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SYNTHETIC MODELING OF STRONG GROUND MOTION ON A CLASS OF ALLUVIAL VALLEYS

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SUMMARY

An approximate method is applied to obtain the seismic response of a class of alluvial valleys under incident SH waves. It is based on the existence of a complete family of rays supported by the basin boundaries. These rays are grouped in bands where one-dimensional wave propagation occurs. The solution is obtained by superposing folded bands and neglecting diffraction. Synthetic accelerograms are presented for several stations on the surface of a model that roughly represents an E-W section of the Valley of Mexico. Hybrid modeling is also considered to include the one-dimensional response of soft superficial layering.

INTRODUCTION

It is well known that geologic site conditions can cause local anomalies of strong ground motion (Ref. 1). These local effects were recently observed, for instance, during the September 19, 1985 Michoacán, México earthquake which caused unprecedented destruction in Mexico City (Ref. 2). The damaged areas of the city suffered the combination of site effects in the basin, and an anomalous flux of energy from a distant source. Indeed, the basin characteristics played a determinant role in the destruction pattern by causing observed spectral amplifications of 10 to 50 times as compared with nearby firm ground observations in the frequency range between 0.2 to 1.0 Hz (Ref. 3).

For layered soils, without lateral irregularities, it suffices to use one-dimensional models (Ref. 4) to account for site effects. However, lateral irregularities can produce significant variations in predicted seismic response for horizontally layered soils. This is mainly due to wave focusing and generation of local surface waves. The understanding of basic wave phenomena involved makes allowance for quantitative analyses of the seismic mobility of a site, assuming certain knowledge of the geological and topographical structures.

Various methods have been used to compute the strong ground motion response of local geological structures. Let us mention the finite-difference method (Ref. 5) the discrete wavenumber method (Refs. 1,6), and the boundary integral method (Ref. 7), among others. The ray method has been used by several investigators. For instance, it is worth to mention Moczo *et al.* (Ref. 8) who utilized a frequency-domain approach in their computations. The Gaussian beam method has also been used and compared with other procedures for this purpose (Ref. 9). Ray methods have become a useful tool in seismic response analysis

due to their speed and accuracy, specially for higher frequencies, as well as their ability to handle rather complicated media.

In this paper, a different ray theory approach is used. The method was presented by Sánchez-Sesma et al. (Ref. 10) based on previous work on the seismic response of a dipping layer (Ref. 11). They described a complete family of rays for a class of basin geometries allowing a simple description of seismic site response. These rays are grouped on bands where one-dimensional wave propagation occurs, and the solution is obtained by superposing folded bands and neglecting diffraction. The proposal is an origami (the art of Japanese paper folding) solution. Its simplicity allows to consider approximately the effect of deformable basin boundaries for arbitrary angles of incident SH waves. A synopsis of the formulation and synthetic accelerograms obtained with a very low computational effort are presented for an alluvial valley formed by opposite dipping layers. Hybrid modeling is also considered to include the response of soft superficial layering through a combination with one-dimensional techniques.

THE PROBLEM

A dipping layer The SH antiplane response of an elastic dipping layer overlying a moving rigid base (Fig. 1) has been recently presented (Ref. 11). It was found that for dip angles of the form $\pi/2N$, where $N = 1, 3, 5, \dots$, the surface displacement for an harmonic motion of the base $v_0 \exp(i\omega t)$, where $i = \sqrt{-1}$, $\omega =$ circular frequency and $t =$ time, is given by

$$\frac{v}{v_0} = \sum_{j=0}^M \epsilon_{M-j} (-1)^j \exp(-ikx \cos \theta_j), \quad (1)$$

where $M = (N-1)/2$, $\epsilon_m =$ Neumann factor ($= 1$ if $m = 0$; $= 2$ if $m \geq 1$), $k = \omega/\beta =$ SH wavenumber, $\beta =$ SH wave velocity, $x =$ abscissa, and $\theta_j = (N-2j-1)\pi/2N$. Here and hereafter the time factor $\exp(i\omega t)$ is omitted. For the case of transient motion of the base, the factor $f(t - x \cos \theta_j/\beta)$, where $f(\cdot)$ is an arbitrary time function, replaces the exponential part in Eq. (1).

Because of the self-similarity of the wedge (i.e., there is no characteristic length), the solution consists of plane waves simultaneously departing from the origin with angles $\pm \theta_j$. For this choice of dip angles, the exact solution can be obtained only by geometrical means (i.e., diffraction does not exist). The Neumann factor stands for the fact that the last two waves are only one because $\theta_M = 0$ (Fig. 1).

A complete family of rays Sánchez-Sesma et al. (Ref. 10) extended the results of the foregoing section to a moving rigid base supporting a symmetrical layer with dip angles of the form $\pi/2N$, where $N = 3, 5, 7, \dots$ (see Fig. 2). For a given value of N , it can be shown that there are $(N+1)/2$ families of ray paths whose origin and end are normal to the basin boundaries. Their lengths are equal to $L_j = 2a \cos \theta_j$, for $j = 0, 1, \dots, M$, where $a =$ semi-width of the basin. A ray path exists for a band width $W_j = L_j \tan \pi/2N$ (Fig. 3). Once the bands are folded, the rays contained in them describe the complete solution when diffraction is neglected from the central vertex and the bands' edges. Hence the origami nature of the method. It must be borne in mind that the edges of the basin do not generate diffraction as the field is continuous there.

Each origami band is, at its extremes, normal to the rigid base boundaries. Therefore, it can be considered as a homogeneous layer with prescribed motion at their ends, and field motion of the form

$$\frac{v}{v_0} = \frac{\cos(\omega S/\beta)}{\cos(\omega a \cos \theta_j/\beta)} \quad (2)$$

which represents a standing wave that satisfies the scalar one-dimensional wave equation and boundary conditions at the band coordinate value $S = \pm L_j/2$. Field motion in the whole domain is readily evaluated taking into account free boundary and rigid base reflections. This can be done by "band folding". However, care must be taken to change the sign of the rigid base reflections, as well as to account for a factor of two for the free surface image reflections.

The total surface displacement field is given by superposition of overlapping folded bands. Then, in order to do this, let us consider a band j with a dip angle θ_ℓ ($\ell \leq j$) at the free surface, $z = 0$, and a band coordinate of the form $S = a \cos \theta_j - (a - |x|) \cos \theta_\ell$. The range of validity of this relationship is given by (Ref. 10)

$$\sum_{K=\ell}^{j-1} \epsilon_{K-\ell} \cos \theta_K \leq \left| \frac{x \sin \theta_\ell}{a \tan \pi/2N} \right| < \sum_{K=\ell}^j \epsilon_{K-\ell} \cos \theta_K. \quad (3)$$

The first member of the inequality must be zero when $\ell = j$. Therefore, the surface ground motion can be written as

$$\frac{v}{v_0} = \sum_{j=0}^M \sum_{\ell=0}^j \epsilon_{M-\ell} (-1)^\ell \frac{\cos(\omega[a \cos \theta_j - (a - |x|) \cos \theta_\ell]/\beta)}{\cos(\omega a \cos \theta_j/\beta)} R_{j\ell}, \quad (4)$$

where $R_{j\ell} = 1$ if the condition of inequality (3) holds; otherwise, $R_{j\ell} = 0$. Equation (4) is the complete solution when diffraction is neglected. However, it has a very restricted application. Its range of application can be extended in an approximate way to consider an elastic boundary and arbitrary incidence angles of incoming plane SH waves. This can be accomplished by considering reflection and transmission coefficients at the basin boundaries.

Finally, assuming that the waves inside of the basin belong to our complete family of rays and are affected by reflection coefficients A_K , the total displacement field in the band j can be obtained considering the phases of all waves and satisfying boundary conditions at points on the left and right bottom boundaries of the basin (Ref. 10). Then,

$$\frac{v}{v_0} = e^{-i\omega a \cos \theta_j/\beta} \left\{ \underset{\rightarrow}{\tilde{a}} \pi e^{-i\omega S/\beta} + \underset{\leftarrow}{\tilde{b}} \pi e^{i\omega S/\beta} \right\}, \quad (5)$$

where

$$\underset{\rightarrow}{\pi} = \begin{cases} \begin{matrix} \ell \\ (\pi A_K) \\ K=0 \end{matrix} & \text{if } S < 0 \\ \begin{matrix} \ell & j \\ (\pi A_K) & (\pi A_K^2) \\ K=0 & K=\ell+1 \end{matrix} & \text{if } S > 0, \end{cases} \quad (6)$$

$$\underset{\leftarrow}{\pi} = \begin{cases} \begin{matrix} \ell & j \\ (\pi A_K) & (\pi A_K^2) \\ K=0 & K=\ell+1 \end{matrix} & \text{if } S < 0 \\ \begin{matrix} \ell \\ (\pi A_K) \\ K=0 \end{matrix} & \text{if } S > 0, \end{cases} \quad (7)$$

and

$$A_K = \frac{\eta - \frac{\sqrt{1 - (\beta_r/\beta)^2 \cos^2(\theta_K + \pi/2N)}}{\sin(\theta_K + \pi/2N)}}{\eta + \frac{\sqrt{1 - (\beta_r/\beta)^2 \cos^2(\theta_K + \pi/2N)}}{\sin(\theta_K + \pi/2N)}}, \quad \text{Im}(\sqrt{\cdot}) \leq 0. \quad (8)$$

Here, \tilde{a} , \tilde{b} = amplitude of the waves emitted from the left - and right-hand side basin boundary, respectively, $\eta = \rho\beta/\rho_r\beta_r$ = impedance ratio, ρ = mass density (subscript r stands for rock), and $A_0 \equiv \tilde{I}$. Superposition of Eq. (5) for overlapping folded bands, as was done before for Eq. (4), grants an approximate calculation of surface response of the alluvial valley under incoming SH waves with arbitrary incidence angle.

NUMERICAL EXAMPLE

The performance of this method has already been illustrated in a previous paper (Ref. 10). To do this, amplitudes of surface displacements were compared with calculations using a more reliable technique which considers diffraction and anelastic attenuation in the valley (Ref. 12). The overall agreement of results was very good, mainly for strong impedance contrast and high frequency.

Calculations in time domain are inexpensive using this method. For instance, we obtain synthetic accelerograms with conventional computer resources (7 secs of CPU time in a Burroughs 7800 computer) for points on the surface of a triangular valley with 6 degrees dipping angles ($N = 15$) and 10 km width. Fig. 4 shows the problem geometry and the material properties for the basement rock (subscript r) and the alluvial material. Anelastic attenuation is considered by multiplying β by the factor $(1+i/2Q)$, where Q = quality factor. This model roughly represents an E-W section of the Valley of Mexico.

Fig. 5 shows synthetic accelerograms for the nine equally-spaced stations without considering the effects of the superficial layer. They clearly display the effects of lateral irregularities; namely, larger amplifications and signal duration increments. However, if we superpose the one-dimensional response of the superficial layer (Ref. 4), through a hybrid procedure in combination with the origami solution, striking results are obtained (Fig. 6) which resemble the observed records in Mexico City during the 1985 Michoacán earthquake. At some stations, the simulated peak ground acceleration is of about 0.25 g which is very close to the maximum value recorded in the city (0.2 g). It is also worth to notice the different kind of behavior due to the increasing thickness of the layer.

CONCLUSIONS

An approximate solution based on ray theory has been applied to study strong ground motion on a class of alluvial valleys. The method is suitable to model the seismic response of this family of basins in time domain given its low cost and virtually no upper frequency limit. Synthetic accelerograms on the surface of an approximate model of an E-W section of the Valley of Mexico show the great importance of two-dimensional response which increases amplitude and duration of input signal. Inclusion of soft superficial layering produces large additional amplifications and indicate that one-dimensional response is of primary importance. However, our results show that the effects of lateral irregularities cannot be disregarded.

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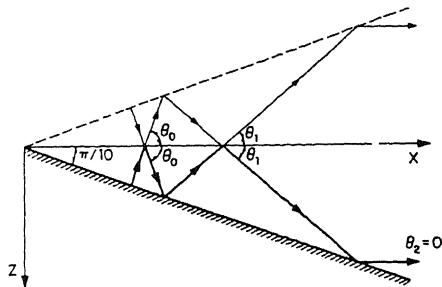


Fig. 1 Dipping layer overlaying a moving rigid base. The rays solution is shown for $N = 5$

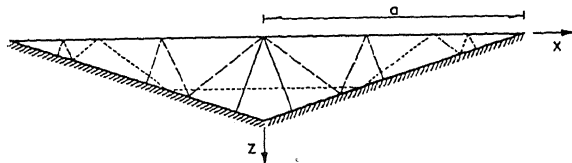


Fig. 2 Alluvial valley formed by opposite dipping layers. Ray paths for $N = 5$

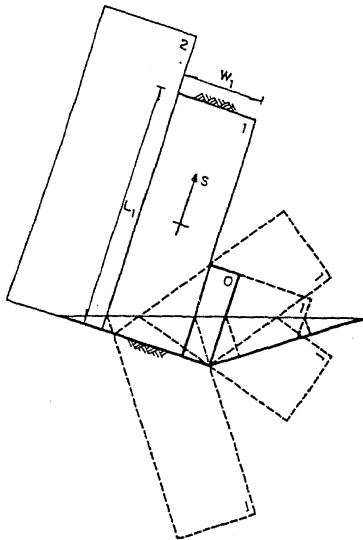


Fig. 3 Origami bands for $N = 5$. Band one foldings are shown with dashed lines. L_j = band length. W_j = band width. S = band coordinate.

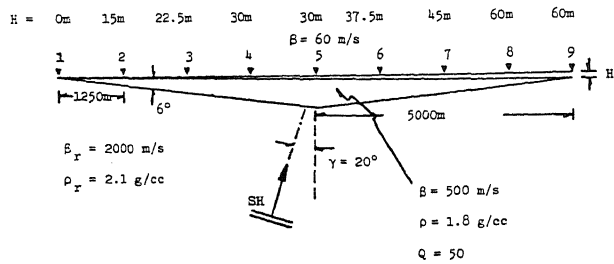


Fig. 4 Alluvial valley with dipping angles of $\pi/2N$, where $N = 15$. Oblique incidence of SH waves. Input for the model is given at station 1 by the N-S horizontal component of the recorded accelerogram at Tacubaya site, a hill zone location in Mexico City, during the September 19, 1985 Michoacán earthquake. A soft superficial layer of variable thickness and uniform low velocity ($\beta = 60$ m/s) has been assumed above the valley.

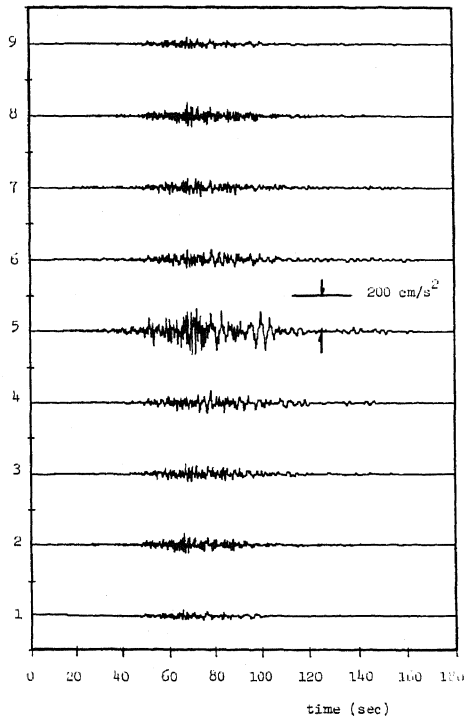


Fig. 5 Synthetic accelerograms for the alluvial valley shown in Fig. 4 without considering the effects of the superficial layer.

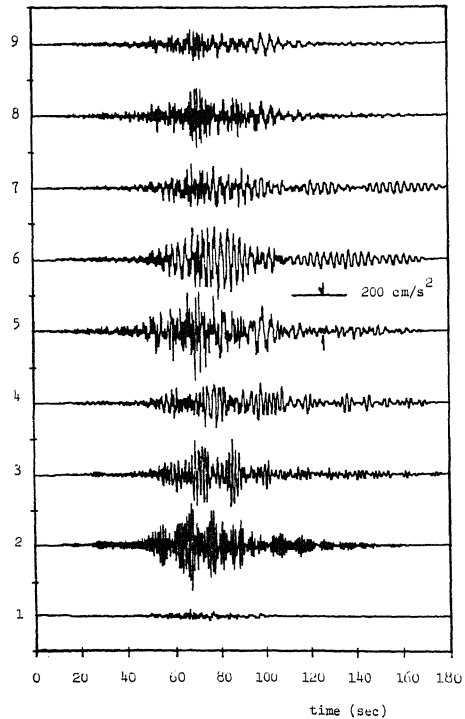


Fig. 6 Synthetic accelerograms for the alluvial valley shown in Fig. 4 considering the effects of the superficial layer.