ACCELERATED LIQUID MASS DAMPERS AS TOOLS OF STRUCTURAL VIBRATION CONTROL

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SUMMARY

The concept is presented on a new form of mass damper for controlling structural vibration, which utilizes fluid flow in small tube rapidly oscillating in accordance with deformation of the structure. The resisting force of the liquid is formulated on the basis of laminar flow theory and theoretical models of vibrating structure with the damper are proposed. The property and the effect of three different types of the damper are discussed. The result of shaking table tests shows that the damper becomes powerful means for suppressing the resonant resonance of structures to earthquakes.

INTRODUCTION

Dynamic absorbers or tuned mass dampers are widely used to suppress the resonant resonance in many structures under various sources of vibration. In tall buildings, auxiliary solid mass dampers have been successfully used to control wind-induced vibration. Also, devices using sloshing of water in tanks are being developed in the same purpose. However, the applicability of the mass dampers for the purpose of earthquake response control is still an open question because of random nature of the vibration.

The concept of the accelerated liquid mass damper was originally presented by the author in 1973 (Ref. 1). It utilizes high inertial resistance of liquid rapidly oscillating in a small tube in accordance with structural vibration. The effective auxiliary mass of the liquid is so large as it can remarkably modify the natural frequency of the structure. Three different forms of its application have been developed. It is shown that, by selecting appropriate type of the damper, the effective control of structural response is possible not only against the vibration by external force of machines, wind, etc., but also against the random ground motion of earthquakes.

Fundamental Form of Accelerated Liquid Mass Damper

The damper unit of fundamental form consists of a pair of cylindrical vessels connected to each other by a conduit tube of small diameter with the cavity filled with any liquid (Fig. 1). The vessels are axially extendible, being made by piston and cylinder or flexible bellows.

When both the vessel heads are subjected to cyclic movement of alternating extension and contraction, oscillating flow is induced in the tube. The liquid is accelerated over the moving speed of the vessel heads by the factor of sectional area of the cylinder divided by that of the tube, leading to large inertial and viscous resistance. Further, the differential pressure at both the
ends of the tube, which forces the liquid to flow, is transmitted in the same intensity to large area of the vessel heads, thus producing large reactions, $R$ and $R'$. Fig. 2 shows an example of the ways in which the damper is provided in a spring-mass system.

Oscillatory Flow of Liquid in Tube and Resistance of Damper Unit

In the following, a formula representing the resisting force of sinusoidally oscillating damper unit is presented, which was resulted from the oscillatory laminar flow theory of viscous fluid in circular tube. For the applicability of the laminar flow solution and the criteria of transition to turbulent flow, refer to Ref. 2.

Under the assumption of laminar flow, the Navier-Stoke's equation for the viscous fluid flow shown in Fig. 3 can be written as follows:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right)$$  (1)

where $u$ : velocity, $\rho$ : density, $\nu$ : kinematic viscosity, $p$ : pressure.

Assuming sinusoidal variation of the pressure gradient as

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = \kappa e^{i\omega t}$$  (2)

Eq.(1) becomes a Bessel's equation. Its solution was originally found by T. Sexl in 1930 and detailed structure of the flow was later reviewed by S. Uchida (Ref. 3). The expression of fluid velocity represented by modified Bessel function in Uchida's solution can be simplified by using the approximation of

$$ber z \simeq \frac{1}{\sqrt{2\pi z}} e^{\kappa \frac{z}{\sqrt{2}}} \cos \left( \frac{z}{\sqrt{2}} - \frac{\pi}{8} \right)$$, etc. for $z \geq 6$  (3)

Based on the relations between the pressure gradient, mean velocity and frictional force at the tube wall surface in the simplified form, we can represent the difference of resultant pressure at the ends of the conduit tube, $\Delta P$ in Fig. 4, in terms of the mean velocity $u_m$. Then, if we consider the ratio of sectional areas of the cylinders and the tube, the difference of the resisting forces, $\Delta R$, is represented in the terms of the velocity and acceleration of the pistons as

$$\Delta R = \left( 1 + \frac{1}{\lambda} \right) \beta^2 m_k \ddot{x} + \left\{ \lambda w / \left( \lambda - 1 \right)^2 \right\} \beta^2 m_k \ddot{x}$$  (4)

where $x$ : displacement of pistons, $m_k$ : mass of liquid within the tube,
\[ \beta = A / a \quad A : \text{sectional area of cylinder} \quad a : \text{sectional area of tube} \]
\[ \lambda = \omega / 2 \nu \quad \nu \cdot R \quad : \text{Stokes parameter, approximation of Eq.(3) being valid for } \lambda \geq 4 \]

**Accelerated Liquid Mass Damper of Direct Form**

Fig. 5 Frame with Direct Damper    Fig. 6 Damped System    Fig. 7 Ground Motion

In this form, the damper is directly incorporated in a structure. Fig. 2 is an example of the direct damper having external supports and Fig. 5 shows another example having internal supports.

Let a spring-mass system with constants of \( k \) and \( m \) be provided with a direct damper. The equation of motion for the system subjected to external force of \( f(t) = fe^{i\omega t} \) is written as

\[ \begin{align*}
(m + M_{eq}) \ddot{x} + C_{eq} \gamma x + \beta x &= \frac{\gamma F}{m} e^{i\omega t} \\
(\text{Eq.(5)}) \quad M_{eq} &= (1 + \frac{1}{\lambda}) \beta^2 m_L \\
(\text{Eq.(6)}) \quad C_{eq} &= \left\{ \frac{\lambda \omega}{(\lambda - 1)^2} \right\} \beta^2 m_L
\end{align*} \]

In Eq.(5) \( M_{eq} \) and \( C_{eq} \) represent the inertial and the viscous resistances of the damper and the damping term of the structure itself was neglected. From Eq.(4) it is obvious that

\[ \begin{align*}
M_{eq} &= \left( 1 + \frac{1}{\lambda} \right) \beta^2 m_L \\
C_{eq} &= \left\{ \frac{\lambda \omega}{(\lambda - 1)^2} \right\} \beta^2 m_L
\end{align*} \]

For easy understanding of the characteristics, the system can be represented by a model shown in Fig. 6, in which the resistances of the damper are modeled by a virtual solid mass and a dashpot. Eq.(5) is reduced to

\[ \ddot{x} + 2h\omega \omega_0 \dot{x} + \omega_0^2 k x = \frac{\gamma F}{m} \]

where

\[ \begin{align*}
\omega_0^2 &= k / \left\{ (1/\alpha + 1 + 1/\lambda) \beta^2 m \right\} \\
h &= \lambda \zeta / \left\{ (2(\lambda - 1)^2 (1/\alpha + 1 + 1/\lambda) \right\} \\
\zeta &= \omega / \omega_0 \\
\alpha &= \beta^2 m_L / m \\
\Lambda &= m / (m + M_{eq}) = 1 / (1 + (1 + 1/\lambda) \alpha)
\end{align*} \]

Eq.(8) indicates that the natural frequency is largely lowered from that of the original structure by the inertia of the damper. It must be also noted that \( M_{eq} \) and \( C_{eq} \), and therefore \( \omega \) and \( h \) are the functions of the forcing frequency \( \omega \). The responding displacement and acceleration are obtained as follows:

\[ \begin{align*}
X &= \left\{ (1 - \zeta^2)^2 + (2h\zeta^2) \right\}^{1/2} \delta_x e^{i(\omega t - \phi)} \\
\dot{X} &= -(\gamma F / m) \left\{ (1 - \zeta^2)^2 + (2h\zeta^2) \right\}^{1/2} e^{i(\omega t - \phi)} \\
\Theta &= \tan^{-1}(2h\zeta / (1 - \zeta^2)) \\
\delta_x &= \frac{\gamma F}{k}
\end{align*} \]

For the damped system subjected to harmonic ground motion of \( \ddot{y} = a e^{i\omega t} \) (Fig. 7), the equation of motion becomes

\[ \begin{align*}
(m + M_{eq}) \ddot{x} + C_{eq} \dot{x} + k x &= -m a e^{i\omega t} \\
(\text{Eq.(15)}) \quad (m + M_{eq}) \ddot{x} + C_{eq} \dot{x} + k x &= -m a e^{i\omega t}
\end{align*} \]

In Eq.(15), active inertia force exerted by the liquid in the tube was neglected because its real mass is very small compared to the mass of structure, i.e. the virtual effective mass of the damper is of passive nature. Eq.(15) is reduced to

\[ \ddot{x} + 2h\omega \omega_0 \dot{x} + \omega_0^2 k x = -\gamma a e^{i\omega t} \]

\[ \ddot{x} + 2h\omega \omega_0 \dot{x} + \omega_0^2 k x = -\gamma a e^{i\omega t} \]

\[ \ddot{x} + 2h\omega \omega_0 \dot{x} + \omega_0^2 k x = -\gamma a e^{i\omega t} \]

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Solving Eq. (16) we get the same factor of relative displacement to \( \delta_s = ma/k \) as Eq. (12). The responding acceleration is given by

\[
\ddot{x} + \ddot{y} = \left[ \left( 1 - \varepsilon \right)^2 + \left( 2\varepsilon \right)^2 \right] \left[ \left( 1 - \varepsilon \right)^2 + \left( 2\varepsilon \right)^2 \right]^{1/2} a e^{(\omega t + \phi)} \tag{17}
\]

where \( \phi = \tan^{-1}\left[ \frac{2\gamma h\varepsilon^3}{\left( 1 - \varepsilon^2 \right)\left( 1 + \varepsilon \right)^2 + \left( 2h\varepsilon \right)^2} \right] \tag{18} \)

When \( h \gg 1 \), the amplification factor of acceleration at the resonant point becomes

\[
\left( \ddot{x} + \ddot{y} \right) / a = \gamma / 2h \tag{19}
\]

This means that the inertial resistance of the damper reduces the resonant acceleration by the factor of \( \gamma = m/(m + M_{eq}) \). Actually, the viscous damping of the damper further diminishes the resonant amplitude.

**EXAMPLE (1)** A model frame with and without the direct damper was subjected to harmonic excitation on a shaking table. The frame and the damper had similar form to the one in Fig. 5 and their parameters are listed below. Figs. 8 and 9 show the examples of comparison of theoretical and experimental resonance curves.

The fundamental character of the observed response is in agreement with the theoretical prediction, though the peak amplitudes are far smaller than the theory. The difference may be attributed to the turbulence in the fluid flow, which is dominant in the range of large amplitude and high frequency.

From the above observation, the direct damper is considered to be very effective against the excitation of external forces by machines or wind, because it suppresses relative displacement and relative acceleration to the minimum for the whole range of frequency.

However, it is not always the best means of controlling the earthquake response as the effect of isolating the structures from the high frequency excitation is lost because of its high damping.

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**FRAME**

- Width and Height: 1m x 1m
- Spring Constant: 47.6 kg/cm
- Mass: 41.8 kg
- Natural Freq.: 5.3 Hz
- Damping Ratio: 0.0025

**DAMPER**

- Radius of Cylinder: 2.2 cm
- Length of Tube: 1.0 m
- Radius of Tube Used Liquid
  - Case (1) 0.6 cm 60% Glycerin \( \nu = 0.0767 \) cm²/sec
  - Case (2) 0.3 cm 40% Glycerin \( \nu = 0.0295 \) cm²/sec

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**Fig. 8 Relative Displacement**

- 120, 60% Glycerin
  - (Max: 17.4 at 4.2 Hz)
- 60% 40% Glycerin
  - (Max: 5.7 at 2.9 Hz)

**Fig. 9 Acceleration**

- 120, 60% Glycerin
  - (Max: 10.7 at 4.2 Hz)
- 60% 40% Glycerin
  - (Max: 1.9 at 2.9 Hz)
Accelerated Liquid Mass Damper of Gapped Form

This form was developed for preventing structures from so called selective resonance caused by random ground motion of earthquakes, whose dominant frequency range is generally unpredictable. In its gapped form, the damper unit is installed in structures providing with small gapped space in the connection point. While the structure is vibrating within the gapped space, the damper exerts no resisting force. Once the amplitude grows beyond the width of the gap, the virtual effective mass of the damper exerts large inertial resistance, thus providing the structure with amplitude-dependent nonlinearity with regard to its natural frequency. For detailed discussion and experimental verification, refer to Ref. 4 and also a separate presentation in the conference, P3A-04 (Ref. 5).

Accelerated Liquid Mass Damper of Open Circulation

Fig.10 Open Circulation Damper

This type of the damper was devised to avoid the occurrence of higher order resonance as in the direct damper and high frequency response from collision of masses as in the gapped damper. As shown in Fig.10, the pistons are provided with a stop valve and a pair of outlet tubes extending from the cylinders are led to the central reservoir having open surface of liquid. By the action of the valve, the liquid is squeezed out through the tube only when the pressure in the cylinder is positive, the flow being one-directional. Consequently, the virtual effective mass of the liquid is attached to the structure at the point of the maximum displacement without impulse and acts through the accelerating phase of vibration. In the following decelerating phase, it is separated away from the system as illustrated in Fig.11. In this form of the damper, provided virtual mass does act to resist acceleration, but does not act to deflect the spring. Therefore, it does not give rise to an additional degree of freedom leading to the higher mode resonance.

The open circulation damper suppresses resonant peak of vibration efficiently by the multiple effects of
1) occurrence of alternating phases of longer and shorter natural periods
2) damping by the release of fluid's kinetic energy out of the system
3) damping by viscous resistance of the fluid

Further, it does not amplify the high frequency components of input excitation by the above reason. Thus, the open circulation damper makes the response of structures minimum both to stationary and random excitations by external forces of machines or wind, and by the earthquakes as well.

EXAMPLE (2) On a shaking table, a model frame with and without the open circulation damper of parameters listed below was subjected to earthquake waves, El Centro NS 1940 and Tohoku Univ. NS 1978, both in real time, 1/2 time axis and 1/4 time axis. The resonant response was well suppressed for all the cases, the peak acceleration being almost the same to or smaller than that of the table. Fig.12 shows the examples of recorded time histories.

<table>
<thead>
<tr>
<th>FRAME</th>
<th>Width and Height : 1m x 1m</th>
<th>DAMPER</th>
<th>Radius of Cylinder : 2.2 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>32.2 kg</td>
<td>Radius of Tube : 0.3 cm</td>
<td></td>
</tr>
<tr>
<td>Spring Constant</td>
<td>6.7 kg/cm</td>
<td>Length of Tube : 25.0 cm</td>
<td></td>
</tr>
<tr>
<td>Natural Freq.</td>
<td>2.2 Hz</td>
<td>Used Liquid : water</td>
<td></td>
</tr>
<tr>
<td>Damping Ratio</td>
<td>2.5 %</td>
<td>Effective Mass : $\approx$ 20.7 kg</td>
<td></td>
</tr>
</tbody>
</table>
CONCLUSION

The accelerated liquid mass dampers provide new tools of controlling response of structures to excitation from various sources of vibration. It was shown that the direct damper is effective to excitation by external forces of machines or wind and the open circulation damper exhibits the most powerful effect to suppress the resonating response to earthquakes as well.

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REFERENCES