## SE-12

# RESPONSE CONTROL BASED ON STRUCTURAL OPTIMIZATION AND ITS COMBINATION WITH ACTIVE PROTECTION

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#### SUMMARY

This paper describes some recent developments at UMR in the areas of structural optimization and the combination of optimum design with active control algorithms. The structural optimization has been developed for designing economical and serviceable building structures with consideration of deterministic or nondeterministic resistance and response. The structural optimization technique is further developed to be combined with optimal control for active protections of tendon devices or mass-dampers or a combination of them for which the buildings are subjected to earthquake excitations.

#### INTRODUCTION

The optimum design concept has been recognized as being more rational and reliable than those that require the conventional trial and error process (Ref. 1). It is because for a given set of constraints, such as allowable stresses, displacements, drifts, frequencies, upper and lower bounds of member sizes, and given seismic loads, such as equivalent forces in the code provisions, spectra, or time-histories, the stiffnesses of members are automatically selected through the mathematical logic (structural synthesis) written in the computer program. Consequently, the strengths of the constituent members are uniformly distributed, and the rigidity of every component can uniquely satisfy the demands of the external loads and the code requirements, such as displacements and drifts. By using an optimum design computer program, one can conduct a project schedule at a high speed and thus increase the benefit because of the time that is saved. An optimum design program can also be used for parametric studies to identify which structural system is more economical and serviceable than the other and assess the principles of various building code provisions as to whether they are as logical as they are intended to be (Refs. 2,3,4,5,6).

Structural control is achieved by using passive or active control devices. The passive devices utilize the fact that energy dissipating mechanisms can be activated by the motion of the structure itself. Active control devices require external energy for their operation. Extensive optimal control algorithms are available (Refs. 7 and 8). Recent studies have recognized the importance of combining structural optimization with control (Ref. 9). The advantages of the approach are quite obvious that it can have all the strong points of both structural optimization and optimal control.

Structural Optimization and Optimum Control Algorithm Structural optimization algorithms may be classified into two major areas of mathematical programming and optimality criteria (Refs. 10,11,12), and can be expressed as

Minimize: 
$$C_{\mathfrak{p}}(x)$$
 (1)

Subject to: 
$$Y_j(x) \leq d_j$$
;  $j = 1,2,...,m$  (2)

and 
$$x^{(k)} \leq x \leq x^{(u)}$$
 (3)

where  $\mathbf{C}_{\mathtt{T}}$  is the objective function of structural weight or structural cost including the costs of materials, construction, maintenance, damages, etc.,  $\mathbf{Y}_{\mathtt{j}}$  consist of m constraint functions for allowable stresses, displacements, buckling capacity, natural frequencies, etc.; and x represent design variables

which can be cross-sectional areas, moments of inertia, and thicknesses of the constituent members of a structural system. Equations (1) and (2) can be formulated for deterministic or nondeterministic resistance and response (Refs. 12 and 13). For the optimum control, the equation of motion of the N-story shear building equipped with a number of active tendons and subjected to an earthquake acceleration record,  $\ddot{\mathbf{x}}_{\mathbf{G}}(\mathbf{t})$ , is

$$[M]{\ddot{x}(t)} + [C]{\dot{x}(t)} + [K]{x(t)} = [\gamma]{u(t)} + {\delta} \ddot{X}_{\sigma}(t)$$
 (4)

where [M] = mass matrix, [C] = damping matrix, [K] = stiffness matrix,  $\{x(t)\}$  = story relative displacements,  $\{u(t)\}$  = control forces,  $[\gamma]$  = location for AT, and  $\{\delta\}$  = excitation influence vector. Equation (4) can be expressed as

$${\dot{z}(t)} = [A]{z(t)} + [B]{u(t)} + {C} \ddot{x}_{g}(t)$$
 (5)

where  $\{z(t)\} = \{\frac{\{x(t)\}}{\{\dot{x}(t)\}}\}\$ , a 2Nx1 state-vector, [A] = plant matrix, [B] = location matrix, and  $\{C\}$  = excitation vector. The optimal control  $\{u^*(t)\}$ , is derived by minimizing an instantaneous time-dependent performance index  $J_p(t)$  defined as

$$J_{p}(t) = \{z(t)\}^{T}[Q]\{z(t)\} + \{u(t)\}^{T}[R]\{u(t)\}$$
 (6)

where [Q] = positive semidefinite weighting matrix, and [R] = positive definite weighting matrix and satisfying the state-equation, Eq. (5). The performance index  $J_p(t)$  is minimized at every time instant t, for all t in the interval  $0 \le t \ge t_f$ , where  $t_f$  is the earthquake duration.

## CONSIDERATIONS IN UMR WORK

Deterministic 2-D Structures A computer program designated as ODSEWS-2D-II (Optimum Design of 2-Dimensional Steel Structures for Static, Earthquake, and Wind Forces-Version II) was developed for the purpose of analyzing and designing two-dimensional structures. The formulation is based on the displacement method and the consistent mass model, and includes second-order P-A forces. The structural systems to which it can be applied are trusses, and unbraced and braced frames. The seismic excitations can be one-dimensional or two-dimensional; one-dimension is horizontal, two-dimensions is horizontal coupled with vertical. The dynamic forces may be 1) seismic excitations at the base, 2) dynamic forces applied at the structural nodes, and 3) wind forces acting on the structural surfaces. The seismic excitations include 1) the records of actual earthquakes, 2) response spectra of Newmark, Seed, and Housner, and those available in the Chinese Seismic Building Code and ATC-3-06, 3) the Uniform Building Code, 4) the Chinese Seismic Building Code, and 5) the

ATC-3-06 provisions including the equivalent lateral forces with or without soil-structure interaction and the modal analysis with or without soil-structure interaction. The constituent members of a system are made of either built-up sections or hot-rolled wide flange sections. The constraints considered are stresses, displacements, story drifts, natural frequencies, maximum differences between relative stiffnesses, and upper and lower bounds of cross sections. The objective is to obtain the minimum weight or minimum cost of a structural system. The minimum cost includes 1) basic steel and extra size prices, 2) painting cost, 3) connection and welding steel, and 4) damages.

Deterministic 3-D Structures In this development (Ref. 4), the structural elements are steel and reinforced concrete members. The steel elements are beams, beam-columns, and braces; and the reinforced-concrete elements are the beam-columns and the flexural panels. The structural model was developed with computational efficiency as its goal. Each structure uses a rigid (in plane) slab system in order to represent the planar response with three degrees of freedom. The slab is assumed to be flexible in the out of plane directions in order to allow vertical deflections at each structural node. A reduced stiffness matrix is found by condensing the rotational degrees of freedom at each structural node. Therefore, a structure can be represented with three degrees of freedom in the plane of each floor and a vertical degree of freedom at each structural node. This model provides a means of studying three-dimensional structures subjected to a variety of loadings including multi-component ground motions.

Dynamic input includes response spectra and code provisions. The response spectra were developed for multi-component excitations. The computer algorithm allows the use of three different response spectra for each seismic analysis which allows both translational degrees of freedom and the vertical degrees of freedom to be excited through the use of their own response spectra. The ATC-03 provisions provide two approaches for seismic analysis, the equivalent lateral force technique and the modal analysis approach. The structural optimization is based on these forces and procedures recommended in the provisions. A computer program called ODRESB-3D was developed for the work.

Nondeterministic Structures The seismic loadings used in this study (Ref. 13) include UBC, NNSRS, and stationary process. Three types of practical and commonly used loading models are employed in this study. The first type is the UBC codified seismic load. The second is the Newmark's nondeterministic seismic response spectrum (NNSRS) including the statistical response results of actual horizontal or vertical earthquake records. The third is a Gaussian random process with a constant or varied seismic spectrum which has been commonly used to represent the seismic random load.

The parameter study in the reliability-based optimum design includes the parameter study for UBC by investigating the sensitivity of some parameters and comparisons of formulations. The parameters studied are the coefficient of variation of column resistance parameters and coefficient of variation for UBC. The formulation comparisons are the probability distributions of response and resistance, the variance approaches, and the zone coefficients in UBC. The parameters and formulations are also studied for NNSRS such as variations of column resistance, peak ground acceleration, and different variance approaches. For the stationary seismic loads, the formulations for various stationary seismic spectra and failure probability expressions are compared in the optimal solutions. Further parameter studies are the influences of nonstructural and expected failure cost on the optimum design results. In the past, four live load models of ANSI (American National Standard Institute), NBS (National Bureau of Standards), UK (United Kingdom), and UNREDUCED (actual) models were proposed. However, no comparison has been performed to show if there is any

difference among these models. In this study the comparison of four live models is investigated.

The cost objective function may have three components: initial construction cost ( $C_{\underline{I}}$ ), future failure cost ( $L_{\underline{f}}$ ), and system probability of failure ( $P_{\underline{f},\underline{m}}$ ). They are expressed as

$$C_{\mathbf{T}} = C_{\mathbf{I}} + L_{\mathbf{f}} P_{\mathbf{f} \mathbf{T}} \tag{7}$$

in which  $C_{\overline{1}} = C_{ui} \Sigma L_i A_i + C_n$ ,  $C_u$  = an unit steel volume cost,  $C_n$  = nonstructural members cost,  $L_f = C_V C_{\overline{1}} + C_L$ ,  $C_V$  = coefficient to describe the ratio of repair cost to initial cost,  $C_L$  = the business and human losses, and  $P_{fT}$  = system probability of failure. Reliability is based on two mathematical models of normal and lognormal distribution with two different variance approaches. The constraints include the reliability considerations for displacement and internal forces of individual members as well as a system.

Structural Optimization Combined With Optimum Control The study (Ref. 9) deals with the optimal design of building structures equipped with active control systems. The control systems considered are the active mass damper, the active tendon system, and a combination of the two systems. The work included the Ricatti closed-loop algorithm based on classical control theory, non-optimal closed-loop control in the frequency-domain, and instantaneous open-loop, closed-loop, and open-closed-loop algorithms in the time-domain. Among all the algorithms mentioned above, the time-domain algorithms have been extensively studied for the combined effect of structural optimization with optimal control. Also included in the work are a critical-mode control algorithm and the resulting spillover effect on the uncontrolled modes, the optimal location of controllers in conjunction with the critical-mode control algorithm, and the time-delay in the application of the control forces.

The structural optimization is formulated as a constrained minimization problem for which the design variables are the floor stiffnesses of the building and certain control parameters. The objective function is the structural weight of the building. The constraints include floor drifts, floor displacements, control forces, and natural frequencies. The critical-mode control algorithm is developed in order to reduce the amount of computation time which is important in the structural optimization scheme. The critical-mode control algorithm is also used to determine the optimal location of a limited number of controllers. Two methods are investigated; the first is based on the modal shapes and the second upon the minimization of the control energy and response performance indices.

## SAMPLE RESULTS

<u>Structural Optimization</u> A five story L-shaped structure, shown in Fig. 1 was designed for frequency (period) constraints. The constraints consist of keeping the first period between the values of 0.75 and 1.0 sec., the second period below 0.50 sec. and the third period 0.40 sec. All three of the period constraints became active with  $T_1 = 1.02 \ \text{sec.}$ ,  $T_2 = 0.50 \ \text{sec.}$ , and  $T_3 = 0.41 \ \text{sec.}$ , as shown in Fig. 2. The ability to maintain certain frequencies or periods is in order that that structures can be controlled into specific regions of the response spectra.

<u>Combining Structural Optimization and Optimum Control</u> The application of structural optimization to an eight-story one-bay shear building using the instantaneous closed-loop algorithm is demonstrated herein. The displacement

constraints of Case 1 are 0.0183 m, 0.0366 m, 0.0549 m, 0.0732 m, 0.0914 m, 0.1097 m, 0.1280 m, 0.1463 m, for 1st through 8th floor, respectively. The maximum control force is 1334.5 kN for all the floors. For Case 2 the displacement constraints are reduced to 70%; the other constraints are the same. At the optimum, the value of the objective function is equal to 187.36 kN and 517.50 kN, respectively. The stiffness distribution is shown in Fig. 3. It can be seen that optimization is not mainly to reduce the structural cost but to achieve optimal structural strength through rational stiffness distribution based on a given set of constraints. An application of control energy optimization using the optimal structure of Case 1 is shown in Fig. 4, in which the control forces of the beginning cycle are the results of Case 1. At the optimum (Iteration 4) the control forces and control energy have been significantly reduced. The maxima displacements of course are still bound by the constraints used in that case.

#### CONCLUSIONS

A number of interesting results have been obtained from which some general remarks are drawn as follows: 1) The combined structural optimization with optimal control algorithm can effectively determine optimal control forces and optimal control devices and locations; using optimal control algorithm alone cannot yield global optimum results. And 2) Using structural optimization alone can be treated as passive control algorithm which can yield desired magnitude in the response spectrum and desired vibration mode shapes so that the design force levels can be reduced and the unfavorable mode shapes can be avoided.

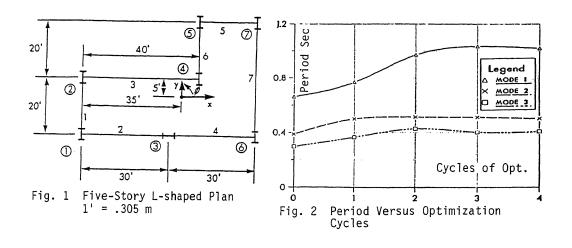
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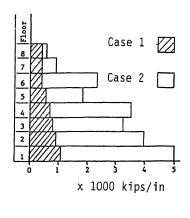


Fig. 3 Optimal Stiffnesses 1 kip/in = 175.13 kN/m

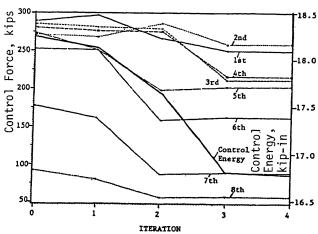


Fig. 4 Maxima Control Forces and Control Energy