fiber representations used in these models allows for a wide variety of physical phenomenon to be accounted for with little need for empirically based analysis parameters. Moreover, such models can provide the user with detailed information regarding response.

When the standard displacement approach is adopted to estimate the internal distribution of deformations, numerical problems have been encountered at or near the point of maximum resistance and the member begins to soften. This has been observed for both section and member models (2, 47, 49, 51, 54). For example, the simple cantilever column shown in Fig. 9 exhibits deformation softening as the concrete at the base begins to spall. In a standard displacement approach for estimating internal deformations, the interior sections have no way of detecting the resulting loss of capacity at the end section. Consequently, they continue to load while the end section sheds load. Thus, equilibrium is violated and results are erroneous or unstable. To correct for this, many studies have adopted unrepresentative material properties, especially for the concrete where elasto-perfectly plastic behavior is often assumed. In such cases, the resulting predictions may be unrealistic, especially if large deformations and damage are expected. A detailed study of the reasons for this unsatisfactory numerical behavior has been recently made (51).

An alternative formulation has been proposed to correct for these numerical problems considering unilateral and bilateral excitations with varying axial loads (2, 51). In this formulation, the tangent stiffness matrix for the member is computed using the flexibility interpolation approach (47, 52). To reduce the number of sections required, flexibilities are assumed to vary linearly between monitored sections. The tangent stiffness is obtained by inversion of the member flexibility matrix. Deformations at the critical end sections are based on variable transformation functions based on the current distribution of flexibilities along the member. To obtain the deformations at internal sections a mixed approach is used. Moments and axial loads at the internal sections are determined by equilibrium considerations. The current state of deformations at these sections is then obtained by iteration on the curvatures and axial strains at the section until the target forces are matched. In this way, equilibrium and stability are preserved. Illustrations of this technique indicate good correlation with experimental data and stability under large deformations as shown in Fig. 9.

Experimental results indicate that a considerable portion of the deformations in a column may be associated with anchorage slip (22, 23, 24). Multi-spring models (44) directly incorporate this source of deformation in their phenomenological rules. In the case of distributed plasticity models, fixed end rotations associated with bond slip in the anchorage regions need be treated with special elements devised for this purpose (2). Figure 10 shows a case where such springs are employed at the base of a simple cantilever column. The response is somewhat softer, as expected, when the bond slip is included. Significantly, for this example, the mode of failure is also different since the bond slip is sufficient to prevent crushing of the concrete at the deformation levels considered.

<u>Short Columns</u> The complexities of the inelastic cyclic behavior of short columns dominated by shear makes precise modeling much more difficult. Pinched and degrading hysteretic rules have been proposed on a phenomenological basis. A conceptual model for predicting deformations has been proposed and has led to some success (32). In addition analytical procedures for estimating the behavioral modes of short columns and their shear capacities have been proposed (30-32).

<u>Other Analysis Approaches</u> Little work has been done to assess the effects of initial conditions associated with aging on the behavior of concrete columns. For static loading conditions, several analytical studies have focused on creep and shrinkage effects on performance (e.g., Ref. 37). While this may not have a profound effect on the seismic response of many structures, it may influence dynamic characteristics as well as the distribution and intensity of damage. Similarly, few studies have examined the effect of

slenderness on column behavior. Studies for static loading (e.g., Ref. 55) have not been extended into the dynamic range. Slenderness effects may have a significant effect on the performance of structures in which columns are subjected to high fluctuations in axial loads due to overturning moments or vertical ground accelerations (9). An assessment of realistic boundary conditions for determining slenderness effects in actual structures would be valuable.

While a variety of analysis procedures have been developed, the limitations and capabilities of these models have not been fully assessed. In part, the analytical models are capable of providing the user with detailed information about the local stress/strain state and damage in the members analyzed. Typically, detailed experimental information of this type is not available for correlation with analytical results. Thus, an integrated approach is needed in which more refined information is obtained in experiments and the analysis results are used to give insight into critical loading conditions for use in formulating experimental programs. A systematic study of the various analytical models available is desirable. Clearly, each of the various approaches will have advantages in certain situations. However, the limits of the ranges of applicability of the models have not been established. These types of studies will also suggest improvements in the modeling procedures, and possibly enable substantial simplifications to be made in some circumstances. Models should be able to reflect the softening behavior associated with spalling or disintegration of the concrete cover. Additional work is needed to devise numerically efficient and stable methods for predicting this behavior. The work in Ref. 51, while providing a practicable solution, does not address the larger theoretical problems related to nonproportional deformations, softening and numerical instability. Similarly, theoretical research is needed to improve our capabilities for predicting response as influenced by shear and bond slip as well as the initial conditions associated with creep and shrinkage. As these models are verified, they should be implemented in general purpose computer programs to enable researchers and designers to assess the effects of bidirectional column behavior on the overall performance of structures.

EFFECTS ON STRUCTURAL BEHAVIOR

While the effect of column behavior on structural performance is treated in Ref. 6, it is useful to look at a few implications of the observed and predicted performance of bilaterally loaded columns. The complex hysteretic relations observed are likely to have significant effects on structural response. For example, consider a simple cantilever column with a constant axial load (about 60% of balanced). The section considered is rectangular. If this column is subjected to an imposed displacement which is skewed with respect to the principal axes of the section, the response is initially, as expected, oriented along the direction of deformation. However, as the base of the column begins to yield, the moment paths in the two directions no longer follow the direction of the imposed tip displacement, as shown in the moment plane projection in Fig. 11. As a result of spalling the moment path diverges quickly form the theoretical interaction surface. The deterioration in strength is preferentially oriented toward the nearest principal axis. This is a consequence of the distribution of damage occurring in the base section. While not shown here (2), such behavior has a complex influence on dynamic response. Simple sinusoidal force excitations at the top of such a cantilever, for example, produce complex and offset elliptical orbits. Thus, skewed unidirectional excitations will produce bidirectional response in the inelastic range.

The dynamic response of the roof of a two-story concrete frame is shown in Fig 12 (2). For the Taft record scaled to produce 60%g in the larger component, the orbital motion of the roof to bidirectional excitations is complex. For this structure, the maximum response obtained in each direction is slightly less than that obtained in that direction if only unidirectional excitation is considered. However, the maximums occur bilateral response occurs almost simultaneously in the two principal directions so that the maximum amplitude of response is significantly worse for the bilaterally loaded case. More significantly, the damage to the columns as inferred from the analysis is also considerably worse. For example, Fig. 13 shows for the bidirectionally excited structure the strain history for one of the base steel reinforcing bars in a column as well as the local energy dissipation history for that column's hinge region. For comparison, the maximum values obtained for a comparable unidirectional analysis is shown as well. This comparison indicates that damage and energy dissipation demands can be substantially larger for biaxially loaded structures than for unidirectionally loaded ones. Additional studies of the implications of the biaxial behavior of columns are needed (see Ref. 6). These examples do indicate that current modeling techniques are capable of providing the user with detailed and meaningful information regarding global response and local damage.

CONCLUSIONS

Recent experimental and analytical research has led to substantial improvements in our understanding of the performance of columns subjected to bilateral flexure and constant or varying axial loads. However, the behavior is quite complex, and influenced by a wide variety of parameters, including the loading history, the boundary conditions, the shape, proportioning and detailing of the column. Results indicate that the biaxial response of columns can be substantially different than that observed for uniaxial bending. Variation of axial load leads to even greater differences. In many cases, in comparison with unilaterally loaded columns, bilateral loading results in a lower stiffness and strength, a more rapid deterioration in strength and stiffness, and greater damage. The consequences of this on design must be addressed by integrated analytical and experimental research as indicated above for individual columns as well as for complete structural systems. Analytical models are capable of providing detailed information on global as well as local response. However, the capabilities and limitations of various methods remain to be fully assessed. On-going developments in computer software and hardware will permit these techniques to be utilized in the design of complex structures and in the development of more reliable design procedures.

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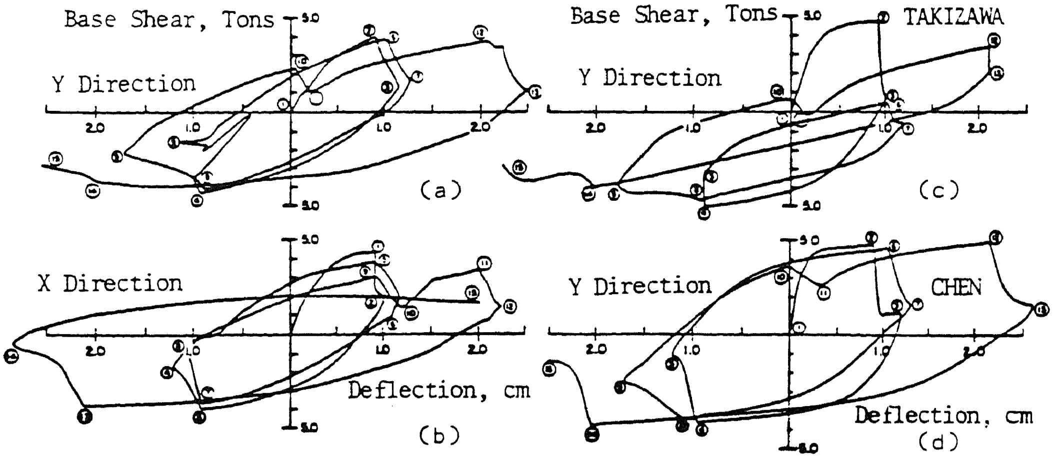


Fig. 1 Comparison of Results for a Bilateral Square Loading Pattern: (a and b) Experimental Results (14) and (c and d) Analytical Results (14, 43).

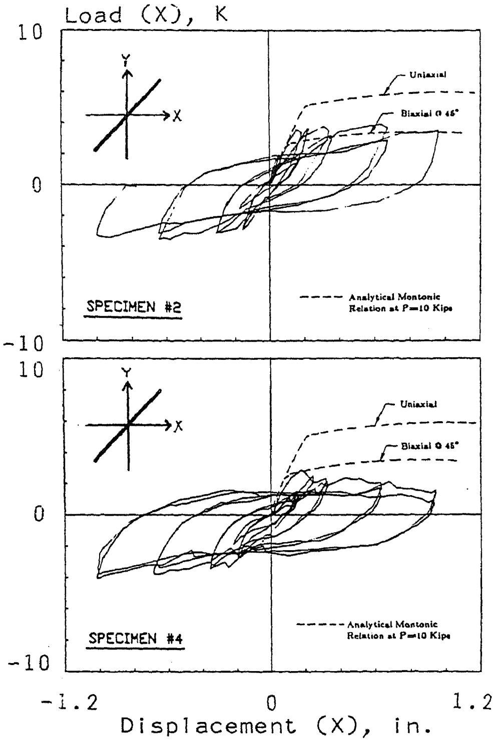


Fig.2 Effect of Skewed Loading (24):
(a) Load equals 10 kips, and
(b) Load averages 10 kips.

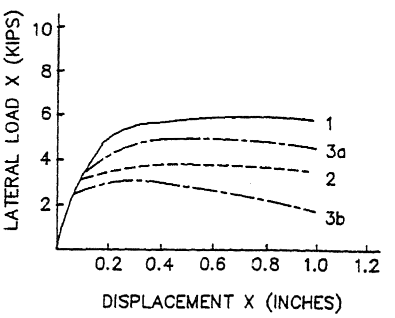


Fig.3 Comparison of Loading Envelopes (24).

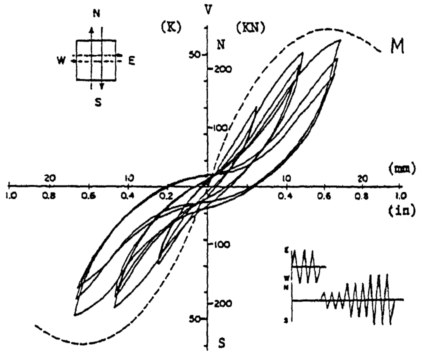


Fig.4 Behavior of Short Columns (30).

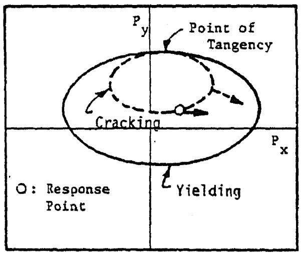


Fig.5 Sliding of Cracking Ellipse along Yielding Ellipse in Plasticity Model (14, 21).

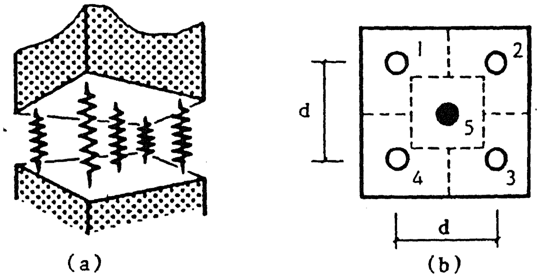
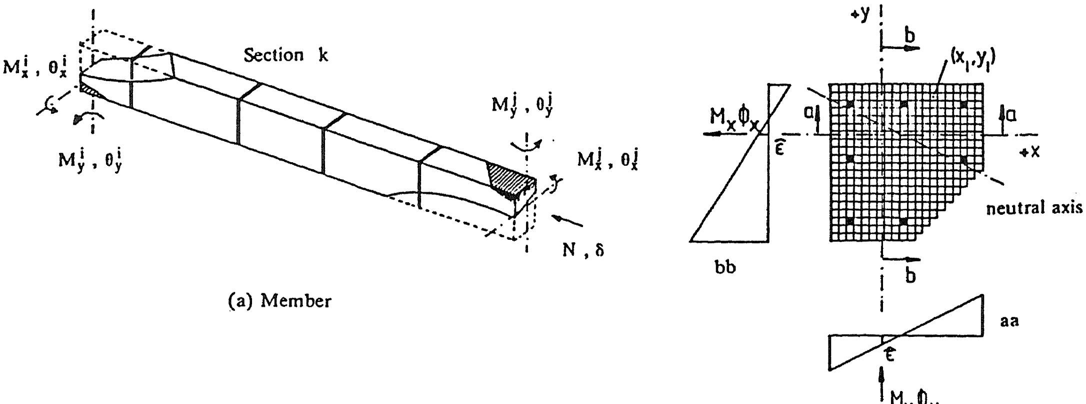


Fig.6 Multi-Spring Model (44,12)



(a) Member

(b) Section

Fig. 7 Stress-Strain Based Fiber Model (2,47,51)

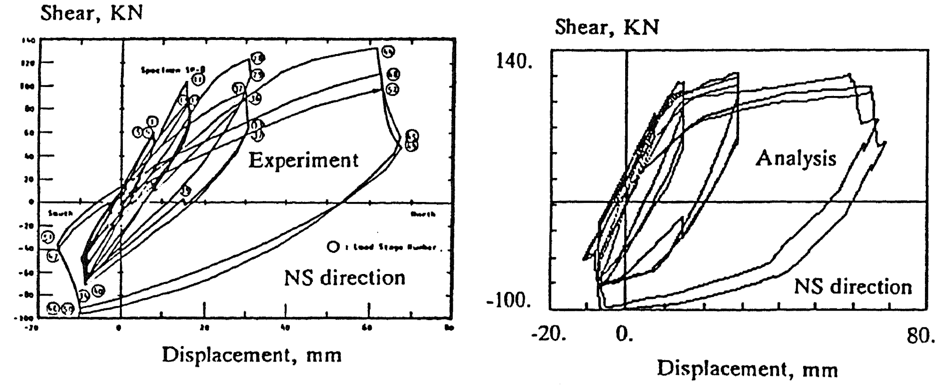


Fig.8 Comparison of Experimental (22) and Analytical (2) Results.

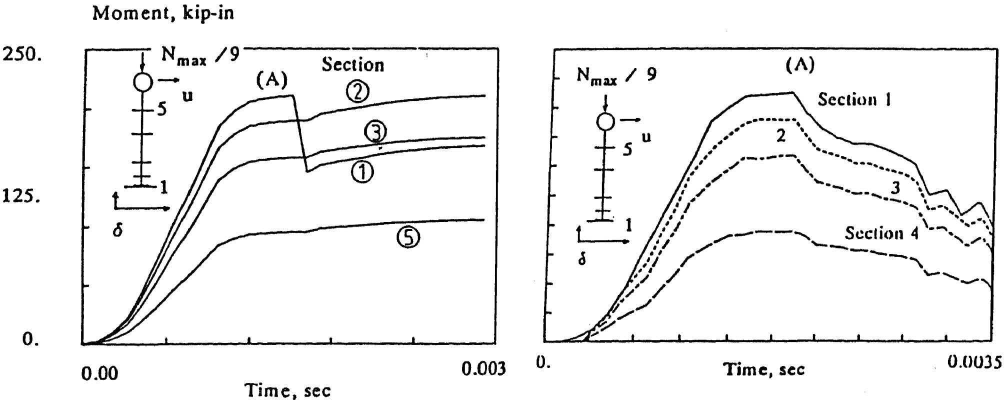


Fig.9 Numerical Problems with FE Models Under Softening Response: (a) Standard Displacement Approach and (b) Improved Mixed Force/Displacement Approach (51).

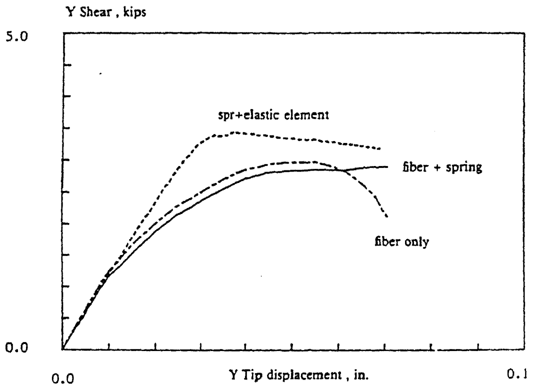


Fig.10 Effect of Fixed End Rotation Spring on Fiber Model Results (2).

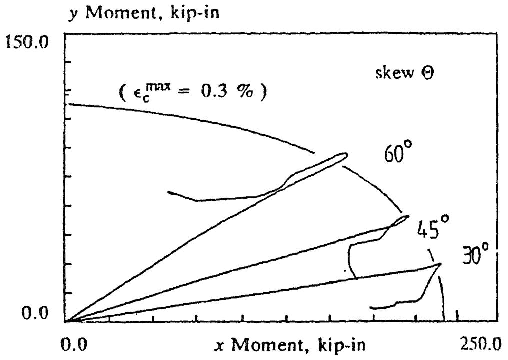


Fig.11 Biaxial Moment Paths For Constant Displacement Inclination Angle (2).

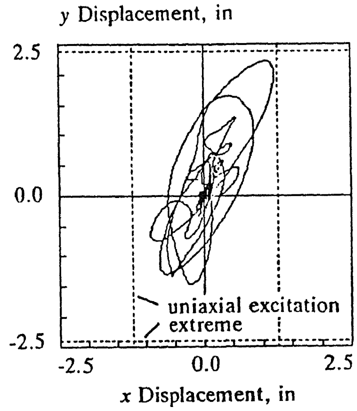


Fig.12 Predicted Particle Motion at Roof of Two Story RC Frame -- Centroid (2)

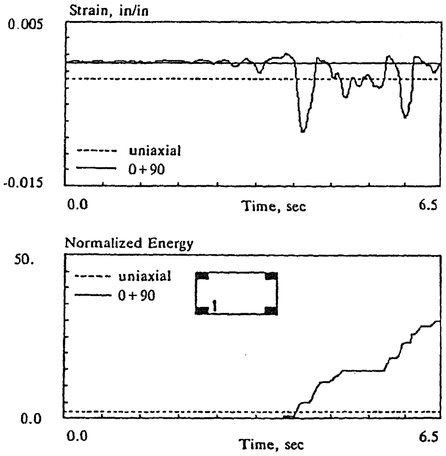


Fig.13 Predicted Steel Strain and Local Energy Dissipation Demands for a Column (2).