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FOUNDATION UPLIFT AND RADIATION DAMPING EFFECTS CALCULATED IN THE LAPLACE DOMAIN

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SUMMARY

In this paper a new method is presented, which permits consideration of effects of wave propagation in the soil and local nonlinearities. By using a step-by-step method in the Laplace domain the calculation will be expanded for problems of nonlinear foundation uplift. The nonlinear calculation is performed by a number of linear steps, where each linear step satisfies the superposition condition. The nonlinearity is dealt with in the time domain, while the remaining calculations are performed in the Laplace domain. Thus the results of every step are transformed from the Laplace to the time domain and vice versa.

INTRODUCTION

Soil-structure interaction problems are most commonly solved in the frequency domain, which permits the representation of geometric damping and also the use of the substructure method in dynamic analysis. That means that soil and structure can be treated separately utilizing different methods. In the simplest case the soil could be modelled as a frequency-dependent spring-dashpot system. Because of the superposition principle involved, linearity has to be presumed. In case of strong earthquake motion the foundation may lift up and nonlinear effects will appear. The general linear soil-structure interaction technique is then no longer valid.

Several investigators have analysed foundation uplift. Wolf and Dabre (Ref.1), for instance, solved the uplift problem iteratively in the time domain. Koh, Spanos and Roesset (Ref.2) used an approximate analytical solution. Kawamoto (Ref.3) and Wolf (Ref.4) considered an analysis using a hybrid Frequency-Time Domain which is similar to the method presented here. A brief review of the previous work in this field can be found in references 2 and 5.

The method presented in this paper calculates the dynamic response of systems with local nonlinearities and will be applied here to the case of foundation uplift. Other applications are presented elsewhere (Refs. 6,7). The technique uses the substructure method, which permits the modelling of the soil and the building separately. Therefore the "true" stress distribution at the soil-structure interface can be used. Different damping models can be used invoking the correspondence principle. The model can also be condensed to the most important degrees of freedom to decrease computer time. In contrast to references 3,4 it uses the Laplace transform which is advantageous in some aspects. In contrast to the Fourier transform it allows the incorporation of initial conditions in a simple manner. Systems with only a small amount of damping or without damping at all can also be investigated without difficulty, which is of special interest here. The nonlinearities will be dealt with in the time domain, while all other calculations will take place in the Laplace domain.

CALCULATION OF LOCAL NONLINEARITIES IN THE LAPLACE DOMAIN

Since superposition is involved the integral transform method is based on the assumption of linearity. Therefore, at first sight it seems impossible to analyse nonlinear problems with integral transforms. But we can, as with most time integration methods, formulate the nonlinear behaviour in an incremental way. For every increment the linearity is satisfied. Within the increment the integral transform method can be applied. The time at which the system changes its linear characteristics, has to be determined in the time domain.

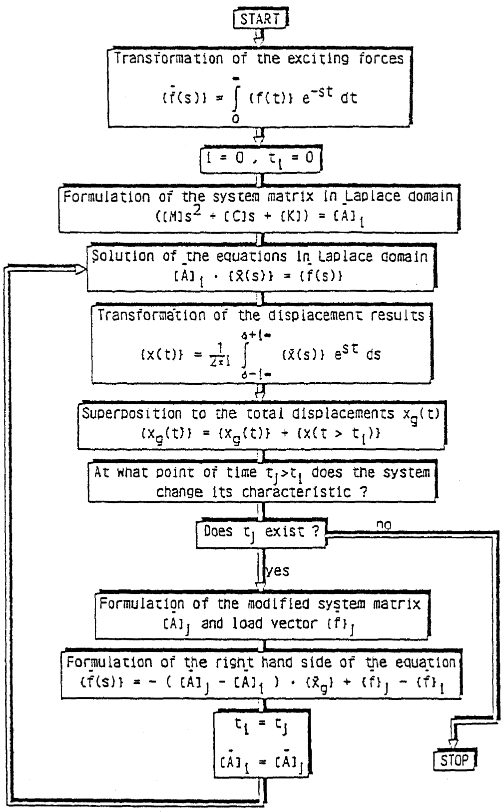

Figure 1: Flow chart of the total-step method

The calculation of nonlinear structures in the Laplace domain will be performed using the total-step method (Ref.10) which has been proved to be advantageous for these problems (Ref.9). The algorithm is presented in a flow chart in figure 1 and can be summarised as follows. The calculation starts with the transformation of the exciting forces and the formulation of the initial system matrix. After having found the solution the displacement results are transformed into the time domain. In the first calculation step the total displacements $x_g(t)$ are identical to the calculated results $x(t)$. After the system change has occurred at time t_j (the following calculation is performed only for times $t>t_j$) the modified system matrix$[\tilde{A}]_j$ and the pseudo-forces $\tilde{f}(s)$ are formulated. The new system will be solved and the results will be added to those of the first step, which completes the second step of the calculation. This iteration is continued until no more changes in the system occur

CONTINUOUS MASS DISTRIBUTION AND MATERIAL DAMPING

Application of the Laplace transform with respect to time enables one to use the exact deformation of a beam element with continuously distributed masses as a shape function in the calculation. This is in contrast to the conventional finite-element method, which is based on the lumped or consistent masses and thus leads to approximate solutions of the problem.

Instead of the separated mass, damping and stiffness matrices ([M], [C], [K]) such a model is described by a frequency-dependent matrix $[\tilde{A}]$ which is valid for linearly elastic material behaviour following Hooke´s law:

$$\sigma = E \cdot \varepsilon \qquad (1)$$

where σ is the stress, ε the strain and E the modulus of elasticity. Transforming eq. (1) into the Laplace domain and replacing the real constant E by a corresponding complex value with the help of the correspondence principle (Ref.8) will result in:

$$\tilde{\sigma} = \tilde{E} \cdot \tilde{\varepsilon} = \frac{q_0 + q_1 \cdot s + q_2 \cdot s^2 + \ldots + q_n \cdot s^n}{1 + p_1 \cdot s + p_2 \cdot s^2 + p_3 \cdot s^3 + \ldots + p_m \cdot s^m} \cdot \tilde{\varepsilon} \tag{2}$$

where $q_o,\ldots,q_n$ and $p_1,\ldots,p_m$ are real constant values, $s=\delta+i\omega$ is the Laplace transform parameter and ($\sim$) indicates a value which belongs to the Laplace domain. In this way equation (2) describes the corresponding viscoelastic behaviour.

In our paper a model is used which is represented by a chain of Kelvin models with linearly increasing spring stiffnesses. It is described by equation (3).

$$\tilde{\sigma} = \varepsilon \cdot E \cdot \left(\frac{B_1 \cdot B_2}{B_1^2 + 4 \cdot B_3^2} + i \cdot \frac{2 \cdot B_2 \cdot B_3}{B_1^2 + 4 \cdot B_3^2} \right) \tag{3}$$

where

$$B_1 = \ln\left\{ \frac{(E_n + \delta)^2 + \omega^2}{(E_1 + \delta)^2 + \omega^2} \right\}, \quad B_2 = \ln\left\{ \frac{(E_n + \delta)^2}{(E_1 + \delta)^2} \right\} \quad B_3 = \arctan\left\{ \frac{(E_n - E_1) \cdot \omega}{(E_n + \delta)(E_1 + \delta) + \omega^2} \right\}$$

More details can be found in reference 9.

EFFECTS OF FOUNDATION UPLIFT

To restrict the following considerations to the most important aspects of the problem only two-dimensional problems and surface foundations are considered here. The substructure technique is used to handle soil and structure separately. While for the structure finite elements are used the soil is modelled with constant boundary elements. Therefore, in order to simplify the problem, the soil is assumed to be homogeneous and linearly elastic.

The dynamic behaviour of the soil can be expressed by equation (4) which describes the relationship between forces and displacements at the soil-structure interface (see figure 2).

$$\left[\tilde{K}^B\right] \left\{\tilde{u}^B\right\} = \left\{\tilde{P}^B\right\} \tag{4}$$

where $\left[\tilde{K}^B\right]$ is the dynamic stiffness matrix of the soil and $\left\{\tilde{u}^B\right\}$, $\left\{\tilde{P}^B\right\}$ are the vectors of the displacements and forces at the soil-structure interface. The corresponding equation for the structure is given by:

$$\left[\tilde{K}^S\right] \left\{\tilde{u}^S\right\} = \left\{\tilde{P}^S\right\} \tag{5}$$

For the coupling of the two substructures "soil" and "structure" it is necessary that the degrees of freedom in both models be the same at the interface (see figure 2). If that is not the case, we have to transform the degrees of freedom of the soil by a transformation matrix [T]:

$$\left\{\tilde{u}^B\right\} = [T] \left\{\tilde{u}^F\right\} \tag{6}$$

where $\left\{\tilde{u}^F\right\}$ is a vector of displacements of the soil equivalent to those in the structure model. By employing the principle of virtual work, which states that the work done by the interaction forces is the same for both displacement vectors, we obtain:

$$[T]^T \left[\tilde{K}^B\right][T] \left\{\tilde{u}^F\right\} = \left[\tilde{K}^F\right] \left\{\tilde{u}^F\right\} = \left\{\tilde{P}^F\right\} \tag{7}$$

The two substructures are described by their dynamic equations (4) and (5). If we divide the degrees of freedom of the structure into those on the soil-structure interface $\left\{\tilde{u}_2^s\right\}$ and the remaining ones $\left\{\tilde{u}_1^s\right\}$, and if we use the compatibility conditions of figure 2 together with the equations (4) and (5) we finally get the dynamic equation of the coupled system:

$$\begin{bmatrix} [\tilde{K}^{s}_{11}] & [\tilde{K}^{s}_{12}] \\ [\tilde{K}^{s}_{21}] & [\tilde{K}^{s}_{22}]+[\tilde{K}^{F}] \end{bmatrix} \begin{Bmatrix} \{\tilde{u}^{s}_{1}\} \\ \{\tilde{u}^{s}_{2}\} \end{Bmatrix} = \begin{Bmatrix} \{\tilde{P}_{1}\} \\ \{\tilde{P}_{2}\} \end{Bmatrix} \tag{8}$$

where $\{\tilde{P}\}$ are the external loads. Equation (8) is only valid for the linear case without taking into account uplift. In the nonlinear case the displacements at the interface are different for the soil and the foundation. Therefore it is necessary to calculate the displacements of the structure and also the displacements of the soil and the interaction forces. As this special calculation method is used, it is only possible to release the contact of one element at a time. The value of the interaction force will determine whether a nonlinear change occurred in the member considered:

a) An element will lose contact if the contact force (at node i) is greater than zero (tension).

b) An element will re-establish contact if the displacement (at node i) of the structure is greater than or equal to the displacement of the soil (at the same node).

These two conditions describe the nonlinear aspect of the problem considered. For the numerical calculation the algorithm presented above is used (flow chart in figure 1). The so-called pseudo-forces describe, in this special case, the difference between the calculated coupling forces and the forces following the true nonlinear behaviour of condition (a) and (b).

NUMERICAL LAPLACE TRANSFORM

Many papers have been published comparing different numerical methods for the Laplace transform (e.g. refs. 11,12). Most of them conclude that methods which work with complex data give the best results. In our paper the method which refers the Laplace transform to the Fourier transform is used:

$$L(f(t)) = \tilde{f}(s) = \int_{t=0}^{\infty} f^{*}(t) \cdot e^{-i\omega t}\, dt \tag{9}$$

where:

$$f^{*}(t) = f(t) \cdot e^{-\delta t} \quad \text{when } t \geq 0$$
$$= 0 \quad \text{when } t < 0$$

For details see reference 13. To reduce the existing numerical errors in the inverse Laplace transformation the time function f(t) is smoothed by multiplying the Fourier coefficients by the factors $\sigma_k = \sin(k \cdot 2\pi/N)/(k \cdot 2\pi/N)$, where $k = 0, \ldots\ldots, N$. For the constant real part, δ, of the Laplace transform parameter, s, a value of about $\delta \approx 8/T$, where T is the total time interval, has been proved to produce the best results (for details see reference 9).

NUMERICAL EXAMPLE

The following numerical example will demonstrate the efficiency of the Laplace-domain method presented. The structure treated is taken from reference 14 and represents the switchgear building of a nuclear power plant (see figure 3). The large length of the building perpendicular to the drawing plane allows the structure to be modelled two-dimensionally. The structure itself is modelled as a beam with continuous mass distribution. The internal damping is represented as a chain of Kelvin models. The soil is assumed to be a viscoelastic halfspace with internal damping (Kelvin chain) and with the characteristic values E, ρ and ν as indicated in figure 3. It is modelled using the boundary-element method with 12 elements over the width of the foundation. Constant-stress, constant-displacement elements are used. For the numerical calculations of the boundary element method the computer program by Willms (Ref.15) has been used. The structure is subjected to earthquake loads in the horizontal and vertical directions. The ground acceleration is identical to the measured Montenegro earthquake of 1979 (measured in Petrovac 15.4.1979, see reference 16). The horizontal values are reduced for this calculation to 40% of the measured values. The peak ground acceleration in the horizontal direction is 1.8 m/sec^2 and in the vertical direction 1.73 m/sec^2. Because of the nonlinear uplift the numerical calculation has to take into account the vertical and the horizontal acceleration simultaneously. For the numerical calculation the following values have been chosen: $s = \delta + i \cdot k \cdot \Delta\omega$, where $k=0,1,.......,128$, $T = 6.40$ sec., $\delta = 8/T = 1.25$, $\Delta\omega = 2\cdot\pi/T = 0.9817$ rad/sec., $\Delta t = 0.025$ sec.. Figure 4 shows the displacement function of the extreme right-hand node of the foundation for the linear and also the nonlinear case. At the top of the building (node 9) the largest horizontal displacement values are 4.145 and -4.198 cm, whereas in the linear case 3.318 and -3.058 cm have been found. The increase for the uplift effect is about 25% - 37% in this case.

CONCLUSIONS

Using the present method the Laplace transformation can be used for the solution of nonlinear problems of foundation uplift. Together with the substructure technique the boundary-element method can be applied to incorporate the effect of radiation damping in the soil. A more realistic representation of a rigid or a flexible base mat can also be treated. For the structure continuous-mass models can be used and the internal damping may be considered with the help of the correspondence principle. Uplift and local nonlinearities in the structure can be considered simultaneously by combining this method with the technique given in reference 7.

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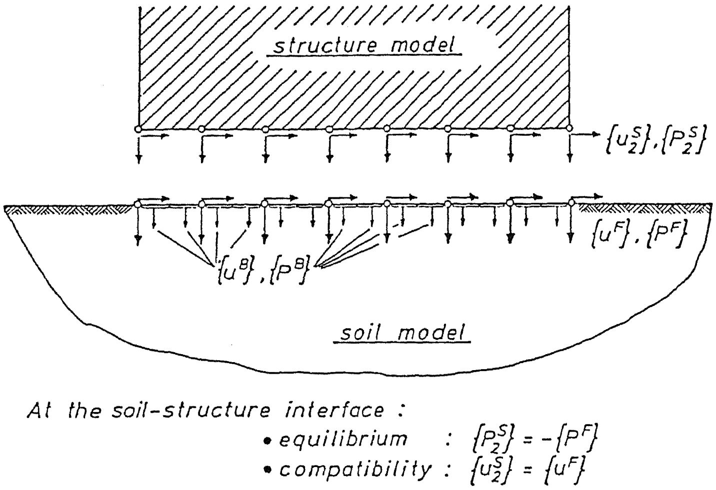

Figure 2: Coupling of the substructures soil and upper structure

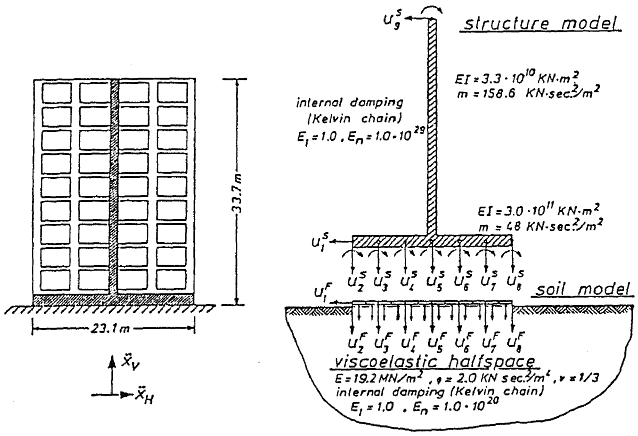

Figure 3: Model of the switchgear building

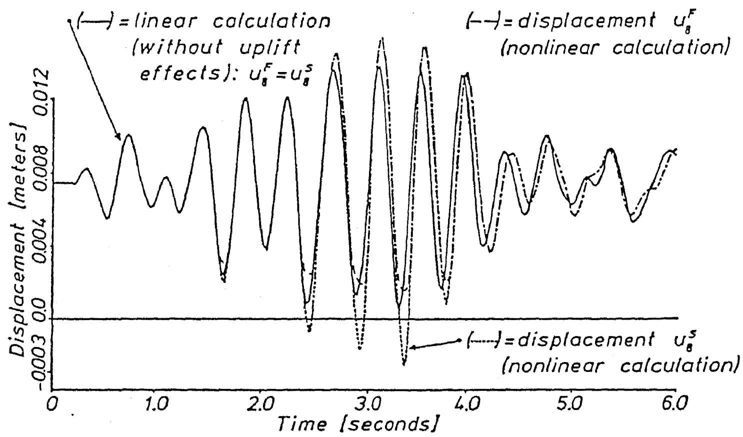

Figure 4: Displacement function at node 8 u^S, u^F