Proceedings of Ninth World Conference on Earthquake Engineering
August 2-9, 1988, Tokyo-Kyoto, JAPAN (Vol.VIII)

SE-15

A QUASI-ANALYTICAL METHOD FOR STRUCTURES WITH ACTIVE CONTROL SYSTEMS

Shinji ISHIMARU[1] and Takahiro NIIYA[1]

[1] College of Sience & Technology, Nihon University,
Chiyoda-ku, Tokyo, Japan

SUMMARY

This paper presents a method to estimate both the response values and optimal control forces of structures with active control devices. A combination of the modal viscous damping factors is selected as the performance criterion of seismic response reduction. The maximum control force is computed as the function with respect to the natural frequencies by making use of both the pole assignment method and a response spectrum procedure, which control force is optimized by a non-linear programming on the basis of the quasi-Newton methods. The proposed method is applied to a MDOF structure. The effectiveness and reliabilty are verified by time-history analyses.

INTRODUCTION

In an engineering sense, it is desirable to be able to judge the effectiveness of a designed control mechanism and the required control force at the early stage of design practice. From the viewpoint, this paper presents a method to evaluate both response values and demanded control energy by making use of the method which combines the pole-assignment procedure with a response spectrum approach and a quasi-Newton method.

Originally, an aim to control structures is to reduce seismic response values of structures by giving control energy as small as possible. One alternative to achieve this purpose is to realize the structures with high damping capacity which are generated by making use of both control mechanisms and control forces. Considering from this viewpoint, the simplest performance criterion of seismic response reduction is to define a combination of the modal viscous damping factors. The required control force can be accordingly estimated from the applied response spectra of velocities and displacements under different viscous damping factors, when the natural frequencies are assumed. Therefore, the feed back gains of the system can be optimized by minimizing the square value of the control force which is the function with respect to the natural frequencies. The applied numerical method for optimazation is a non-linear programming on the basis of the quasi-Newton methods. Consequently, we found that the presented method is useful for judging the effectiveness of a designed control mechanism from both these mode shapes with real numbers and the applied response spectra.

MATHEMATICAL FORMULATION

<u>A Single Degree of Freedom System</u> A SDOF system is introduced to demonstrate the

conceptual flow. The state space description is given by

$$\dot{\mathbf{q}} = \mathbf{A}\,\mathbf{q} + \mathbf{b}\,(f/m_o) - \mathbf{i}\,\ddot{g} \tag{1}$$

$$\mathbf{q} = \begin{Bmatrix} v \\ d \end{Bmatrix} \quad \mathbf{A} = \begin{bmatrix} -2h\omega, & -\omega^2 \\ 1 & \end{bmatrix} \quad \mathbf{b} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \quad \mathbf{i} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \tag{2}$$

in which, d is the displacement of the system, v the velocity, m_o the total mass of the structure, g the ground displacement, h and ω the viscous damping factor and the natural circular frequency, respectively. The symbols **b** and **i** indicate the location vectors of control force f and excitation, respectively. According to the pole assignment method, the control force is determined by the inner product of the feed back gains vector **p** and the state variables vector **q** which is defined as follows.

$$(f/m_o) = \mathbf{p}^T\mathbf{q} = [\ -2h\omega + 2h_c\omega_c, -\omega^2 + (\omega_c)^2]\ \mathbf{q} \tag{3}$$

Substituting Eqs.(2) and (3) into Eq.(1), then Eqs.(4) and (5) are obtained.

$$\dot{\mathbf{q}} = \mathbf{\Lambda}_c\mathbf{q} - \mathbf{i}\,\ddot{g} \tag{4}$$

$$\mathbf{\Lambda}_c = \begin{bmatrix} -2h_c\omega_c, & -(\omega_c)^2 \\ 1\ , & \end{bmatrix} \tag{5}$$

Thus, the properties of the controlled structure vary from ω to ω_c in the natural frequencies and from h to h_c in the viscous damping factors. In the next place, the square value of control force is computed as follows.

$$J = (f/m_o)^2 = \mathbf{p}^T\mathbf{q}\ \mathbf{q}^T\mathbf{p} \tag{6}$$

The expectation of Eq.(6) is given by

$$E(J) = \mathbf{p}^T E[\mathbf{q}\ \mathbf{q}^T]\mathbf{p} = \mathbf{p}^T \begin{bmatrix} E_{vv}, & E_{vx} \\ E_{xv}, & E_{xx} \end{bmatrix} \mathbf{p} = \mathbf{p}^T \begin{bmatrix} \sqrt{E_{vv}} & \\ & \sqrt{E_{xx}} \end{bmatrix} [\,\rho\,] \begin{bmatrix} \sqrt{E_{vv}} & \\ & \sqrt{E_{xx}} \end{bmatrix} \mathbf{p} \tag{7}$$

in which $[\,\rho\,]$ is the coefficient matrix of correlation.

$$[\,\rho\,] = \begin{bmatrix} 1 & \rho_{vx} \\ \rho_{vx} & 1 \end{bmatrix} \qquad \rho_{vx} = \frac{E_{vx}}{\sqrt{E_{vv}\ E_{xx}}} \tag{8}$$

The coefficients ρ_{vx} are computed from the random process theory of the structures subjected to the white-noise. In addition, it was pointed out that the maximum response values v_{max} and x_{max} are approximately proportional to the standard deviations $\sqrt{E_{vv}}$ and $\sqrt{E_{xx}}$, respectively(Ref.1), and hence the maximum value of Eq.(7) can be approximated by

$$F = E(J_{max}) = \mathbf{p}^T \begin{bmatrix} v_{max} & \\ & x_{max} \end{bmatrix} [\,\rho\,] \begin{bmatrix} v_{max} & \\ & x_{max} \end{bmatrix} \mathbf{p} \tag{9}$$

Thus, the maximum response values and the control force can be estimated from the response spectra of a selected ground motion. Since the viscous damping factor is used as the performance criterion of response reduction, the optimazation of the objective function of Eq.(9), which is accordingly assumed as the function of frequency, is carried out by using the Newton method. This approach requires the differentials of the first order and the second one of the function **F** with respect to the variable ω_c, including the derivatives of the maximum response values. For this reason, the spline functions of the third order(Ref.2) are used to provide the continuous functions of response spectra which are computed from the same discrete response values as ordinary response spectra.

<u>A Multi-Degree-of-Freedom System</u> A state equation of an n-degrees of freedom structure can be rewritten by substituting the following terms defined by Eq.(10) into Eq.(1) instead of the terms **q**, **A**, **b** and **i** of Eq.(2).

$$\mathbf{x} = \begin{Bmatrix} \mathbf{v} \\ \mathbf{d} \end{Bmatrix} \quad \mathbf{A} = \begin{bmatrix} -\mathbf{M}^{-1}\mathbf{C}\ , & -\mathbf{M}^{-1}\mathbf{K} \\ \mathbf{I}\ , & \mathbf{0} \end{bmatrix} \quad \mathbf{b} = \begin{Bmatrix} m_o\mathbf{M}^{-1}\mathbf{b}' \\ \mathbf{0} \end{Bmatrix} \quad \mathbf{i} = \begin{Bmatrix} \mathbf{i}' \\ \mathbf{0} \end{Bmatrix} \tag{10}$$

where, $\mathbf{v} = \dot{\mathbf{d}}$, and **M**, **C** and **K** represent the mass, viscous damping and stiffness matrices. The symbols **b** and **i** are the location vectors of the control force and excitations. Now, if the state variables vector **x** is assumed by

$$\mathbf{x} = \mathbf{T}\ \mathbf{R}[\diagdown \phi \diagdown]\,\mathbf{z} \tag{11}$$

, the state equation can be rewritten as follows.

$$\mathbf{R}\lceil\phi\rfloor\dot{\mathbf{z}} = \mathbf{T}^{-1}\mathbf{A}\,\mathbf{T}\,\mathbf{R}\lceil\phi\rfloor\mathbf{z} + \mathbf{T}^{-1}\mathbf{b}\,(f/m_o) - \mathbf{T}^{-1}\mathbf{i}\,\ddot{g} \tag{12}$$

in which the similar transformation $\mathbf{T}^{-1}\mathbf{A}\,\mathbf{T}$ is carried out under the condition that the 2n-th column vector of $\mathbf{T}$ satisfies Eq.(13), in order to provide the companion matrix which has the same form as Eq.(17) with the coefficients a_m instead of the elements b_m. And the control force factor (f/m_o) is assigned by Eqs.(14) and (15).

$$[\mathbf{T}^{-1}\mathbf{b}]^{T} = [\,0, 0, 0, \ldots\ldots, 0, 1] \tag{13}$$

$$(f/m_o) = \mathbf{p}^T\mathbf{x} = \mathbf{p}^T\mathbf{T}\,\mathbf{R}\lceil\phi\rfloor\mathbf{z} \tag{14}$$

$$\mathbf{p}^T\mathbf{T} = [\,a_1-b_1, a_2-b_2, a_3-b_3, \ldots\ldots, a_{2n}-b_{2n}]\ \mathbf{x} \tag{15}$$

Substituting from Eq.(13) to Eq.(15) into Eq.(12) and assuming that the similar transformation $\mathbf{T}^{-1}\mathbf{A}\,\mathbf{T}$ has the companion form, Eqs.(16) and (17) are given.

$$\lceil\phi\rfloor\dot{\mathbf{z}} = \mathbf{R}^{-1}\mathbf{\Lambda}_z\,\mathbf{R}\lceil\phi\rfloor\mathbf{z} + \mathbf{R}^{-1}\mathbf{T}^{-1}\mathbf{i}\,g \tag{16}$$

$$\mathbf{\Lambda}_z = [\mathbf{T}^{-1}\mathbf{A}\,\mathbf{T} + \mathbf{T}^{-1}\mathbf{b}\,\mathbf{p}^T\mathbf{T}\,] = \begin{bmatrix} & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \\ -b_1, & -b_2, & -b_3, & \ldots\ldots, & -b_{2n} \end{bmatrix} \tag{17}$$

Futhermore, the shape mode matrix $\mathbf{R}$ and the above companion matrix $\mathbf{\Lambda}_z$ satisfy the following relationships.

$$\mathbf{\Lambda} = \mathbf{R}^{-1}\mathbf{\Lambda}_z\,\mathbf{R} = \mathrm{diag.}\,[\lambda_1, \bar{\lambda}_1, \lambda_2, \bar{\lambda}_2, \cdots\cdots, \lambda_n, \bar{\lambda}_n] \tag{18}$$

$$\lambda_m = -h_m\omega_m + i\cdot\omega_m\sqrt{1-h_m^2}\ \cdot\ \bar{\lambda}_m = -h_m\omega_m - i\cdot\omega_m\sqrt{1-h_m^2} \tag{19}$$

where h_m and ω_m are the assigned viscous damping factor and circular frequency of the m-th mode. It is needless to say that the coefficients b_m of the characterestic equation of the controlled structure are determined from the assigned pole-values. And $\mathbf{R}$ has the form of Vandermonde matrix.

$$\mathbf{R} = \begin{bmatrix} \lambda_1, & \bar{\lambda}_1, & \lambda_2, & \bar{\lambda}_2, & \cdots\cdots, & \lambda_n, & \bar{\lambda}_n \\ \lambda_1^2, & \bar{\lambda}_1^2, & \lambda_2^2, & \bar{\lambda}_2^2, & \cdots\cdots, & \lambda_n^2, & \bar{\lambda}_n^2 \\ \cdot & \cdot & \cdot & \cdot & \cdots\cdots & \cdot & \cdot \\ \lambda_1^{2n}, & \bar{\lambda}_1^{2n}, & \lambda_2^{2n}, & \bar{\lambda}_2^{2n}, & \cdots\cdots, & \lambda_n^{2n}, & \bar{\lambda}_n^{2n} \end{bmatrix} \tag{20}$$

In the next place, the diagonal matrix $\lceil\phi\rfloor$ is defined as follows.

$$\mathbf{R}^{-1}\mathbf{T}^{-1}\mathbf{i} = \lceil\phi\rfloor\mathbf{1}, \qquad \mathbf{1}^T = [1, 1, 1, \cdots\cdots, 1, 1] \tag{21}$$

The following equation can thus be obtained.

$$\dot{\mathbf{z}} = \mathbf{\Lambda}\,\mathbf{z} + \mathbf{1}\,\ddot{g} \tag{22}$$

However, it is difficult to figure out the total image of the behavior of the system from the participation functions $\mathbf{TR}\lceil\phi\rfloor$, since the mode shape matrix is composed of complex numbers. In order to improve this, the transformation of Eq.(23) is introduced, and Eq.(24) is derived as a result.

$$\mathbf{z} = \mathbf{D}^{-1}\mathbf{q} \tag{23}$$

$$\dot{\mathbf{q}} = \mathbf{D}\,\mathbf{\Lambda}\,\mathbf{D}^{-1}\mathbf{q} + \mathbf{D}\,\mathbf{1}\,\ddot{g} \tag{24}$$

in which the matrix $\mathbf{D}$ is defined as follows.

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_1 & & & \\ & \mathbf{D}_2 & & \\ & & \ddots & \\ & & & \mathbf{D}_n \end{bmatrix} \qquad \mathbf{D}_m = \frac{1}{\lambda_m - \bar{\lambda}_m}\begin{bmatrix} \lambda_m, & -\bar{\lambda}_m \\ 1, & -1 \end{bmatrix} \qquad \mathbf{D}^{-1} = \begin{bmatrix} 1, & -\bar{\lambda}_m \\ 1, & -\lambda_m \end{bmatrix} \tag{25}$$

The transformation $\mathbf{D}\,\mathbf{\Lambda}\,\mathbf{D}^{-1}$ leads to the canonical form.

$$\mathbf{D}\,\mathbf{\Lambda}\,\mathbf{D}^{-1} = \begin{bmatrix} \mathbf{\Lambda}_1 & & & \\ & \mathbf{\Lambda}_2 & & \\ & & \ddots & \\ & & & \mathbf{\Lambda}_n \end{bmatrix} \qquad \mathbf{\Lambda}_m = \begin{bmatrix} -2h_m\omega_m, & -\omega_m^2 \\ 1, & \end{bmatrix} \tag{26}$$

Furthermore, the following relation is also obtained from the definition of $\mathbf{D}$.

$$[\mathbf{D}\,\mathbf{1}]^{T} = [1, 0, 1, 0, \cdots\cdots, 1, 0] = \mathbf{1}_D^{\,T} \tag{27}$$

Considering the definition of $\mathbf{D}$, it is evident that $\mathbf{R}\,\mathbf{D}^{-1}$ becomes the matrix with real numbers. For this reason, the transformation introduced by

$$\mathbf{x} = \mathbf{T}\,[\mathbf{R}\,\mathbf{D}^{-1}]\ [\mathbf{D}[\ulcorner \phi \lrcorner]\mathbf{D}^{-1}]\ \mathbf{q} = \mathbf{R}_0\mathbf{q} \tag{28}$$

gives the advantage that the computations can be carried out with real numbers only. The required control force is consequently represented by substituting Eq.(28) into the term $\mathbf{x}$ of Eq.(14), and the expectation of its square value becomes the following objective function $\mathbf{F}$.

$$\mathbf{F} = E(J_{max}) = E\,[\mathbf{p}^T\mathbf{x}\mathbf{x}^T\mathbf{p}] = \mathbf{p}^T\mathbf{R}_0\ E\,[\mathbf{q}\mathbf{q}^T]\ \mathbf{R}_0{}^T\mathbf{p} \tag{29}$$

As stated before, since the viscous damping factors of the modal coordinates are fixed as the mesure of response reduction, the optimal modal frequencies for the objective function $\mathbf{F}$ are obtained by the following recurrence formula.

$$\boldsymbol{\omega}_c|_{n+1} = \boldsymbol{\omega}_c|_n - [\nabla^2\mathbf{F}(\boldsymbol{\omega}_c|_n)]^{-1}\,[\nabla\mathbf{F}(\boldsymbol{\omega}_c|_n)] \tag{30}$$

In order to obtain the inverse matrix of the Hesse matrix $\nabla^2\mathbf{F}(\boldsymbol{\omega}_c|_n)$, the Broydon-Fletcher-Goldfarb-Shanno method is applied, which is a typical procedure among the quasi-Newton methods(Ref.3). Furthermore, the gradient vector $\nabla\mathbf{F}(\boldsymbol{\omega}_c|_n)$ can be analytically computed, because the matrix of mode shapes is explicitly expressed in terms of frequencies.

NUMERICAL EXAMPLES

The objectives of this section is to verify the reliability of the procedure andto examine the dependence of the derived feed back gains on the properties of applied ground motions. From the viewpoint, the presented method is examined by applying this to a three-story structure subjected to the following ground motions: A = the El Centro acceleration record(Imperial Valley Earthquake, 1940, N-S, 340 gal), B = the Hachinohe Harbor one(Tokachi-oki Earthquake, 1968, N-S, 225 gal) and C = the Tohoku University one (Miyagi-ken-oki Earthquake, 1978, N-S, 258gal). Figure 1 shows the spectra of response velocities with the viscous damping factor of h=0.25 under the applied accelerograms.

The sample srtructure is modeled on the basis of the philosophy that active controlled structures should be mainly composed of passive controllers, partly installed with actuators, furthermore the active controllers should be designed to make full use of damping capacity of passive controllers by optimizing the feed back gains. The proportion of the structure is accordingly described as follows; the weights of all stories are 100 tonf, the distribution of spring constants is 3 tf/cm, 5 tf/cm and 6 tf/cm in order from the uppermost story. The mathematical model of the control mechanisms is given in Fig.2, like the inverted pendulum, where the levers are attached auxiliary masses at the one ends and pinned to the floors at the other ends as the fulcurm(pin-support). However, the actuator is only installed at between the 2nd mass and the 3rd one. Fig.3 shows the practical mechanism which has several link mechanisms attached to the lever, in order to eliminate the unexpected effect due to the arc-motions of the lever which gives the disadvantage of geometrical nonlinear behavior to the control actuator. More detailed description of the theoretical background and the experimental results of the SDOF structure with the similar mechanism can be found in Ref.4. Any way, the matrices $\mathbf{M}$, $\mathbf{C}$, $\mathbf{b}'$ and $\mathbf{i}'$ are written as follows.

$$\mathbf{M} = \begin{bmatrix} a_1, & a_2 & \\ & a_3, & a_2 \\ sym & & a_3 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} b_1, - & b_1 & \\ & b_2, - & b_1 \\ sym & & b_2 \end{bmatrix} \quad \mathbf{b}' = \begin{Bmatrix} 1 \\ -1 \\ 0 \end{Bmatrix} \quad \mathbf{i}' = \begin{Bmatrix} i_1 \\ i_2 \\ i_2 \end{Bmatrix} \tag{31}$$

$$\begin{aligned} a_1 &= m + \beta^2 m_d,\ a_2 = -\beta(\beta-1)m_d,\ a_3 = m + (\beta-1)^2 m_d + \beta^2 m_d \\ b_1 &= (\beta-1)^2 c,\ b_2 = 2(\beta-1)^2 c,\ i_1 = m + \beta m_d,\ i_2 = m - (\beta-1)m_d + \beta m_d \end{aligned} \tag{32}$$

where the lever ratios β = b/a are 7.0 (see Fig.2), the weights of auxiliary masses m_d are 0.01 times the ones of the floor masses m and the values of 0.01tf・sec/cm are assigned to the viscous damping constants of c in order to equivalently convert the effects of the friction between the walls and the auxiliary masses.

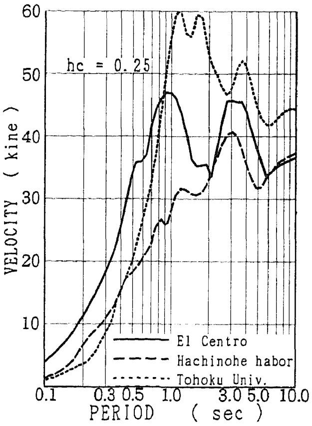

Fig. 1 Response spectra

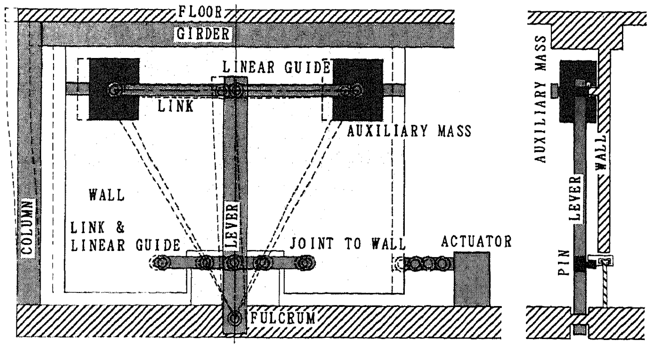

Fig. 3 Illustration of the practical mechanism. Several link mechanisms are installed to elliminate unexpected effect due to arc-motion of the lever.

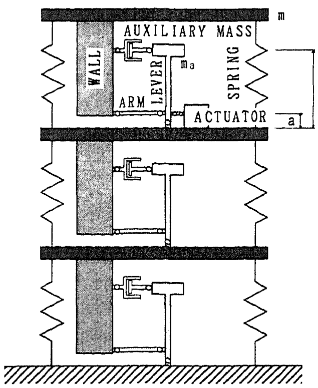

Fig. 2 Mathematical model of the sample structure.

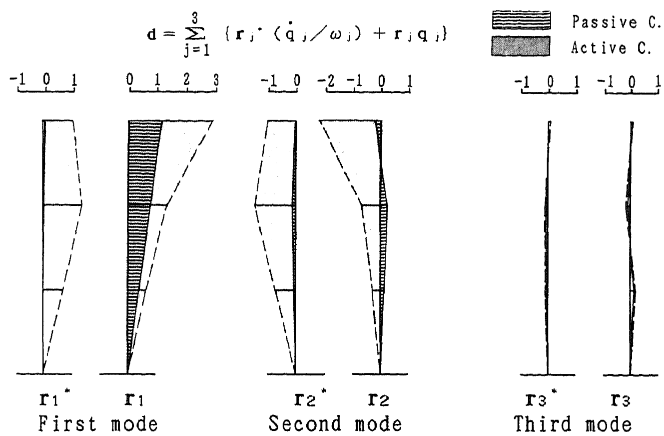

Fig. 4 Comparison between the participation functions of the passive controlled system and the ones of the active controlled structure for the case A1.

Table 1. Properties of the structure

Model	Viscous damping factor			Natural frequency [rad]		
	1st	2nd	3rd	1st	2nd	3rd
Uncontrolled model	0.032	0.077	0.122	3.16	7.75	12.25
Passive controlled model	0.117	0.302	0.286	2.97	5.89	8.07

Table 2. Optimaized results

	Optimal feed back gains $\mathbf{P}^T$						Control F.	Natural Freq. [rad]		
	v_3	v_2	v_1	d_3	d_2	d_1	f/m_0 [Gal]	1-st	2-nd	3-rd
A1	-0.106	-0.075	0.251	-2.274	2.302	1.805	21.6(24.1)	2.88	3.00	8.05
B1	-0.092	-0.073	0.247	-2.213	2.228	1.742	18.4(20.7)	2.92	3.08	8.05
C1	-0.107	-0.075	0.250	-2.277	2.278	1.864	25.4(39.1)	2.88	2.99	8.06
A2	-0.098	0.204	0.290	-1.325	2.344	-0.370	39.1(57.0)	2.97	4.46	7.95
B2	-0.127	0.177	0.301	-1.535	2.463	-0.006	30.0(48.2)	2.97	4.16	7.97
C2	-0.246	0.193	0.243	-2.380	7.368	-7.395	17.1(11.8)	3.05	2.92	7.20

Table 3. Comparision between Response results

	Response Velocities			Response Displacements			C. E.
	v_3 [cm/sec]	v_2 [cm/sec]	v_1 [cm/sec]	d_3 [cm]	d_2 [cm]	d_1 [cm]	[W/tf]
A1	53.9(50.0)	44.1(44.6)	27.0(24.6)	17.5(17.3)	13.1(13.1)	6.9(7.1)	50.9
B1	43.3(37.9)	40.4(35.8)	23.2(22.0)	14.7(13.5)	11.1(9.7)	5.8(5.0)	27.5
C1	64.8(70.7)	56.1(59.8)	32.3(30.7)	15.1(15.9)	12.8(14.4)	6.9(7.3)	78.7
A2	32.7(31.5)	43.8(50.4)	26.3(27.7)	11.3(11.0)	12.1(12.0)	6.6(6.6)	197.6
B2	31.8(32.9)	35.8(35.8)	20.2(20.7)	8.5(8.6)	9.4(9.3)	5.1(4.6)	66.4
C2	75.6(79.8)	55.8(54.8)	33.7(27.8)	17.7(18.4)	12.4(12.7)	6.8(6.5)	35.8

The eigen values of the passive controlled structure, as listed in Table1, vary from 3.16 rad. to 2.97 rad. in the first natural frequency, from 3% to 11.7% in the viscous damping factor of the first mode, when the controllers without an actuator are installed at the all stories. Table 2 indicates the results for the optimal feed back gains of the active controller, when the two combinations of the modal damping factors as the performance criteria are assigned, i.e, the first is 25%, 35% and 30% in order from the first mode(which is expressed with the suffix of '1'), the second 25%,10% and 30% in the same order(which is expressed with the suffix of '2'). The required control forces are also listed in Table 2, in which the values in the parentheses are the results computed from the time history analyses for the controlled structures with the obtained feed back gains. The mode shapes normalized by the particapation factors are drawn in Fig.4, where the symbols $\mathbf{r}_j'$ and $\mathbf{r}_j$ express the vectors with respect to the ratio of the response velocity of the j-th mode to the correspond-ing natural circular frequency and the corresponding response displacement, respectively. In the figure, the solid lines indicate the results for the passive system and the dashed lines for the case of A1. Table 3 compares the response results obtained from this procedure with the ones (values in parentheses) derived from the time history analyses. The required control energy ratios(watt/tonf) is also listed in the right-hand side of this Table.

CONCLUSION

Based on the numerical results, the following conclusions were derived.

1)The estimated response values are in good agreement with the results obtained from the time history analyses(Table-3). On the other hand, almost all the evaluated control forces are slightly smaller than the ones derived from the simulations(Table-2), which are probably caused by making use of the values of correlation based on the white-noise not the applied ground motions.

2)Considering that the mode shapes of active controlled structures are remarkably changed owing to the combinations of the modal viscous damping factors(Fig.4), the presented method is very useful for developing a new control mechanism, because we can easily track the changes of required control forces from mode configurations with real numbers and natural frequencies.

However, it is needless to say that we have to statistically proceed to the further investigation, including the research on what kinds of combinations of modal viscous damping factors shall be optimal under the specified spectra.

REFFERENCES

1. E. L. Wilson, et al. : A Replacement for SRSS Method in Seismic Analysis, Dynamic Analysis of Soil/Structure Seminor 1981, in Japan
2. K. Ichida and F.Yoshimoto:Spline Function and Its Application, Kyoiku Ssuppan(1979)
3. D. F. Shanno:Conditioning of Quasi-Newton Methods for Function MInimizations, Math. Comp 24, pp 641-656(1970)
4. S. Ishimaru:Feasibility of Active Controlled Structures, The U.S.A.-Japan Seminar on Seismic Structure Engineering and Natural Hazard Reduction, at the College of Technology & Science, Nihon University, 2nd August 1988